



## Equations for Mix Design of High Strength Concrete

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### ملخص البحث:

يهدف البحث إلى الحصول على نموذج من المعادلات لتصميم خلطه خرسانيه عاليه المقاومه طبقاً للكود الأمريكي بدلاً من استخدام الجداول و الرسومات البيانيه الموجوده بالكود و ذلك للحد من الأخطاء البشريه التي من المحتمل أن تحدث أثناء عملية التصميم مثل الأخطاء التي تحدث في عملية تحديد و قراءة النقاط و القيم المستخدمه في تصميم الخلطه الخرسانيه و أيضاً للحد من الوقت المُستغرق و الجُهد المبذول في عملية التصميم. و لقد تم ذلك من خلال تحويل جميع الجداول المستخدمه في عمليه التصميم و الموجوده بالكود إلى رسومات بيانيه و ذلك باستخدام برنامج الإكسيل و من ثم إختيار المعادلات المناسبه للرسم البياني بناءً على معامل دقة المعادله ( $R^2$ ) ، كما تم التحقق من دقة النموذج و ذلك بتصميم خلطه خرسانيه عاليه المقاومه يدوياً و أيضاً باستخدام نموذج المعادلات المُستحدث ومن ثم مقارنة و تحليل نتائجهم. و قد أظهر النموذج نتائج جیده بأخطاء تصل إلى 0.14 % كحد أقصى ، كما أظهر النموذج المُستحدث نتائج ملحوظه في توفير الوقت و الجُهد المبذول أثناء عملية تصميم الخلطات الخرسانيه عاليه المقاومه.

### Abstract.

In this study, a model of equations was developed based on tabular data and charts in American Concrete Institute Code (ACI 211.4R.93) to optimize the high-strength concrete mixture design in terms of time, effort and cost. The model of equations was developed with the aid of Microsoft excel sheets after tried various models and choose the most accurate model that adequately represents the data based on the regression coefficient and its predictive capability. The model has the capability of predicting the optimum design of high-strength concrete in an optimum time and it reduces the errors that could happen due to human errors. This model of equations is useful for the adjustment of mixture proportioning of high strength concrete as it can be used instead of data in ACI. Moreover, the validation of this model was performed analytically by design a specific high-strength concrete mixture and compared with the results obtained by the model. The maximum percentage error of the program was about 0.14 % and this is considered a fair result.

**Keywords** concrete mix design, optimization, high strength concrete, mathematical equations

## 1. Introduction

Concrete is one of the most commonly used material on the Earth. It is considered the most important construction material in the world (Krishnaswami, 2009) [1]. Concrete is categorized into different types, one of these types is High strength concrete (HSC) which is widely used in the recent construction industry due to its capability to reduce the sections of structural elements, increase rental space, decrease the amount of steel and it also extends the service life of structural element (Papadakis, 2001) [2]. Moreover, HSC can be used for reducing the self-weight and size of construction members, decreasing axial shortening of compression members, giving the capability for construction longer spans with fewer beams, reducing the number of supports due to long-span, and reducing the thickness of floor slabs and beam sections (Revanth & Kumar, 2017) [3].

Pointing to the historical background and in the 20th century, scientists called concrete with compressive strength of 25 MPa as high strength concrete. After that and in 1980s scientists considered concrete with compressive strength of 50 MPa as HSC. And nowadays technology enables the scientist to produce a concrete with compressive strength higher than 120 MPa which they called ultra-high strength concrete (Mendis, 2001) [4].

Using HSC for the construction of multi-story structures became very common in developing countries. As an example, in Australia, the majority of medium-rise and high-rise of buildings were constructed using high-strength concrete such as 43-story high Casselden Place, Rialto project in Melbourne, and 120 Collins street (Smith & Rad, 1989) [5]. Also, for the Pacific First Centre in Seattle USA, the compressive strength of the used concrete was 125MPa (Randall & Foot, 1989) [6]. Moreover, in Frankfurt HSC was first used in high-rise buildings in 1992. Also, in 1992 HSC up to 100 MPa was used in the Petronas towers in Kuala Lumpur.

Two approaches can obtain HSC mixture; first, by de-flocculation of cement grains by using chemical super-plasticizers to reduce cement particles' tendency in size. Thus, it can flock together, increasing the homogeneity, which in turn will reduce the demanded water; Second, by using extremely fine materials such as silica fume and fly ash. That will fill the micro-voids between particles leading to a dense microstructure while reducing the demanded water in concrete (Malier, 1992) [7].

The two previous approaches could be called a concrete mix design for high strength concrete. concrete mix design, in general, could be defined as the process of selecting suitable ingredients of the desired concrete and accurately calculating their relative quantities to obtain a specific strength and durability with an optimum cost (Santhosh, 2017) [8]. Accordingly, any concrete design process consumes time and human energy during the process and costs money after the design process due to the cost of raw

material, mixing machine, etc. so many studies were done by scientists to optimize the pervious factors; cost, time and human energy.

In 2011 Gupta, Mittal and Saini used MATLAB program to inference a computerized model to design a high strength concrete mix with an ability to achieve strength between 25 Mpa to 45 Mpa (Gupta, Mittal, & Sainai, 2011) [9], Also, in 2013 Zealakshmi, Ravichandran and Kothandaraman developed m-file for high strength concrete mix design based on the guideline given in ACI 211.4 R -93 [10]. The program was developed in MATLAB environment using arithmetic operators, conditional statements and relational operators to achieve the optimum design for high strength concrete with strength up to 60 Mpa (Zealakshmi, Ravichandran, & Kothandaraman, 2013) [11]. Moreover, in 2017 Joun Paul and Makenya develop a m-file using shacklock curves to design a high-strength concrete mixture up to 60 MPa faster than the manual design and the model showed a fair result comparing with the manual approach (Joun & Makenya, 2017) [12].

So, this study focuses on developing a model of equations that can design a high strength concrete by giving accurate quantities of high strength concrete ingredients. Also, the model could optimize the human error that probably happened due to interpolation of points or determine values at the process of design and that will prevent any additional costs resulted from form adding un accurate quantities of mixture ingredients. Moreover, the model could optimize the human energy and time consumed in the design process comparing with the manual design.

## **2. Methodology**

For seeking the objective of the paper, the methodology is divided into three sections; the first section is developing the model of equations from ACI 211.4R.93 [10] by converting the tabular data and charts into equations with the aid of manual calculations and computer programs. The second section is focusing on the procedures of ACI design of high strength concrete. The last section is the verification of the model by design a high strength concrete with a specific compressive strength manually and compared the results with those obtained by the model.

### **2.1 Mix design equations**

Microsoft excel sheets was used to develop the model of equations by plotting the tabular data and by adding a trend line to the curve the accurate equation should be chosen according to correlation factor ( $R^2$ ).

#### **2.1.1 Mixing water**

From table 4.3.4 named with the First estimate of mixing water requirement and air content of fresh concrete based on using sand with 35 percent voids in ACI 211.4R.93 [10] (Table 1), a graph for this tabular data was plotted to develop three equations (1,2,3) which

give the capability for estimating the mixing water according to the desired slump and the maximum size of used aggregates.

**Table 1 First estimate of mixing water requirement of fresh concrete**

Aggregate max Size	9.5 mm	12.5 mm	19.0 mm	25.0 mm
Slump mm	mixing water kg/m <sup>3</sup> of concrete			
25 – 50	184	175	169	166
50 – 75	190	184	175	172
75 – 100	196	190	181	178

Equations (1,2,3) could be used to estimate the quantity of mixing water in kg/m<sup>3</sup> according to the maximum size of used aggregates and the desired slump (25-50) mm, (50-75) mm, (75-100) mm respectively.

$$y_1 = -0.0119 x_1^3 + 0.7074 x_1^2 - 14.208 x_1 + 265.36, \quad R^2=1 \quad (1)$$

$$y_1 = 0.0004 x_1^3 + 0.0489 x_1^2 - 3.2176 x_1 + 215.82, \quad R^2=1 \quad (2)$$

$$y_1 = 0.0004 x_1^3 + 0.0489 x_1^2 - 3.2176 x_1 + 221.82, \quad R^2=1 \quad (3)$$

where  $y_1$  is mixing water weight in kg/m<sup>3</sup>,  $x_1$  is maximum size of the used aggregates in mm

### 2.1.2 Entrapped air

From the same table 4.3.4 in ACI 211.4R.93 [10] (Table 2) and with the same manner by plotting the tabular data, Equation (4) was developed to estimate the percentage of entrapped air in fresh concrete according to the maximum size of used aggregates.

**Table 2 First estimate of entrapped air of fresh concrete**

Aggregate Size (mm)	Entrapped air % when using HRWR
9.5	2.5
12.5	2
19	1.5
25	1

Equations (4) could be used to estimate the percentage of entrapped air according to the maximum size of used aggregates.

$$y_2 = - 0.0006 x_1^3 + 0.0385 x_1^2 - 0.7194 x_1 + 6.6549, \quad R^2=1 \quad (4)$$

where  $y_2$  is the percentage of entrapped air in fresh concrete.

### 2.1.3 volume of dry coarse aggregates

From table 4.3.3 named with the Recommended volume of coarse aggregate per unit volume of concrete in ACI 211.4R.93 [10] (Table 3), a graph for this tabular data was plotted to develop equation (5) which gives the capability for estimating the dry volume of coarse aggregates according to the maximum size of used aggregates.

**Table 3 Recommended volume of coarse aggregate per unit volume of concrete**

Optimum coarse aggregate contents for nominal max sizes of aggregates to be used with sand with F.M of 2.5 to 3.2				
Nominal maximum size (mm)	9.5	12.5	19	25
Fractional volume of oven dry rodded coarse aggregate	0.65	0.68	0.72	0.75

Equations (5) could be used to estimate the fractional volume of oven-dry rodded of the used coarse aggregate.

$$y_3 = - 2 * 10^{-5} x_1^3 - 0.0012 x_1^2 + 0.0297 x_1 + 0.4614 , \quad R^2 = 1 \quad (5)$$

where  $y_3$  is the fractional volume of oven dry rodded coarse aggregates in  $m^3$ .

### 2.1.4 water per cement ratio $W / (C + P)$

From table 4.3.5 (b) named with Recommended maximum  $w / (c + p)$  ratio for concretes made with HRWR in ACI 211.4R.93 [10] (table 4), a graph for this tabular data was plotted to develop four equations (6,7,8,9) which give the capability for estimating the  $w / (c + p)$  according to the field stress ( $f_{cr}$ ).

**Table 4 Recommended maximum  $w / (c + p)$  ratio for concretes made with HRWR**

Field strength $f_{cr}$ MPa		$W / (C + P)$			
		Maximum size of coarse aggregate (mm)			
		9.5	12.5	19	25
48.32	28 - days	0.5	0.48	0.45	0.43
55.12		0.44	0.42	0.4	0.38
62		0.38	0.36	0.35	0.34
68.9		0.33	0.32	0.31	0.3
75.79		0.3	0.29	0.27	0.27
82.68		0.27	0.26	0.25	0.25

Equations (6,7,8,9) could be used to estimate the  $W / (C + P)$  according to the required field stress of concrete after 28 days ( $f_{cr}$ ).

$$y_4 = -0.435 \ln(0.9x_2) + 2.1811, \quad R^2 = 0.99 \quad (6)$$

$$y_4 = -0.411 \ln(0.9x_2) + 2.0669, \quad R^2 = 0.99 \quad (7)$$

$$y_4 = -0.383 \ln(0.9x_2) + 1.9332, \quad R^2 = 0.99 \quad (8)$$

$$y_4 = - 1.34 \ln(0.9x_2) + 1.7165, \quad R^2 = 0.99 \quad (9)$$

where  $y_4$  is the  $W / (C + P)$ ,  $x_2$  is the required field stress ( $f_{cr}$ ).

## 2.2 Mix design procedures

The ACI highlighted the procedures of high strength concrete as the following sequences.

At the beginning the designer should specify the compressive strength of the HSC mix as the code recommended by the equation (10);

$$f_{cr} = \frac{f_c + 9.646}{0.9}, \quad (10)$$

Where  $f_c$  is the designed strength,  $f_{cr}$  is the field strength in (MPa).

Then the desired slump of fresh concrete should be specified by the designer according to the recommendation of the code in the table it's recommended to add a high range water reducer (HRWR) to adequate the amount of water at mix proportions. Accordingly, the recommended initial slump for this adding should range from 25 mm to 50 mm.

After that, the mixing water of the mix should be estimated using equations (1 – 3). Moreover, mixing water could be estimated if the used fine aggregates` voids were equal to 35%. But if the voids were not equal to 35 %, the quantity of mixing water should be adjusted by the following equation;

$$y_5 = (v - 35) * 4.744 , \quad (11)$$

where  $y_5$  is the adjustment of mixing water in (Kg/m<sup>3</sup>) and  $v$  is the percentage of voids at fine aggregates %.

Moreover, the fine aggregates voids could be calculated by the following equation;

$$v = \left(1 - \frac{x_3}{x_4 * 1000}\right) * 100 , \quad (12)$$

Where  $x_3$  is the oven dry unit rodded weight of fine aggregate in (Kg/m<sup>3</sup>) and  $x_4$  is bulk specific gravity factor of fine aggregates.

Also, the entrapped air equation (4) could be used to estimate the percentage of entrapped air according to the maximum size of used aggregates. Moreover, equations (6 – 9) could be used to calculate  $w / (c + p)$  depending on the maximum size of used aggregates. Accordingly, the volume of cement and pozzolan material could be calculated. Moreover, the oven-dry coarse aggregate volume could be used using equation (5).

Referring to the volume of fine aggregates and after estimating the volume of cement, coarse aggregates, water and entrapped air, the volume of fine aggregates could be calculated using equation (13).

$$v_{f.agg.} = 1 m^3 - (v_{cement} + v_{C.agg.} + v_{water} + v_{air}), \quad (13)$$

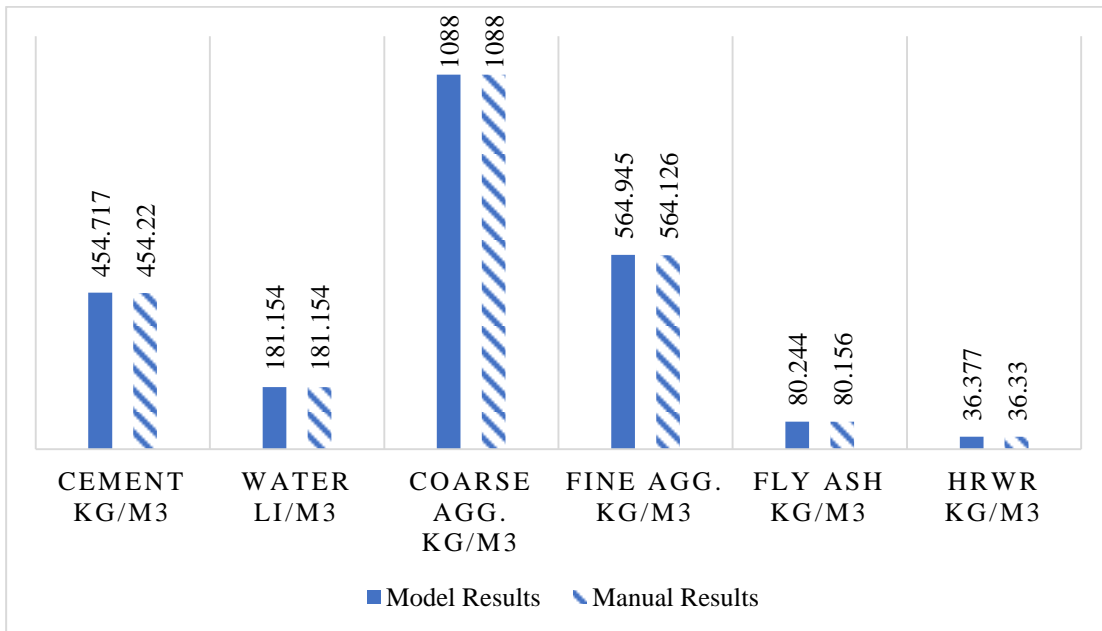
Pointing to the weight of fly ash, it could be calculated as a percent of cement. As an example, fly ash of Class (F) is recommended to replace 15 percent of cement weight. After that, the ingredients` weight of the mix could be adjusted. Also, for the high range water reducer (HRWR) it could be estimated as 8 % of cement weight.

### 2.3 Verification of the model

To verify the model of equations, a high-strength concrete mix with 70 MPa was designed manually and also it was designated by the model of equations. After that, the results from manual design are compared with results obtained by the model.

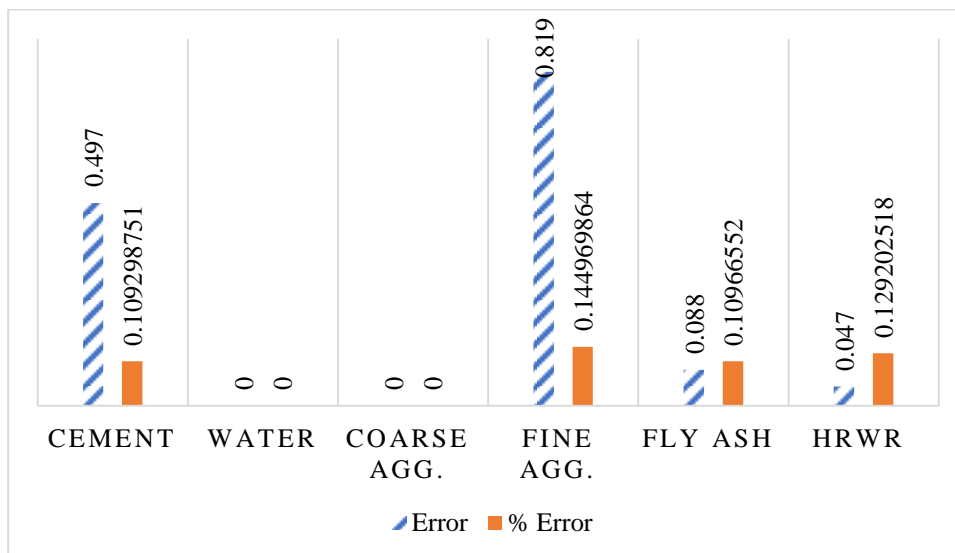
### 3. Results and Discussion

The obtained results from manual design and model design are shown in figure (1), the results are slightly the same and that shows that the accuracy of the model is high.



**Figure (1) Comparison between Model Results and Manual Results**

Moreover, the error of the program is too low. The maximum percentage error was about 0.14 % that considered a fair result. Also, figure (2) shows the difference between the model results and manual results moreover it shows the percentage error for this difference.



**Figure (2) shows the difference between the mode of equations and % error of this difference**

Also, using the model of equations instead of the manual design saves human energy and also it saves consumed time during the design process.

## **4. Conclusion**

Tabular data and charts that related to the design of high strength concrete mix design in ACI 211.4R.93 [10] were converted to a model of equations with the aid of Microsoft excel sheets after tried various models and choose the most accurate model that adequately represents the data based on the regression coefficient and its predictive capability to optimize the high strength concrete mixture design in terms of time, effort and cost. Moreover, the model was validated by design a specified high strength concrete mix with the model and manually. The maximum percentage error was about 0.14% and it was considered a fair result.



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