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DYNAMIC RESPONSE OF INCLINED BRIDGE SPANS DUE TO VEHICLE VIBRATIONS

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الملخص العربى:

يعد التداخل بين الكباري و المركبات احد اهم العوامل التي تؤثر على رد الفعل الديناميكي للكباري، فى هذا البحث تم تطوير برنامج على الحاسب الالي لحساب رد الفعل الديناميكي للكباري مع الاخذ في الاعتبار تاثير التداخل بين الكباري والمركبات. المفهوم الاساسي للبرنامج الحسابي المستخدم مبني على مبدأ تجزئة المنشأت و اختزال المصفوفات. هناك العديد من العوامل التي تؤثر على التداخل بين المركبات والكباري مثل سرعة المركبة, ميول مطالع و منازل الكباري, اتصال بحور الكباري ببعضها, وجود تعرجات في سطح الكوبري، تم عمل دراسة بارامترية لدراسة تاثير هذه العوامل المختلفة. تم حساب قيم معامل التكبير الديناميكي بناءا على النتائج المحسوبة من البرنامج و تم مقارنتها بالقيم المنصوص عليها في الكود المصري.

الكلمات الدالة : رد الفعل الديناميكي, تداخل المركبات والكباري, ميول المطالع والمنازل, معامل التكبير الديناميكي, تجزئة المنشئات.

ABSTRACT:

The dynamic response of bridges is extremely affected by the vehicle-bridge interaction. A software program was modified and used to calculate the bridge dynamic response including the vehicle-bridge interaction. The analytical model used is based on substructuring and matrix condensation. There are many factors affecting the vehicle-bridge interaction such as vehicle speed, slopes of bridge entrance and exit, the bridge spans continuity, and the existence of irregularities in the road surface. A parametric study was conducted in order to investigate the effects of these different factors on the bridge dynamic response. Dynamic Amplification Factors (DAFs) were calculated based on the results obtained through the analysis and these values were compared to their corresponding values specified in the Egyptian Code of Practice.

KEYWORDS: Dynamic Response, Vehicle-Bridge interaction, Entrance and Exit Slopes, Dynamic Amplification Factors, substructuring.

1- INTRODUCTION

The dynamic response of bridges under moving vehicles is affected by various parameters. The interaction between vehicle and bridge systems is among the most significant of these parameters. Throughout the last decades, the modeling of the bridge-vehicle system has witnessed unprecedented advancement as a result of the dramatic increase in computational capabilities of modern electronic computers. The finite element models used to simulate the vehicle-bridge system have progressed from the simple idealization for the whole vehicle as an unsprung single mass into more complex vehicle models describing the various vehicle elements including axle masses, tires, and suspension stiffnesses, and their corresponding damping coefficients. The basic concept of modeling bridge and vehicle systems is based on substructuring and matrix condensation. Previous research at the Structural Engineering Department of Ain Shams University developed an analytical model for the vehicle-bridge system to study the interaction between vehicle and bridge using software package (Microsoft Excel & Visual basic). Many parameters affect this interaction between the vehicle and bridge systems, such as the effect of vehicle vibrations resulting from the passage of the vehicle on several consecutive spans, the continuity of bridge spans, the upward and downward slopes at bridge entrance and exit, vehicle speed, and bridge and vehicle damping and the presence of irregularities in the road surface. The Egyptian Code of Practice uses impact factor formulas not relying on these factors.

2- LITERATURE REVIEW

Zeng and Bert [1] presented a semi-analytical technique to solve the bridge-vehicle interaction problem. A parametric study was conducted on Walnut Creek Bridge in Arizona. The bridge was skewed 45 to the roadway. Many traffic conditions were examined. The highest DAF was obtained when two vehicles were running at the same lane with one axle spacing between them, and the smallest DAF was obtained when two vehicles were running with a span length between them. It was also observed that with increasing the skew angle, DAF values increased.

Fafard et al [2] studied the dynamic response of an existing bridge under vehicle loads. A 3D analytical model was used for the vehicle. A comparison was held between the used models and the experimental results. The experimental results were found to be in good agreement with the F.E.M results. The results showed that DAFs computed for displacements are lower than those of straining actions. Fafrad (1998) recommended using the DAFs for deflections in the serviceability limit state and the DAFs for moments in the collapse limit state for bending.

Yang and Lin [3] developed an efficient procedure to simulate the dynamic response of the bridge-vehicle system based on the concept of matrix condensation. The vehicle was modeled as a lumped mass supported by dashpots and springs. The bridge was modeled as a three-dimensional beam element containing 12 DOFs. An interaction element was defined which consisted of a bridge element and the vehicle suspensions in contact with this element. The developed procedure was found to be computationally efficient and it was especially convenient for the simulation of the bridge-vehicle system with different vehicle flow patterns.

Nguyen-Xuan et al [4] examined the effect of road unevenness on the dynamic vehicle-bridge interaction. A study was performed on the Nguyen-Tri-Phuong bridge in Vietnam. The numerical results were found to be in good agreement with the test results. The findings showed that the road surface condition has major effects on the dynamic impact factor.

Pagnoncelli and Miguel [5] introduced a simple methodology to calculate the dynamic analysis of the bridges considering the vehicle-bridge interaction and applied the methodology to a concrete bridge which was built on the Rio Santos Highway. The bridge was simply supported and had a box girder section. Four simplified models were used for the vehicle model. The results revealed substantial differences between vehicle models. These differences indicated that increasing vehicle model complexity brings the dynamic response closer to reality.

3- BRIDGE VEHICLE-INTERACTION MODEL

Increasing the complexity of the bridge-vehicle dynamic model to simulate their actual behavior leads to more accurate results. The vehicle-bridge model used here is the software program developed by El-Badrawy [6]. The concept of solving the interaction problem between the bridge and vehicle is based on the substructuring and matrix condensation.

3.1 Vehicle model

The vehicle model used in this study is a 3-axle truck. This vehicle model was used by Hassan [7] and El-Badrawy [6] in their previous research work. All the 3 axles are mounted on a common suspension which consists of a multi-leaf spring. The vehicle model consists of the sprung body mass, the multi-leaf suspension, front and rear axles, and the vehicle tires. The sprung body mass [M] is supported on the common suspension. This sprung mass is subjected to a rigid body motion in which the vertical displacement and the pitching rotation are considered. The front and rear suspensions are simulated as springs (K_1, K_2). The unsprung masses (the axles masses m_1, m_2, m_3) are supported on the tires which are simulated as springs (K_{tf} , K_{tr}). Viscous dampers (C_1, C_2, C_{tf} , C_{tr})are introduced to the suspension and tire springs to include the damping characteristics. The free-body diagram of the vehicle can be shown in Figure 1.

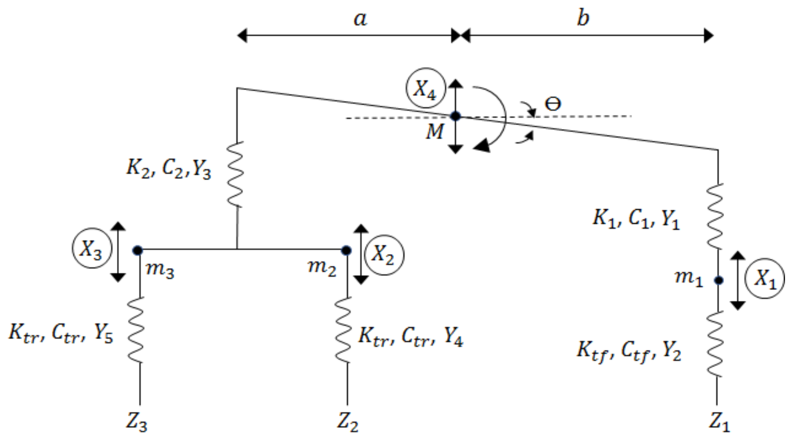


Figure 1: Free body diagram for the vehicle components

3.2 Bridge finite element model

The bridge model used in this study is a two-dimensional beam element. The effect of the transverse vibration modes on the response is neglected. Only the case of symmetric transverse loading is considered. The whole structure is divided into small segments of equal length. The nodes of these elements are numbered in sequential form. The local stiffness matrix for each element can be obtained easily using the standard beam element stiffness matrix. The consistent mass matrix is used to represent the local mass matrix.

3.3 Tire stiffness matrix

The combined vehicle bridge stiffness matrix contains three matrices, the vehicle stiffness matrix$[k_v]$, the condensed bridge stiffness matrix$[k_{bcp}]$, and the tire stiffness matrix. $[k_{tire}]$. Let the vehicle axle location lie between the bridge nodes which have the degrees of freedoms (6&7). Assume that the axle location is at a distance x from the first bridge node and the distance between the two bridge nodes is of a length L as shown in Figure 2.

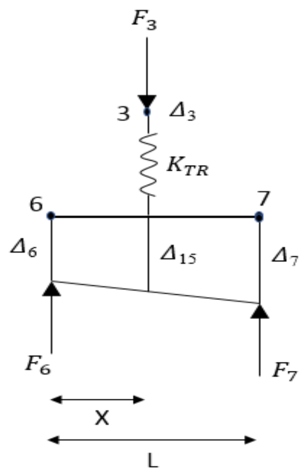


Figure 2: Displacements relations between the bridge contact points and axle DOF.

the following relations can be obtained

$r_1 = x / L$ (1)

$r_2 = (L-x) / L$ (2)

$\Delta_{15} = \Delta_6 . r_2 + \Delta_7 . r_1$ (3)

Where Δ_{15} is the displacement of the bridge at the axle position. And Δ_6 is the displacement of the first bridge node. And Δ_7 is the displacement of the second bridge node.

Also, the following relations can be obtained:

$F_3 = k_{tire} . (\Delta_3 - \Delta_{15}) = k_{tire} . (\Delta_3 - \Delta_6 . r_2 - \Delta_7 . r_1)$ (4)

$F_{15} = - F_3 = k_{tire} . (\Delta_{15} - \Delta_3)$ (5)

$F_{15} = k_{tire} . (\Delta_6 . r_2 + \Delta_7 . r_1 - \Delta_3)$ (6)

Where Δ_3 is the axle displacement, F_3 is the axle force, and F_{15} is the bridge reaction at the axle position and k_{tire} is the tire stiffness.

$F_6 = F_{15} . r_2 = k_{tire} . (\Delta_6 . r_2^2 + \Delta_7 . r_1 . r_2 - \Delta_3 . r_2)$ (7)

$F_7 = F_{15} . r_1 = k_{tire} . (\Delta_6 . r_2 . r_1 + \Delta_7 . r_1^2 - \Delta_3 . r_1)$ (8)

Where F_6 is the bridge reaction at the first bridge node and F_7 is the bridge reaction at the second bridge node. Using the previous equations, the relations between the forces and displacements of the vehicle DOFs and connection points DOFs could be obtained, which is called the tire stiffness matrix $[k_{tire}]$.

$$\begin{bmatrix} F_3 \\ F_6 \\ F_7 \end{bmatrix} = \begin{bmatrix} 0 & -r_2 & -r_1 \\ -r_2 & r_2^2 & r_1 . r_2 \\ -r_1 & r_1 . r_2 & r_1^2 \end{bmatrix} \begin{bmatrix} \Delta_3 \\ \Delta_6 \\ \Delta_7 \end{bmatrix}$$

(9)

3.4 Bridge response calculations

Because of the complexity of the bridge-vehicle interaction problem, the bridge response can be determined in three stages as follows:

3.4.1 Stage 1 displacements

In this stage, the dynamic response of the bridge is determined due to the various dynamic forces acting along the bridge. These dynamic forces result from the inertia forces and the damping forces due to the initial conditions of the bridge at the beginning of the time step. These forces are applied to the bridge model to find the bridge displacements at this stage {displacement 1}.

3.4.2 Stage 2 displacements

In this stage, a group of imaginary supports is introduced at the bridge-vehicle connection points forcing them to have zero vertical displacements. At first, the reactions of the imaginary supports are calculated by multiplying the bridge-vehicle combined stiffness matrix by the vertical displacements (calculated in stage 1) but at the connection points only.

Then, the imaginary reactions vector calculated in the last step is extended to form the whole bridge force vector with the imaginary reactions at the contact points and zeroes at other bridge nodes. Then, this force vector could be applied to the bridge model to calculate the bridge displacements {displacement 2}.

Summation of stages 1&2 result in the calculation of the bridge response due to the dynamic forces along the bridge assuming imaginary supports at the bridge-vehicle connection points preventing them from the vertical displacements as shown in Figure 3.

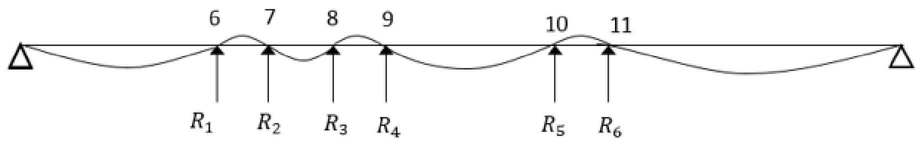


Figure 3: The bridge response due to various dynamic forces along the bridge while applying imaginary reactions at contact points.

3.4.3 Stage 3 displacements

In this stage, the effect of the imaginary supports is canceled and the final bridge response can be obtained. The imaginary reactions calculated in stage 2 can be inverted and introduced into the bridge-vehicle model. These inverted reactions should be added to axles' static loads and applied at the bridge-vehicle connection points in the combined model. Therefore, the forces acting at vehicle DOFs do not equal zero as the vehicle is affected by its initial conditions at the beginning of the time step.

The connection points forces can be accompanied by the vehicle forces to form the whole forces vector in the combined model. Then, the displacements of the vehicle and the connection points can be obtained. The displacements obtained in this stage are the exact values of the displacements for the vehicle and the bridge-vehicle connection points. However, the response of all other bridge nodes has yet to be determined. In order to do that, a set of forces is calculated at the bridge-vehicle connection points, which, if applied on the bridge model, would result in the same response at the contact points.

These forces can be extended to form the whole force vector and applied to the bridge model to obtain the displacements of the other bridge nodes { displacement 3 }.

The summation of the bridge response at the previous 3 stages results in the final response of the bridge.

{ final displacement} = { displacement 1 }+{ displacement 2}+{ displacement 3} (10)

These displacements can be used as the initial conditions of the next time step. Also, the vehicle displacements obtained from the combined model can be used as the initial conditions for the vehicle in the next time step.

3.5 Element and node numbering

The bridge model is divided into a large number of elements. Each element has two nodes. At the supports, there are 2 cases, continuous spans, and simple spans. For the continuous spans case, the element just before and the element just after the support location have a common node at which the support is defined. For the simple spans case, the two elements just before and after the support should be separated. In order to do that, two different nodes are defined at the support location, each node is connected with a different span and two supports are defined separately at each node. For the simple spans case, a modification in the axle search routine should be applied as the two nodes at the support location have no element. The search routine was modified by Galal [8] to search for elements instead of nodes to avoid the presence of the axle at the no-element location.

3.6 Simulation of the bridge profile

For the vehicle-bridge system, the displacements of the bridge contact points and the vehicle wheels should have the same value to attain the compatibility of the displacements at the bridge vehicle contact points. In order to take the effect of the bridge profile on the vehicle vibrations and the bridge dynamic response, upward vertical forces are applied at the axles DOFs and downward vertical forces are applied at the bridge contact points as shown in Figure 4. These upward and downward forces have the same values which equal the height of the bridge profile multiplied by the stiffness of the tire connected with this axle. Sometimes the values of these forces are very large, especially in the case of simulating the bridge entrance and exit slopes. The change of the axle displacements resulted in an overall change in the displacements for the whole vehicle model which counteracts the effect of these large forces and then only the difference between these effects produces the dynamic effect of the vehicle on the bridge. The effect of bridge profile on the stiffness coefficients of the bridge elements is ignored as these slope values are too small to affect these stiffness coefficients.

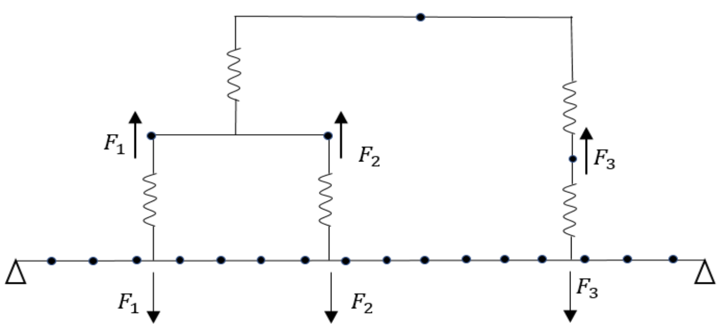


Figure 4: Equivalent Vertical forces due to the effect of bridge profile

4- PARAMETRIC STUDY

4.1 Bridge description

A parametric study was conducted on the Kilo-21 bridge which is located on Alexandria-Matruh highway road. The bridge has a total length of 660m and a 12.8m width. The bridge vertical profile is shown in Figure 5.

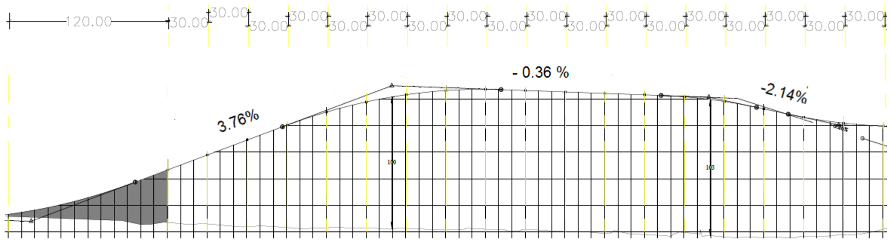


Figure 5: Longitudinal profile for Al Kilo-21 bridge

The whole bridge consists of 18 spans divided into 5 parts as shown in Figure 6. The first two parts are separated from each other by an expansion joint. Each part consists of 3 continuous spans. The last three parts are also separated from each other by the expansion joints. Each part has 4 continuous spans.

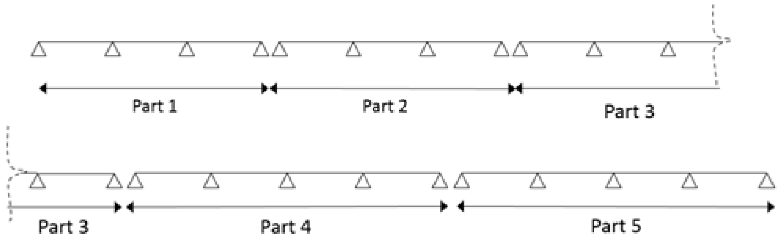


Figure 6: Span's continuity for the actual case of the bridge

The bridge cross-section is a box girder section that has a total width of 12.8 m and a total height of 2 m. Its detailed dimensions are shown in Figure 7. The box section properties are shown in table 1. The section properties were calculated from one box section.

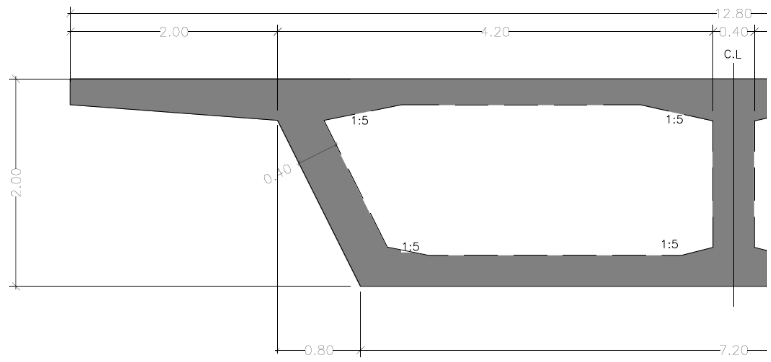


Figure 7: Cross-section for AL Kilo-21 bridge

Table 1: Properties of the studied bridge

Mass (M)	Inertia (I)	Young's Modulus (E)
10.3 t /m	0.9 m^4	3.13 * 10^6 (t/m^2)

4.2 Selected cases study

Several cases were introduced to study the effect of the different parameters on the bridge dynamic response. All the studied cases are shown in Table 2. The effect of different slopes was studied, including the case of a flat surface, actual bridge profile which has curved slopes, case of slopes equal to 1.5 times the original slopes, and the case of straight-line slopes. The effect of vehicle speed on the dynamic bridge response was also studied. A study was also performed on the continuous spans case and the simple spans case to study the effect of supports continuity on the dynamic response of the bridge. The Case of the existence of bumps at the expansion joints was also examined to investigate the effect of road surface irregularities on the bridge dynamic response. The properties of the used vehicle model are shown in Table 3 [9].

Table 2: Runs performed on the studied bridge

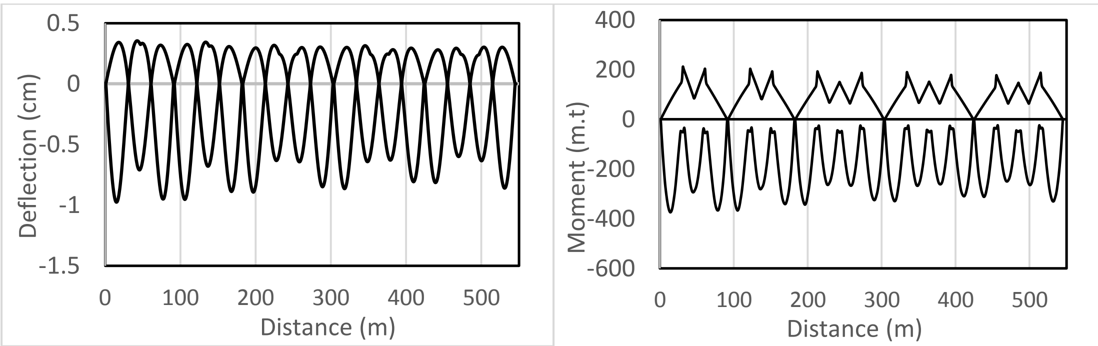
	Support condition	Slopes	Vehicle speed	Bump	Vehicle
1	Continuous	Actual profile	5 km/h	No	Vehicle 1
2	Continuous	Actual profile	40 km/h	No	Vehicle 1
3	Continuous	Actual profile	70 km/h	No	Vehicle 1
4	Continuous	Actual profile	100 km/h	No	Vehicle 1
5	Continuous	Flat	100 km/h	No	Vehicle 1
6	Continuous	1.5 * Actual profile	100 km/h	No	Vehicle 1
7	Continuous	Straight lines	100 km/h	No	Vehicle 1
8	Simple	Actual profile	5 km/h	No	Vehicle 1
9	Simple	Actual profile	100 km/h	No	Vehicle 1
10	Continuous	Actual profile	100 km/h	at 120 m	Vehicle 1
11	Continuous	Actual profile	100 km/h	at 210 m	Vehicle 1

Table 3: Properties of the studied vehicles

Properties	Vehicle 1
Front axle load (A)	223750 N
Intermediate axle load (B)	188125 N
Rear axle load (C)	188125 N
Distance between axle(A) and(B)	3 m
Distance between axle(A) and(C)	2 m
Front tire stiffness	1748000 N/m
Intermediate tire stiffness	3496000 N/m
Rear tire stiffness	3496000 N/m
Front axle stiffness	4×10^6 N/m
Rear axle stiffness	6×10^6 N/m
Body mass (M)	57000 kg
Rotational moment of inertia	118750 kg. m^2
Axle (A) mass	1000 kg
Axle (B) mass	1000 kg
Axle (C) mass	1000 kg
Distance between the vehicle body and the front axle	2.5 m
Distance between the vehicle body and the rear axle	1.5 m

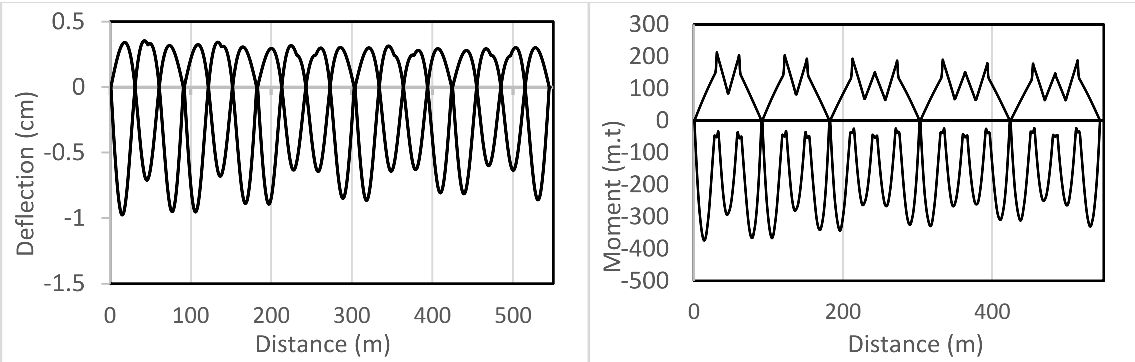
4.3 Analysis results

The results of the previously studied cases are shown in Figure 8 to Figure 16. The moment and deflection values for cases 1,4,5, and 6 are sown in Figure 8 to Figure 11. The effect of changing bridge entrance and exit slopes is shown in Figure 12 and Figure 13.



a) Maximum deflection **b) Maximum bending moment**

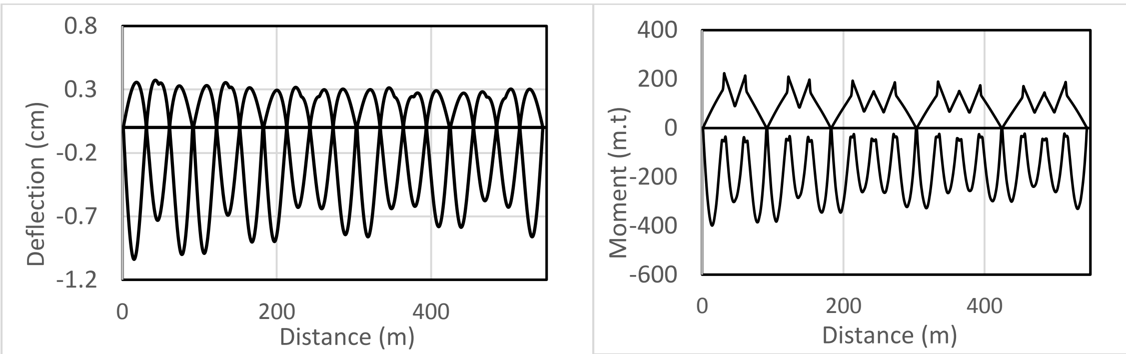
Figure 8: Variation of the maximum deflection and bending moment (Actual bridge profile, Crawl speed, continuous spans)



a) Maximum deflection **b) Maximum bending moment**

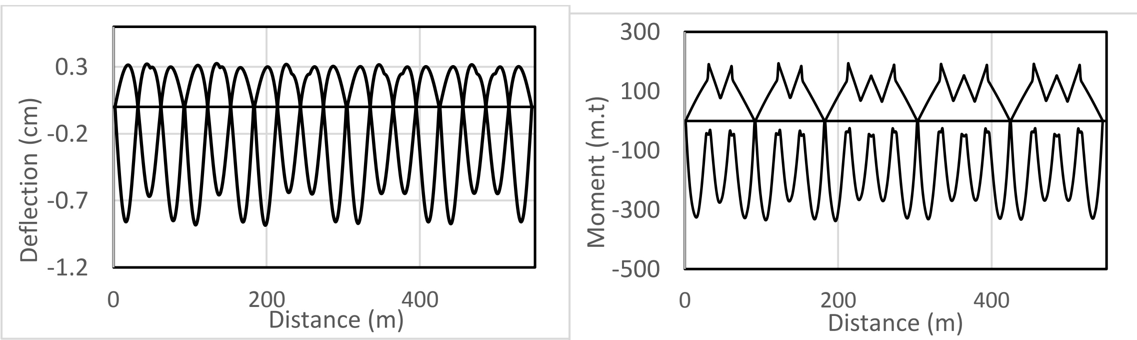
Figure 9: Variation of the maximum deflection and bending moment (Actual bridge profile, 100km/h, continuous spans)

The effect of changing vehicle speed is shown in Figure 14. The effect of bridge spans continuity is shown in Figure 15. The effect of the existence of bumps at the bridge entrance and expansion joint is shown in Figure 16.



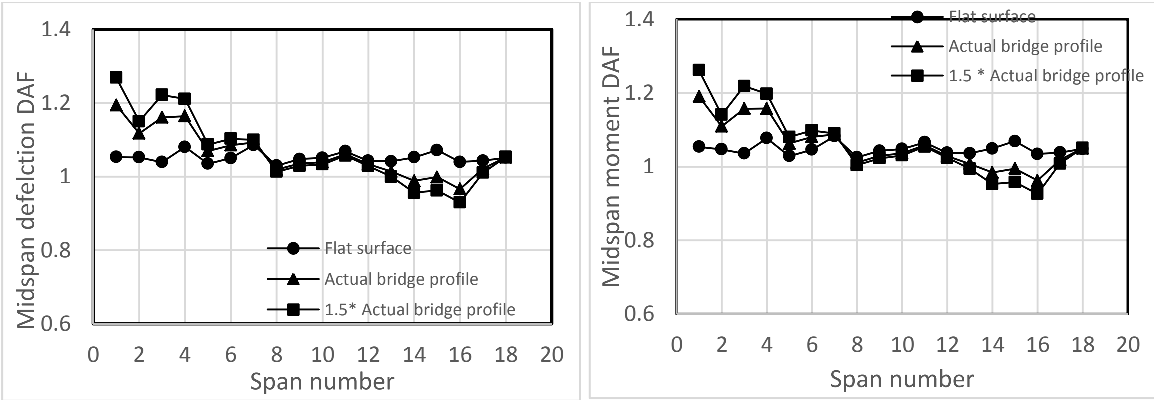
a) Maximum deflection **b) Maximum bending moment**

Figure 10: Variation of the maximum deflection and bending moment (Case of one and a half the actual bridge profile, 100km/h speed, continuous spans)



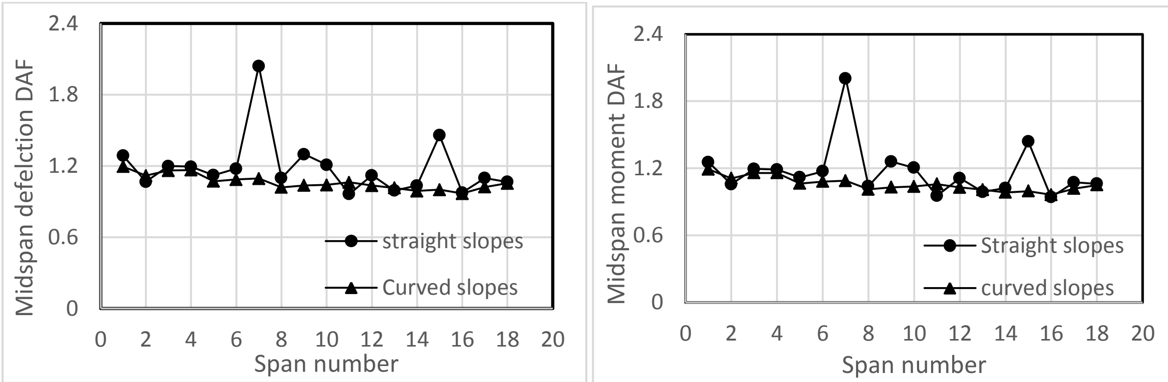
a) Maximum deflection **b) Maximum bending moment**

Figure 11: Variation of the maximum deflection and bending moment (Case of Flat surface, 100km/h speed, continuous spans)



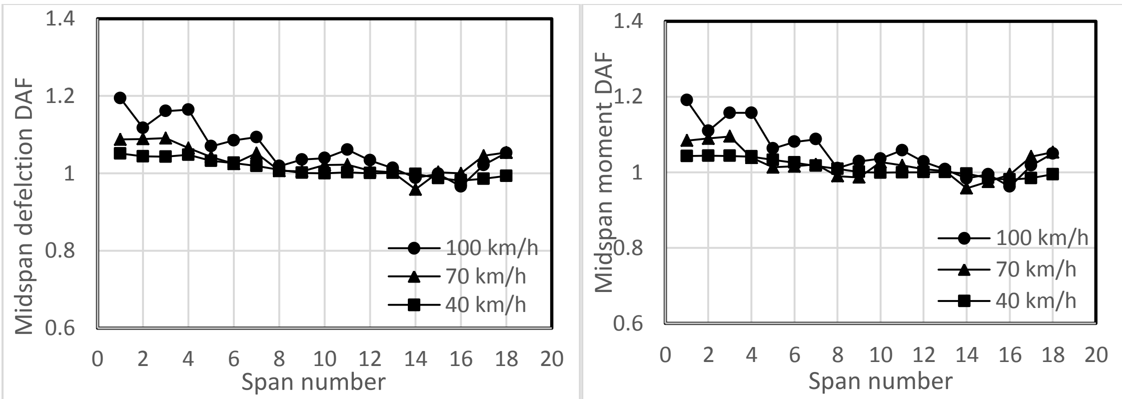
a) Deflection **b) Bending moment**

Figure 12: Effect of changing the bridge profile on the DAF values for deflection and bending moment (case of continuous spans, V = 100km/h)



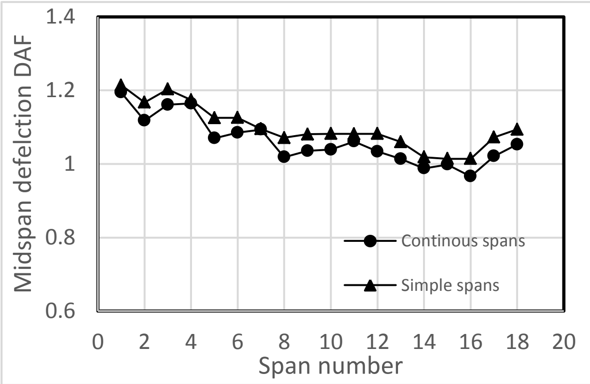
a) Deflection **b) Bending moment**

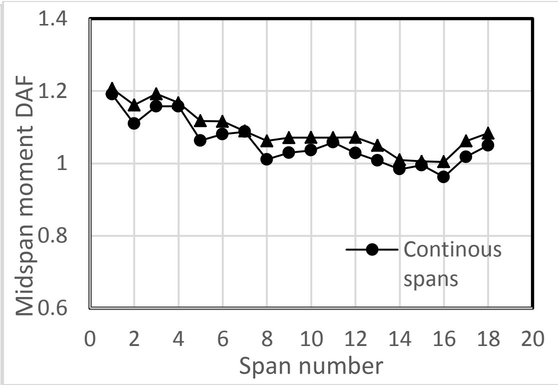
Figure 13: Effect of changing slope pattern on DAF values for deflection and bending moment (case of continuous spans, v=100km/h)



a) Deflection **b) Bending moment**

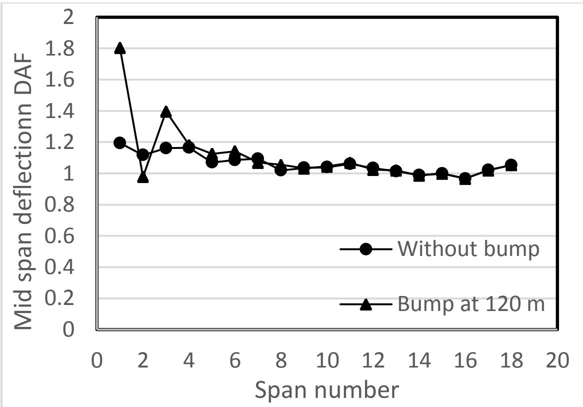
Figure 14: Effect of changing the vehicle speed on the DAF values for deflection and bending moment (Actual bridge profile, continuous spans)

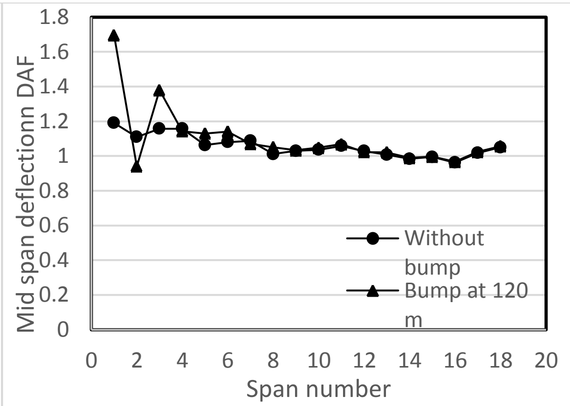




a) Deflection **b) Bending moment**

Figure 15: Effect of bridge support continuity on the DAF values for deflection and bending moment (Actual bridge profile, V = 100km/h)





a) Deflection **b) Bending moment**

Figure 16: Effect of a bump existing at 120 m and 210 m from bridge entrance on the DAF values for deflection (100 km/h, continuous spans)

4.4 Discussion of the results

1. It can be observed that for the case of curved slopes (actual bridge profile), when the slopes increase, DAF values increase in the first spans where the slopes are upward. In the last spans, DAF values decrease where the slopes are downward. The increase in DAF values in the first spans can be explained by the fact that each time the vehicle moves on the upward slopes, it gained more excitations which caused more vehicle vibrations and consequently a higher effect on the bridge dynamic response. For the downward slopes, the decrease in DAF values could be explained by the fact that the vehicle has what is called the flying mood resulting in a decrease in the tire forces. For the case of straight slopes, the values of DAF for the bridge are higher than those of the curved case, as the sudden change in the bridge profile at the points where the slope changes its directions resulted in more increase in the vehicle vibrations, which leads to an increase in DAF values. DAF values also increase at the point where the slopes change from the nearly horizontal case to the downward case in contrast to the case of curved slopes where DAF values tend to decrease. This can also be attributed to the large increase in vehicle vibrations in the case of straight slopes.

2. It can be noted that for cases of different vehicle speeds, by increasing vehicle speed, DAF values generally increased but not for all cases. The increase in DAF values can be illustrated by the fact that increasing vehicle speed causes more vehicle vibrations and consequently higher bridge response. In some cases, by increasing vehicle speed, DAF values decreased. This result can be interpreted by the phase angle of the vehicle as it moves along the span and the relation between the vehicle and bridge periods of vibration and the time taken by the vehicle to cross this bridge span.
3. For the case of different support conditions, it is observed that the simple spans case has higher DAF values than the actual supports conditions which consist of a number of continuous spans. This can be attributed to many reasons, firstly the fact that the supports discontinuity caused larger values for bridge deflections which caused higher vehicle vibrations then went back to affect the bridge response causing higher DAF values. Secondly, the effect of the change in slope at the supports in the simple spans case caused more vibrations for the vehicle. That is not the case for the continuous spans case in which the support maintains its slope just after and before it. Thirdly, the effect of slope continuity for continuous spans case at the supports resulted in a negative starting deflection value for the span next to the loaded one. This negative deflection value led to a decrease in the final displacement value for bridge spans.
4. It can be observed that DAF values for the bending moment are slightly less than those for the deflection. These extremely close values can be attributed to the fact that the bridge's first mode of vibration dominates both responses. The higher bridge modes which tend to affect the bending moment more than the deflection have very little effect on the results.
5. The effect of the existence of bumps at the expansion joints was obviously found to increase the response in the first span after the bump locations. The DAF values reached 70% over the static response which exceeded the DAF values specified in the Egyptian Code of Practice.

5- Summary and conclusion

This paper presented a research work that deals with the bridge-vehicle interaction problem. A software program was used to solve this dynamic problem using the substructuring & matrix condensation concepts. A parametric study was conducted on Al Kilo 21 bridge, including the different factors affecting this interaction, focusing on the effect of bridge entrance and exit slopes. Based on the analysis results, a number of conclusions have been obtained as follows:

1. A number of special considerations should be taken into account in the design of bridge entrance slopes as DAFs were found to increase at the spans with upward slopes.
2. The road profile should be designed to have curved slopes rather than straight slopes since the case of the curved slopes resulted in significantly lower DAF values than the case of the straight slopes.

3. The Construction Supervising Engineer must pay attention to the Pavement work especially at the bridge entrance and exit because any irregularities in it lead to high DAFs.
4. The case of simple spans resulted in higher DAF values than the continuous spans case, that is why special considerations are highly recommended in the design of these types of bridges.
5. The existence of bumps was found to dramatically increase DAF values to the extent that. in some cases, it reached 80% over the static response.
6. DAF values obtained in case of surface irregularities were found to exceed DAF values specified in the Egyptian Code of Practice, especially at the bridge entrance. Egyptian Code formulas do not depend on these factors, so a thorough comprehensive study of the effects of these factors on different types of bridges is highly recommended.

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