



Using The Technique of Between Satellite Single Difference To Improve the Convergence time of the GNSS PPP Solution Prof. Dr. Essam M. Fwaz⁽¹⁾ & Prof. Dr. Adel Esmat⁽¹⁾ & Dr. Mahmoud Salah⁽²⁾ & Eng .Zahraa Mohmmmed⁽¹⁾

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الملخص

هذا البحث يتضمن شرح طريقة "بين الأقمار الصناعية" وهذه الطريقة تستخدم للحصول على احداثيات نقاط التحكم الارضية المستخدمة في العديد من المشروعات الهندسية و كذا مقارنتها بالطريقة التقليدية وايضا اظهار مزايا استخدام هذه الطريقة عن الطرق التقليدية وينقسم هذا البحث الى جزئين في الجزء الأول منة يتم دمج الارصاد المختلفة الناتجة من الأقمار الصناعية المختلفة "القمر الامريكى - القمر الصينى- القمر الفرنسى" كمرحلة أولى و في المرحلة الثانية يتم تصحيح كافة الاخطاء باستخدام النماذج المختلفة وتطبيق الطريقة الثانية موضوع البحث التى تستخدم فى ازالة تأثير فرق الوقت حتى يمكن التخلص من تأثير طبقه الايونوسفير فنحصل على دقة اعلى

ABSTRACT

This paper will discuss the "Between-Satellite-Single-Difference" technique (BSSD) ionosphere-free linear combination of pseudorange and carrier phase measurements from GNSS constellation namely "GPS, GALILIO and BeiDou". Inter system Biases will be removed from both code and phase. The using of BSSD technique can improve the precision of the latitude, longitude and altitude components, This can be done by comparing these results with the traditional un-differenced technique. The study has shown that there are better positioning precision and it will present an efficient model for precise point positioning (PPP). BSSD can cancel out the receiver hardware delay, receiver clock error. The PPP solution will be improved by using our BSSD-based model by comparing with traditional un-differenced PPP model.

1-Introduction

Precise Point Positioning has been investigated by a number of research groups in the last two decades (e.g, Zumberge et al 1997, Kouba and Héroux 2001, Gao and Chen 2004). The Between Satellite **Single Difference** ionosphere free linear combination of pseudorange and carrier phase measurements from GNSS constellation namely "GPS, GALILIO and BeiDou". Inter system Biases will be removed **from both code and phase**. The using of BSSD technique will improve the precision of the latitude, longitude and altitude components, in comparison with the traditional un-differenced technique. The **result** showed better positioning precision is obtained by using BSSD technique. The receiver related biases from both code and phase GNSS observations can be cancelled out **from both code and phase GNSS observations**. However, In BSSD technique, the satellite differential code biases are still affecting the phase ambiguities due to the dissimilarities of satellites code biases produced from the signals spectrum dissimilarities in the filtering and correlation processes (Phelts 2007). In multi-GNSS observations level, multiple GNSS satellites use the satellite observation references which is referred to loose BSSD combining. Theoretically, un-differenced and BSSD PPP solutions should be statistically equivalent if stochastic errors are modelled

correctly. However, due to the time varying nature of the receiver biases, this technique requires as a minimum two satellites to be available in each GNSS system which sometimes is still not guaranteed especially for Galileo and BeiDou systems. GPS satellite is taken as a reference satellite for the other GNSS satellites observations here in which is called tight BSSD combining. The drawback of using the tight combining is that the receiver DCBs will not be completely removed due to the difference between the receiver DCB of GPS and other GNSS satellites as a result, GNSS PPP model is developed, which combines the observations of GPS, Galileo and BeiDou systems, for precise applications. Both un-differenced and BSSD ionosphere-free linear combinations of pseudorange and carrier phase GNSS measurements are processed using precise clock and orbital products obtained from the multi-GNSS experiment MGEX (Montenbruck et al 2014) . The performance of the developed PPP techniques is assessed using a number of IGS MGEX GNSS stations data. It is shown that the positioning accuracy is improved when the observations of the constellations are combined. In addition, the positioning accuracy of BSSD IF model is more accurate than the traditional un-differenced model.

2-GNSS Observations Equations

The general ionosphere-free equations for pseudorange and carrier-phase can be written as (Hofmann-Wellenhof, B., Lichtenegger, H., and Walse, E. 2008 ,Leick, A. 2004)

$$P_3 = \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2} = \rho + cdt_r - cdt^s + T - c(Ad_{r1} - Bd_{r2}) + c(Ad^{s1} - Bd^{s2}) + e \quad (1)$$

$$\Phi_3 = \frac{f_1^2 \Phi_1 - f_2^2 \Phi_2}{f_1^2 - f_2^2} = \rho + cdt_r - cdt^s + T + c(A\delta_{r1} - B\delta_{r2}) - c(A\delta^{s1} - B\delta^{s2}) + (\overline{\lambda N}) + \varepsilon \quad (2)$$

Where P_1 and P_2 are GNSS pseudorange measurements on L_1 and L_2 , respectively; Φ_1 and Φ_2 are the GNSS carrier phase measurements on L_1 and L_2 , respectively; dt_r and dt^s are the clock errors for receiver and satellite, respectively; d_r and d^s are frequency-dependent code hardware delay for receiver and satellite, respectively; δ_r and δ^s are frequency-dependent carrier phase hardware delay for receiver and satellite, respectively; e, ε are relevant system noise and un-modeled residual errors; and $\overline{\lambda N}$ is the ambiguity term for phase measurements. For the un-differenced ionosphere free linear combination, this term is not integer due to the non-integer nature of the combination coefficients,

$$\overline{\lambda N} = \frac{f_1^2 P_1 N_1 - f_2^2 P_2 N_2}{f_1^2 - f_2^2}$$

where N_1 and N_2 are the L_1 and L_2 non-integer ambiguity parameters, including the initial phase biases at the satellite and the receiver, respectively; λ_1 and λ_2 are the wavelengths of the L_1 and L_2 carrier frequencies, respectively; c is the speed of light in vacuum; T is the tropospheric delay component; ρ is the true geometric range from the antenna phase center of the receiver at reception time to the antenna phase center of the satellite at transmission time. A and B are frequency dependent factors

$$A = \frac{f_1^2}{f_1^2 - f_2^2} \text{ and } B = \frac{f_2^2}{f_1^2 - f_2^2}$$

3-Standard Un-differenced GNSS PPP Technique

Using Equations (1) and (2) and considering GPS time as a reference time system, the un-differenced ionosphere-free linear combinations of GPS, GLONASS, Galileo and BeiDou observations can be written as (Rabbou, M. A., & El-Rabbany, A. 2015, Cai, C., Gao, Y., Pan, L. and Zhu, J. 2015)

$$P_{3G} = \rho_G + c[dt_r + B^r_G] - c[dt^s_G - B^s_G] + T_G + e_G \quad (3)$$

$$P_{3E} = \rho_E + c[dt_r + B^r_G] - c[dt^s_E - B^s_E] + T_E + c[ISB_E] + e_E \quad (4)$$

$$P_{3C} = \rho_C + c[dt_r + B^r_G] - c[dt^s_C - B^s_C] + T_C + c[ISB_C] + e_C \quad (5)$$

$$\Phi_{3G} = \rho_G + c[dt_r + B^r_G] - c[dt^s_G - B^s_G] + T_G + (\overline{\lambda N} + \Delta B^r - \Delta B^s)_G + \varepsilon_G \quad (6)$$

$$\Phi_{3E} = \rho_E + c[dt_r + B^r_G] - c[dt^s_E - B^s_E] + T_E + c[ISB_E] + (\overline{\lambda N} + \Delta B^r - \Delta B^s)_E + \varepsilon_E \quad (7)$$

$$\Phi_{3C} = \rho_C + c[dt_r + B^r_G] - c[dt^s_C - B^s_C] + T_C + c[ISB_C] + (\overline{\lambda N} + \Delta B^r - \Delta B^s)_C + \varepsilon_C \quad (8)$$

where G , E and C refer to GPS, Galileo and BeiDou systems observations, respectively; ISB is the inter-system bias; B^r , B^s are ionosphere-free differential code biases for receiver and satellites, respectively ΔB^r is the difference between receiver differential code and phase biases; ΔB^s is the difference between satellite differential code and phase biases. As can be seen from Equations (7) to (10), the un-calibrated biases such as ΔB^r and ΔB^s are lumped with the ambiguity parameters.

Table 1 shows the mathematical equations for the different GNSS biases.

$B^r_G = [Ad_{r1} - Bd_{r2}]_G; B^s_G = [Ad^{s1} - Bd^{s2}]_G$
$B^r_E = [Ad_{r1} - Bd_{r2}]_E; B^s_E = [Ad^{s1} - Bd^{s2}]_E$
$B^r_C = [Ad_{r1} - Bd_{r2}]_C; B^s_C = [Ad^{s1} - Bd^{s2}]_C$
$\Delta B^r = c(B^r_\Phi - B^r_P); \Delta B^s = c(B^s_\Phi - B^s_P)$
$ISB_E = B^r_E - B^r_G; ISB_R = B^r_R - B^r_G; ISB_C = B^r_C - B^r_G$

4-Between Satellites Single Difference GNSS PPP Technique

To completely remove the receiver related biases from both the code and phase GNSS observations, between-satellite-single-difference (BSSD) ionosphere-free PPP technique can be used for combined GNSS observations model. For each system, a reference satellite is selected while the other GNSS satellites observations are subtracted from it. To develop the mathematical equations of BSSD technique, four GNSS satellites are selected mainly GPS I, Galileo n and BeiDou o, to be reference satellites to the four constellation systems observations. Following (Rabbou, M. A., & El-Rabbany, A. 2015, Li, X., Zhang, X., Ren, X., Fritsche, M., Wickert, J., & Schuh, H. 2015). GNSS-BSSD model can be written as.

$$P_{3G} - P^l_{3G} = \rho_G - \rho^l_G - c[(dt^s_G + B^s_G) - (dt^s_G + B^s_G)^l] + T_G - T^l_G + e_G - e^l_G \quad (9)$$

$$P_{3E} - P_{3E}^m = \rho_E - \rho_E^m - c[(dt_E^s + B_E^s) - (dt_E^s + B_E^s)^m] + T_E - T_E^m + e_E - e_E^m \quad (10)$$

$$P_{3C} - P_C^o = \rho_C - \rho_C^o - c[(dt_C^s + B_C^s) - (dt_C^s + B_C^s)^o] + T_C - T_C^o + e_C - e_C^o \quad (11)$$

$$\Phi_{3G} - \Phi_{3G}^l = \rho_G - \rho_G^l - c[(dt_G^s + B_G^s) - (dt_G^s + B_G^s)^l] + T_G - T_G^l + [(\overline{\lambda N} + \Delta B^s) - (\overline{\lambda N} + \Delta B^s)^l] + \varepsilon_G - \varepsilon_G^l \quad (12)$$

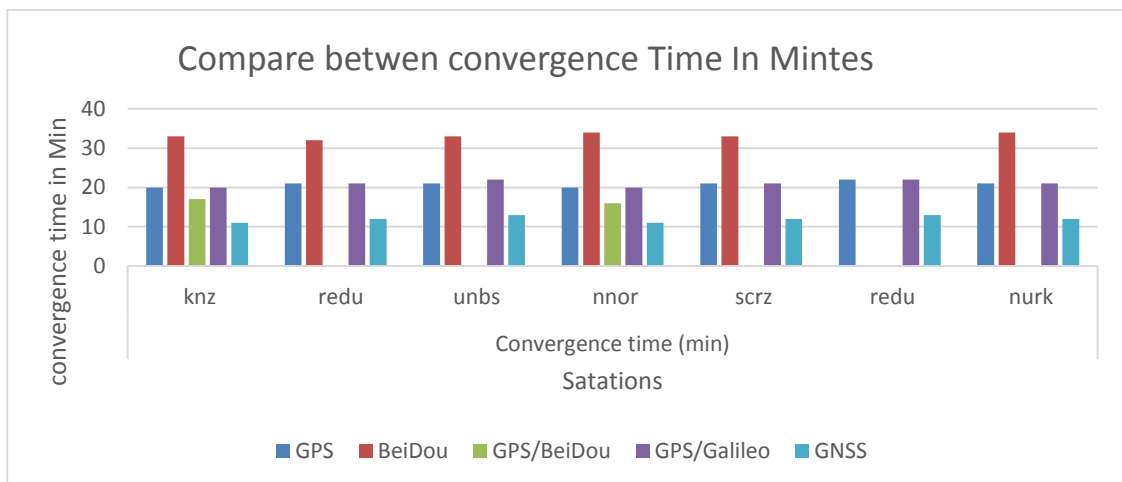
$$\Phi_{3E} - \Phi_{3E}^m = \rho_E - \rho_E^m - c[(dt_E^s + B_E^s) - (dt_E^s + B_E^s)^m] + T_E - T_E^m + [(\overline{\lambda N} + \Delta B^s)_E - (\overline{\lambda N} + \Delta B^s)_E^m] + \varepsilon_E - \varepsilon_E^m \quad (13)$$

$$\Phi_{3C} - \Phi_C^o = \rho_C - \rho_C^o - c[(dt_C^s + IFCD_C^s) - (dt_C^s + IFCD_C^s)^o] + T_C - T_C^o + [(\overline{\lambda N} + \Delta B^s)_C - (\overline{\lambda N} + \Delta B^s)_C^o] + \varepsilon_C - \varepsilon_C^o \quad (14)$$

the mathematical correlation between the observations should be taken into account when forming the observation weighted matrix for this model (Abd Rabbou, El-Rabbany 2014, Cai, Gao. 2007, Abd Rabbou, & El-Rabbany, 2015).

5-Analysis and result

GNSS PPP techniques, namely the undifferenced, between satellite single differences (BSSD) were developed to process the multi-constellations GNSS observations. The BSSD model can cancel out the receiver related biases and errors from both GNSS code and phase measurements. the contribution of BeiDou observations can be considered geographically dependent based on the BeiDou satellite availability in each station. GNSS PPP technique present comparable convergence time compared with the standard un-differenced technique due to the lack of code and phase-based satellite clock products and the mathematical correlation between the positioning and clock-ambiguity parameters. The availability of signals on three or more frequencies produced from multiple GNSS constellations offer a good chance for enhancing precise point positioning (PPP) convergence time and accuracy, compared to dual- frequency observations from a single constellation. after three-hours of GNSS processing data. Compared with undifferenced, It can be seen that the BSSD model enhances the convergence time and the positioning accuracy during the convergence time .while the BSSD PPP presents comparable positinoning accuracy to the un-differenced PPP at the end of the three-hours GNSS processing in figure (1,2,3,4,5)



Figure(1) : Convergence time for used statins by using BSSD technique

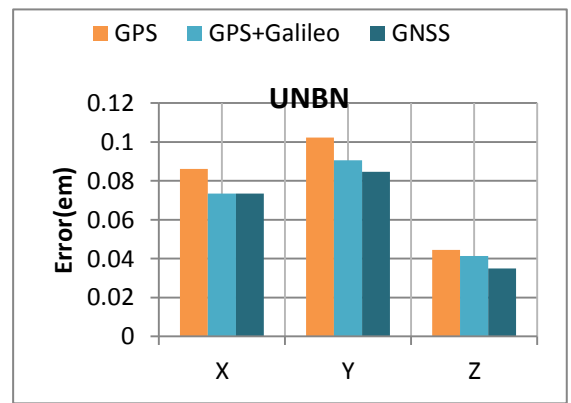
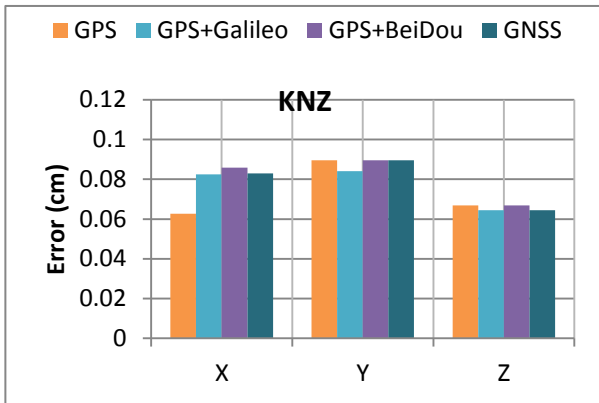
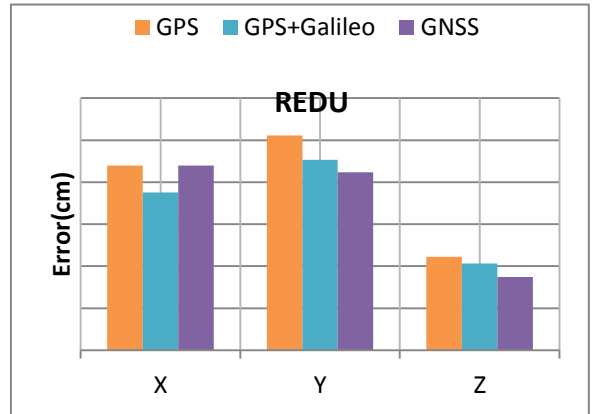
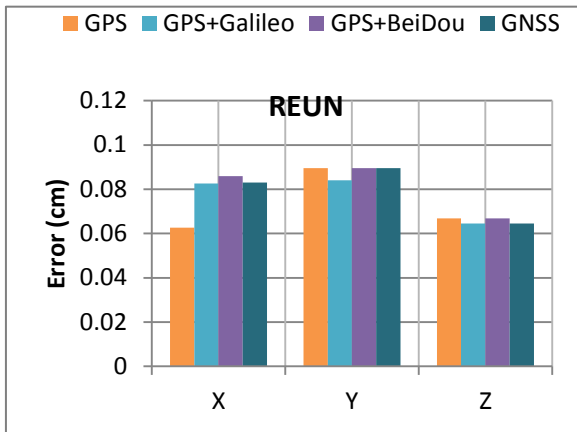
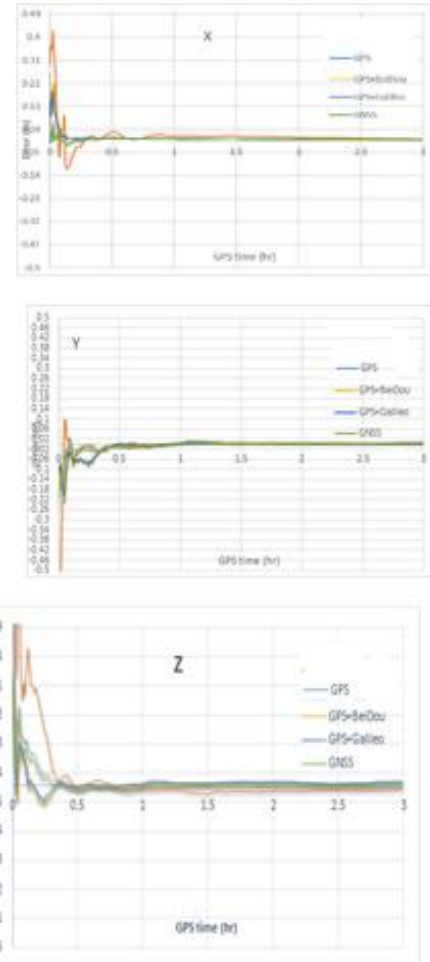
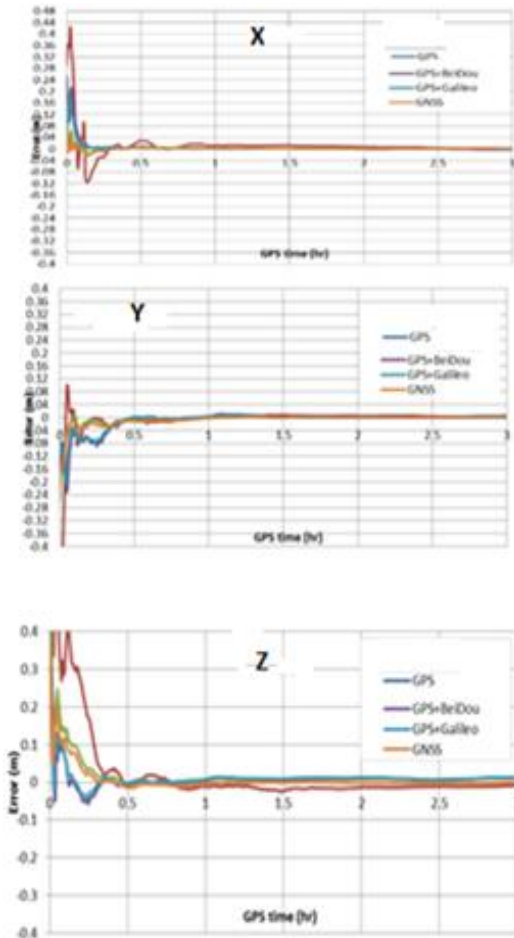


Figure (2): Positioning accuracy summary for different GNSS combinations for multiple point



Station: KNZ at DOY 1/4, 2016

Station: REDU at DOY 1/4, 2016

Figure (3):BSSD-PPP positioning error for different GNSS Combinations for stations Redu and KNZ

The positioning accuracy for different GNSS PPP combinations after 20 minutes processing this will be illustrate in table 2 .

Table 2: The positioning accuracy for different BSSD GNSS PPP combinations after 20 minutes processing

GNSS-PPP Combinations	X (m)		Y (m)		Z (m)	
	RMSE	Max	RMSE	Max	RMSE	Max
GPS	0.09	0.23	0.06	0.21	0.16	0.29
BeiDou	0.16	0.48	0.09	0.35	0.22	0.43
GPS/ BeiDou	0.07	0.13	0.03	0.15	0.09	0.20
GPS/Galileo	0.08	0.22	0.05	0.20	0.17	0.28
GNSS	0.04	0.11	0.01	0.13	0.06	0.18

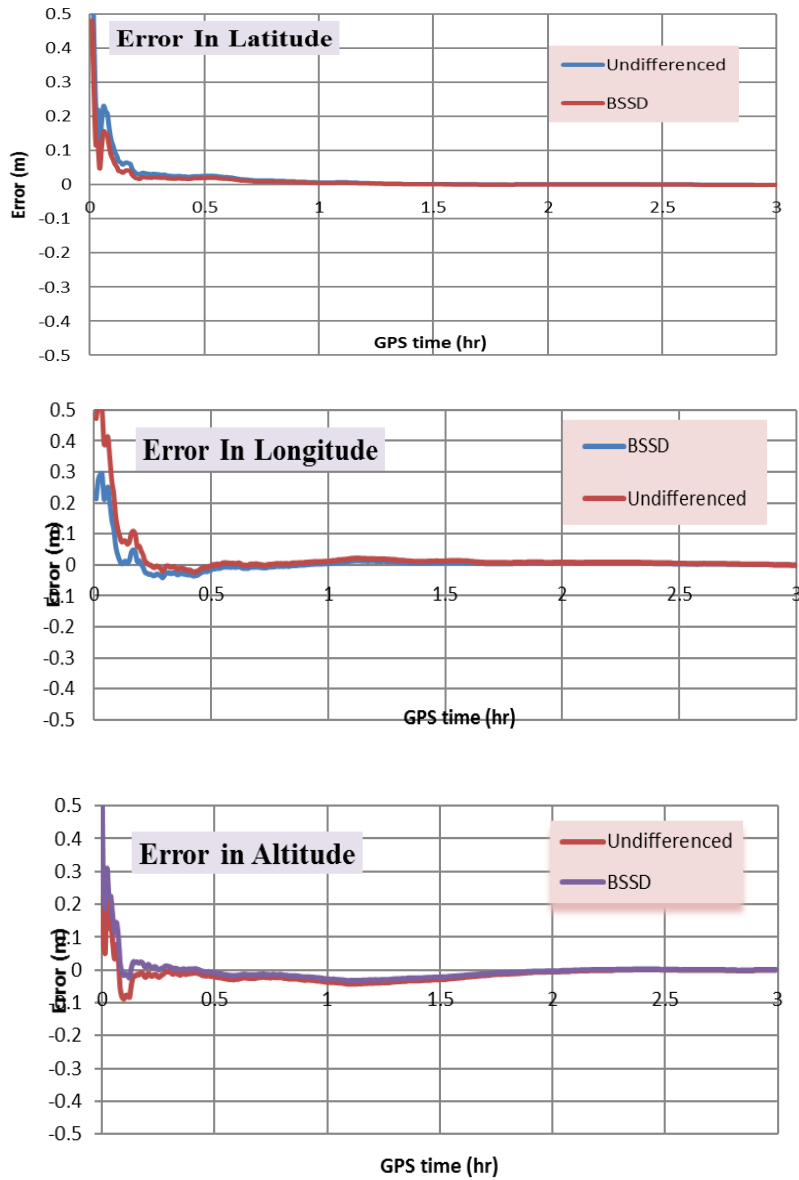


Figure 4: The positioning accuracy for the two GNSS PPP techniques for station REDU

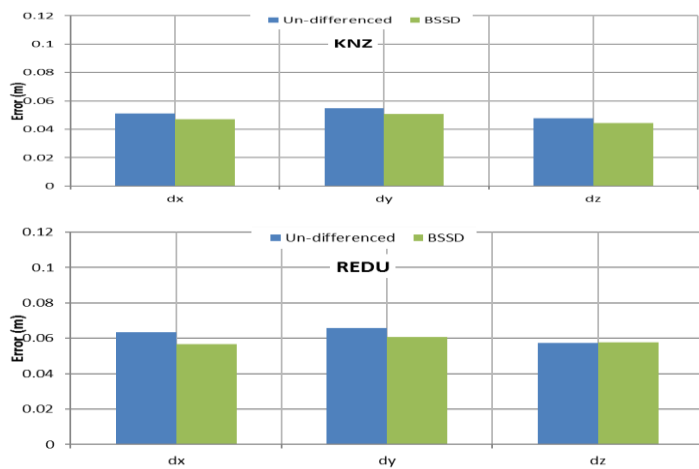


Figure 5: The positioning accuracy for the Two GNSS PPP techniques for station KNZ and REDU

5-conclusion

The use of BSSD linear combination improved the convergence time of the GNSS PPP solution about 50%, by comparing with the un-differenced GPS-only PPP model. By applying BSSD technique, each satellite will be selected as a reference satellite (GPS, Galileo, and BeiDou). However, by combining the observations of multi-GNSS constellations comes at the expense of introducing additional biases to the observation mathematical models. These include the GPS to Galileo time offset, GPS to BeiDou time offset and the hardware delays of both Galileo and BeiDou. The test results showed improvement of the PPP in both solution precision and convergence time. However, those studies were limited to the post-processing mode. The values of the ISB have been obtained for various days and receiver types almost identical. The Results obtained from both of the un-differenced and BSSD modes have shown that the values of the ISB are largely stable over the observation time periods. There is slight improvement in the solution of the convergence time obtained with the loose combination in comparison with the tight combination.

References

1. Gao, Y.; Chen, K. 2004 Performance analysis of precise point positioning using real-time orbit and clock products. *J. Glob. Position. Syst.* 2004, 3, 95–100.
2. Cai, C., Gao, Y., Pan, L. and Zhu, J. 2015. Precise point positioning with quad-constellations: GPS, BeiDou, GLONASS and Galileo. *Advances in Space Research*, 56(1), 133-143.
3. Cai, C. and Gao, Y. (2007) Precise Point Positioning Using Combined GPS and GLONASS Observations. *Positioning*, 1, 0.
4. Abd Rabbou, M. and El-Rabbany, A. (2015) PPP Accuracy Enhancement Using GPS/GLONASS Observations in Kinematic Mode. *Positioning*, 6, 1-6. <http://dx.doi.org/10.4236/pos.2015.61001>.
5. Rabbou, M. A., & El-Rabbany, A. (2015). Precise Point Positioning using Multi-Constellation GNSS Observations for Kinematic Applications. *Journal of Applied Geodesy*, 9(1), 15-26.
6. Li, X., Zhang, X., Ren, X., Fritsche, M., Wickert, J., & Schuh, H. (2015). Precise positioning with current multi-constellation Global Navigation Satellite Systems: GPS, GLONASS, Galileo and BeiDou. *Scientific reports*, 5.
7. Abd Rabbou M, El-Rabbany A (2014) Tightly Coupled Integration of GPS Precise Point Positioning and MEMS-Based Inertial Systems. *GPS Solution*, doi:10.1007/s10291-014-0415-3.
8. Phelts, R. E. (2007). Range Biases on Modernized GNSS Codes, Proceedings of European Navigation Conference GNSS/TimeNav, Geneva, Switzerland. <http://waas.stanford.edu/~wwu/papers/gps/PDF/PheltsENC07.pdf>
9. Hofmann-Wellenhof, B., Lichtenegger, H., and Walse, E. (2008). GNSS global navigation satellite systems: GPS, GLONASS, Galileo, and more, Springer, New York.
10. Leick, A. (2004). GPS satellite surveying, 3rd Ed., Wiley, New York.
11. Montenbruck, O., Steigenberger, P., Khachikyan, R., Weber, G., Langley, R. B., Mervart, L., Hugentobler, U. (2014). IGS- MGEX: Preparing the Ground for Multi-Constellation GNSS Science, *Inside GNSS*, 9(1):42-49.
12. Kouba, J. and Héroux, P. 2001 Precise point positioning using IGS orbit and clock products. *GPS Solution*, 5, 12–28.

13. Zumberge, J. F., Heflin, M. B., Jefferson, D. C., Watkins, M. M. and Webb, F. H. 1997. Precise point positioning for the efficient and robust analysis of GPS data from large networks. *Journal of Geophysical Research*, 102(B3): 5005-5018.