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COMPARATIVE STUDY BETWEEN DIFFERENT ORIFICE SHAPES IN TERMS OF THE EFFECT OF PRESSURE HEAD ON THE DISCHARGE COEFFICIENT

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ملخص البحث:
تعتبر دراسة سلوك سريان المياه خلال الفتحات ذات أهمية كبيرة جداً في مجالات عديدة مثل أجهزة قياس التدفق والأجهزة الطبية والسدود ويعتبر معامل التصرف خلال الفتحات من أهم المؤثرات التى يجب دراستها بعناية كبيرة حيث أنه يعبر عن مدى كفاءة السريان ويلعب دوراً مهما في التصميمات الهيدروليكية. وفي هذا البحث تم عمل دراسة معملية حول تأثير تغير الضغط الواقع عموديا على مركزالفتحات على معامل تصرف المياه خلالها لأشكال هندسية مختلفة ومقارنتها ببعضها.

تم اقتراح واستخدام أربعة أشكال للفتحات وهي الشكل الدائري والمربع والمثلث متساوي الأضلاع والمستطيل كما تم اختبار أربعة قيم مختلفة للضغط الرأسي وهي (35 سم و 40 سم و 45 سم و50 سم) وهي (9.25 مم و 7.5 مم و 6.75 مم و 4 مم و 2 مم)، وجدير بالذكر أنه تم استخدام أربعة مساحات مختلفة وهي: (78.54 مم^2 و 176.71 مم^2 و 314.16 مم^2 و 490.87 مم^2) وهذه المساحات هي المكافئة لدوائر أقطارها (10 مم و 15 مم و 20 مم و 25 مم) على التوالي. و قد تم عمل 16 شريحة مصنوعة من الاكريلك الشفاف (أبعاد الشريحة: 7 سم x 7 سم) وتم قطع الفتحات بالليزر عليها.

ولإجراء هذه الاختبارات تم بناء نموذج معملي يتكون من خزان تجميع للمياه المتدفقة خلال الفتحة ويتم إعادتها مرة أخرى عن طريقة المضخة إلى الخزان العلوي والخزان العلوي تم تصميمه للمحافظة على منسوب المياه أثناء تدفقها خلال الفتحة بالإضافة إلى أنبوبتين داخل بعضهما تم وضعهما بشكل رأسي داخل الخزان العلوي بحيث تكون الأنبوبة الخارجية قابلة للحركة صعوداً وهبوطاً للتحكم في منسوب المياه والأنبوبة الداخلية مثبتة في قاع الخزان فوق فتحة دائرية نافذة إلى خزان التجميع وذلك لإعادة تدوير المياه الزائدة عن المنسوب المطلوب إلى خزان التجميع وتم وضع تجويف (مجرى) على فتحة مربعة على جدار الخزان العلوي وذلك لسهولة وضع وإزالة شرائح مختلفة السماكة تحتوي على فتحات مختلفة الأشكال والسماكات كما سبق تفصيله ولمعايرة التدفق تم استخدام وعاء مدرج وساعة إيقاف لمعايرة التدفق خلال الفتحات وذلك لمقارنته بالتدفق المحسوب نظريا من معادلة برنولي ومن ثم حساب معامل تصرف المياه خلال الفتحات.

وتوصلت الدراسة إلى أنه في حال ثبات مساحة الفتحة فإن زيادة نسبة ضغط عمود الماء الواقع رأسياً على الفتحة إلى سمكها تزيد قيمة معامل التصرف لوغارتمياً وفي جميع المقارنات التحليلية التي تمت في هذه الدراسة تم ملاحظة أن الفتحات الدائرية تعطي أعلى قيم لمعامل التصرف مقارنة بجميع الاشكال الأخرى تليها الفتحات المثلثية متساوية الأضلاع ثم الفتحات المربعة وأخيراً لوحظ أن الفتحات المستطيلة تعطي أقل قيم لمعامل التصرف.

ABSTRACT
The purpose of this paper is to experimentally investigate the effect of pressure head above the orifice centerline on the discharge coefficient for water flow though different orifice shapes. To achieve this objective, a laboratory physical model was constructed and operated in the Hydraulic Laboratory of the Faculty of Engineering, Al- Azhar University. Orifice shapes have been investigated were: circle, square, rectangle and equilateral triangle.

In order to study these parameters of orifice flow, 16 orifice plates (7cm. x 7cm.) each was manufactured and installed in the designed model. Bernoulli's equation was used as a theoretical basis for calculating the discharge coefficient.

The study concluded that the effect of pressure head above the orifice centerline on the

discharge coefficient was investigated and it was concluded that when the ratio (h/d) increases, the discharge coefficient decreases in logarithmic manner. Additionally, in the context of comparing the four tested orifice shapes, it was found that circular shape always gives a higher value of the C_d followed by the equilateral triangular shape then the square shape and finally the rectangular shape gives the least value of the discharge coefficient.

1. INTRODUCTION

The orifice flow performance has a significant importance in several fields, such as flow measurement devices, dams, medical instrumentations, piping systems, and fuel injection into combustion engines [1]. The coefficient of discharge (C_d) of an orifice is very substantial parameter as it is considered the efficiency of the orifice flow. Also, Orifice flow have been used extensively for flow control and flow restriction.

2. RESEARCH OBJECTIVES

The objective of this study is to investigate the effect of pressure head above the orifice centerline on the discharge coefficient (C_d) and presenting analytical comparison between four (4) different orifice shapes (circle, square, equilateral triangle and rectangle).

3. COEFFICIENT OF DISCHARGE

The discharge coefficient is a dimensionless number defined as the ratio of the amount of water discharged through an orifice to the amount theoretically possible at that flow conditions. Discharge coefficient primarily illustrates what proportion the particular flow defers from the ideal flow. A smaller value of discharge coefficient shows that the actual discharge is smaller compare to the theoretical or ideal value and vice versa.

4. ORIFICE

An opening, in a tank, through which the liquid flows out is known as orifice. This hole or opening is termed an orifice, with the condition that the liquid level of the upstream side is always above the top of the orifice. The use of an orifice is the flow measurement. Also, an orifice can be exist in the vertical side of a tank or in the base. Orifices can be utilized in an extensive range of applications, as these plates are existing in a variety of materials and in several designs, such as concentric, eccentric, or segmental. Additional advantage is that the orifice plate can be badly worn or damaged, yet it will still offer a sensibly repeatable output, albeit expressively imprecise. Another very useful feature of the orifice type installation is the capability to service or replace the readout or transmitter without the need to take out the orifice or to interfere the flow process [2]**.**

5. APPLING BERNOULLI'S EQUATION TO THE EXPERIMENTAL MODEL

Bernoulli's equation was applied between two points ***Figure 1*** where point (1) is being somewhere at the free water surface and point 2 is being at the vena contracta. It can be assumed that the velocity and the gauge pressure at point (1) are zeroes as it is subjected to atmospheric pressure. In addition, the gauge pressure at point (2) is also zero because water is discharged into the atmosphere.

$$\frac{P_1}{\gamma} + \frac{{v_1}^2}{2g} = \frac{P_2}{\gamma} + \frac{{v_2}^2}{2g} + h_L \qquad \textbf{Eq. 1}$$

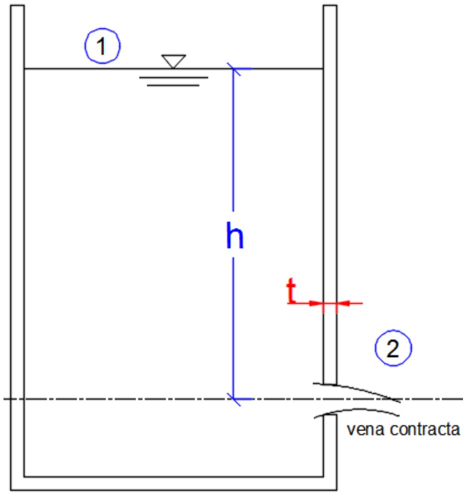

Figure 1: Definition sketch of the two points in Bernoulli's equation

Two separate coefficients are the components of the discharge coefficient, C_c and C_v. C_c is the coefficient of contraction and should be calculated at the vena contracta of the water jet (Water jet is a continuous stream of water that comes out or flows out of an orifice).

Because of the phenomenon of vena contracta, the actual area of flow from an orifice is smaller than the area of the orifice itself. ***Vena contracta can be defined as*** the ratio of the flow area at the vena contracta to the area of the orifice. [3]**.**

These two areas may be jointed together by the formula:

$$A = C_c A_o \qquad \textbf{Eq. 2}$$

$$v_2 = \sqrt{2g(h - h_L)} \qquad \textbf{Eq. 3}$$

Where A is the jet cross-sectional area at the vena contracta, Cc is the coefficient of contraction and A_o is the orifice area. [4]**.**

Although it is not easy to determine precisely the location of the vena contracta, some researchers investigated that for a circular orifice, the vena contracta is located nearly half diameters downstream the orifice plate inner face. ***(Brater et al. 1969).***

The coefficient of velocity, C_v represents head loss that can be generated as water moves out of the tank through an orifice. Taking the origin form of the energy equation between two points as shown in *Error! Reference source not found.*, point 2 is located at the vena contracta while point 1 is being some point inside the tank at the same level of the orifice, the following equation can be written as:

$$\frac{P_1}{\gamma} + \frac{{v_1}^2}{2g} = \frac{P_2}{\gamma} + \frac{{v_2}^2}{2g} + h_L \qquad \textbf{Eq. 4}$$

Coefficient of velocity have been investigated experimentally in one of the previous studies for (2– 6) cm diameter orifices and for heads between (0 – 30) m. The investigation concluded that C_d varies from 0.951 to 0.993 and as the head decreases, the velocity coefficient slightly decreases. ***(Smith and Walker 1923).***

6. STANDARD EQUATION FOR DISCHARGE CALCULATION

Discharge can be defined the product of the velocity and area of flow at the vena contracta and can be written as:

$$Q = Av_2 \qquad \textbf{Eq. 5}$$

By substituting in *Error! Reference source not found.* from equations Error! Reference source not found. , and *Error! Reference source not found.* , the following equation can be written:

$$Q = C_c C_v A_o \sqrt{2gh} \qquad \textbf{Eq. 6}$$

By combining the two coefficients C_c and C_v into one coefficient C_d, standard equation for estimating a flow through small orifice discharging the water into the atmosphere under constant head can be written as:

$$Q = C_d A_o \sqrt{2gh} \qquad \textbf{Eq. 7}$$

(Small orifice means that we can neglect the pressure distribution across the orifice, i.e., diameter of the orifice is low compared to the head).

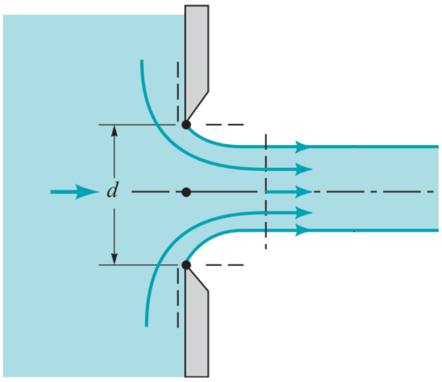

Figure 2: Vena contracta effect for a sharp-edged orifice. [7]

7. DIMENSIONL ANALYSIS

In this study, the discharge coefficient (C_d) is the dependent variable. It can be stated as a function of all other independent variables as follows:

$$C_d = f(\rho, v, d, \mu, t, h, g) \qquad \textbf{\textit{Eq. 8}}$$

Where, ρ is the density of fluid, g is the gravity acceleration, μ is the dynamic viscosity, v is the velocity, d is the orifice diameter, t is the orifice thickness and h is the pressure head.

No of variables (n) = 7 variables.

No. of fundamental dimensions (m) = 3 dimensions (L, M, and T).

No. of π - terms = n – m = 7 – 3 = 4.

No. of repeating variables = 3.

Selected repeating variables: g, d, and ρ

According to Buckingham Pi-theorem, we get the following equation:

$$C_d = \mathrm{f}\left(\frac{h}{d}, \frac{t}{d}, \frac{v}{\sqrt{gh}}, \frac{\mu}{\rho h\sqrt{gh}}\right) \qquad \textbf{Eq. 9}$$

By taking into confederation $\underline{\pi_1}$ and $\underline{\pi_2}$ (geometric factors), we get following equation:

$$C_d = \mathrm{f}\left(\frac{h}{d}, \frac{t}{d}\right) \qquad \textbf{Eq. 10}$$

Where,

$\frac{h}{d}$ *is the head to diameter ratio.*

$\frac{t}{d}$ *is the thickness to diameter ratio.*

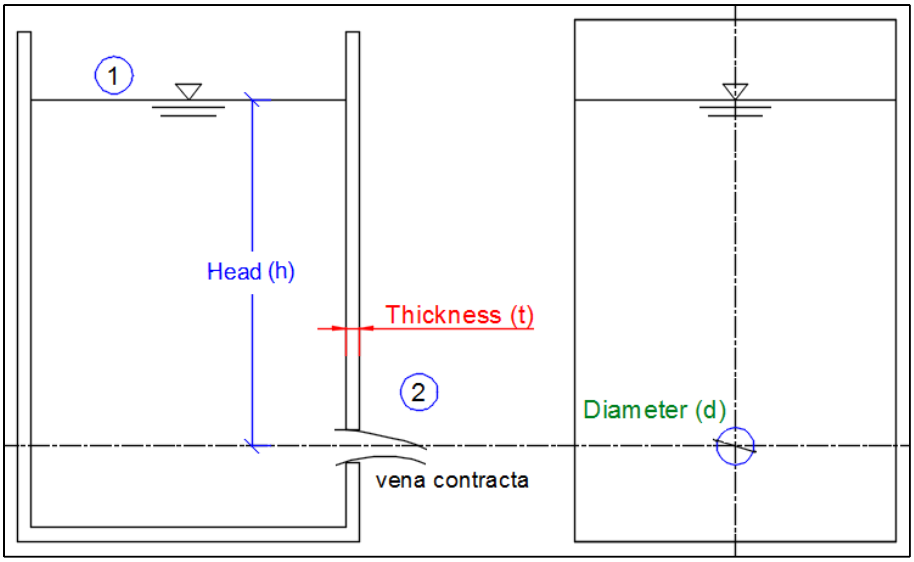

Figure 3 : Geometric parameters taken into consideration in dimensional analysis

8. PHYSICAL MODELING

In order to achieve the aim of the experimental work, a laboratory physical model was constructed Figure ***4.*** The model mainly consists of two tanks, lower and upper. Both tanks are formed of transparent acrylic plastic to be easy in laser cutting and to allow visual observation of the water flow.

The lower tank (collecting tank) was designed to collect water passing through the orifices and recycle it into the upper tank. To do that, a centrifugal pump was installed to raise water from the lower to the upper tank. The upper tank was designed to keep constant water head above the orifice.

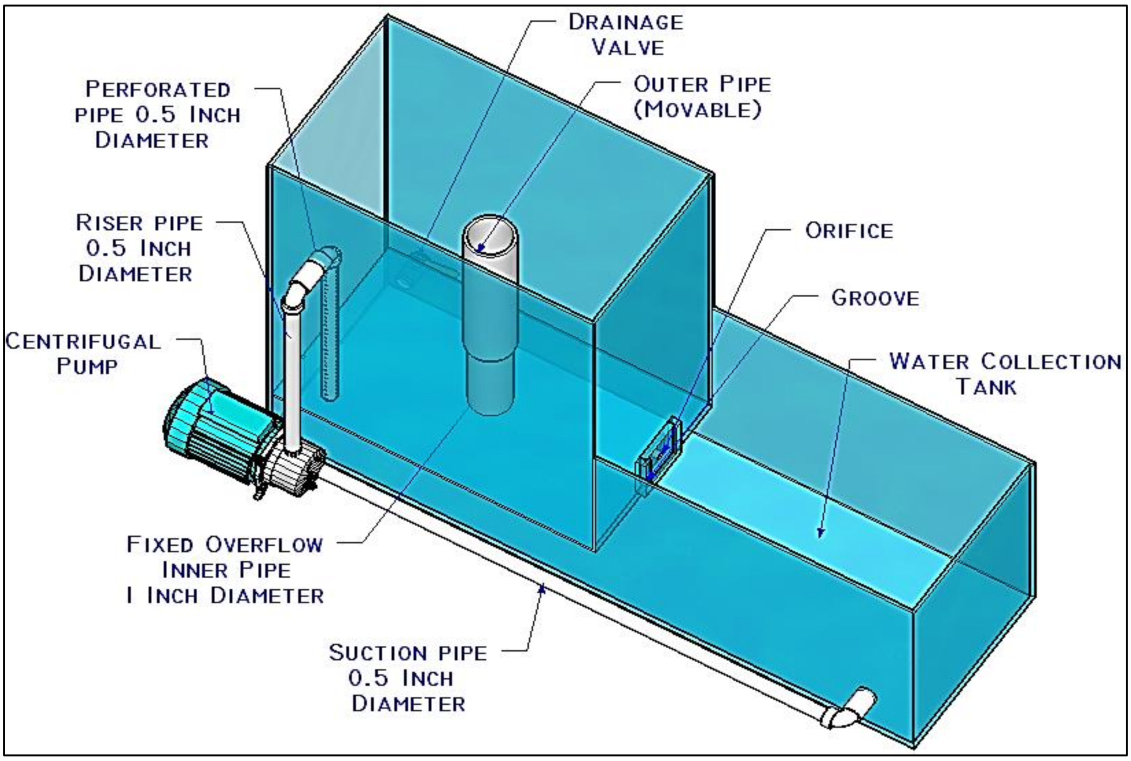

Figure 4: Model isometric definition sketch

To control water head, two concentric pipes were installed in the middle of the upper tank in order to return water back to the lower tank. The inner pipe was designed to be fixed on the bottom of the tank and the outer pipe was installed to be moveable in order to control water head above the orifice.

In order to study the effect of orifice thickness on the discharge coefficient, 24 orifice plates (7cm. x 7cm.) each was manufactured from transparent acrylic plastic. Orifices were cut using laser technology. The shown groove ***Figure 5*** was designed to be fixed into the tank wall to facilitate changing the orifice plates.

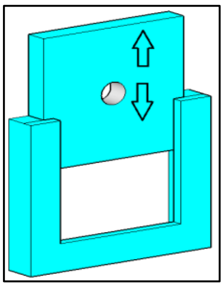

Figure 5: Groove to change the orifice plate

In order to supply the system with the needed flow rate, a 0.25 HP centrifugal pump ***Figure 6*** was installed to withdraw water from the lower to the upper tank. The function of the pump is to substitute the water discharged from the orifice to keep constant head pressure above the orifice.

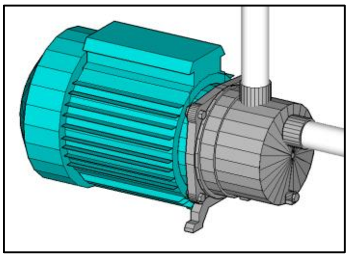

Figure 6: Centrifugal Pump (0.25 HP)

The discharge passing through the orifices was calibrated using a graduated container and stop watch. The graduated container was used to collect passing water in a certain time.

9. DISCHARGE MEASUREMENT

The discharge was calculated by measuring the water volume discharged out of an orifice in a known time. This volume was determined by collecting water in a graduated container and the discharge was calculated by dividing the collected volume by the time. Each run was carried out three (3) times and the average discharge was taken.

10. ANALYSIS AND DISCUSSIONS

This study was done to scrutinize what effect, if any, there would be on the discharge coefficient (C_d) when the pressure head above the orifice centerline (h) was changed. To achieve this endeavor, the relationship between head to diameter ratio (h/d) and the (C_d) was plotted in which the ***X-axis*** represents the ratio (h/d) and the ***Y-axis*** represents the discharge coefficient C_d. These experiments were tested at four (4) orifice areas with constant orifice thicknesses (9.25 mm). Four (4) figures were plotted with (16) measured C_d

Figure 7 to ***Figure 10*** and ***Table 1*** to ***Table 4.***

The obvious phenomenon from the data representation was the decrease of the discharge coefficient (C_d) as the ratio (h/d) increase. The observed details were the following:

Figure 7 to ***Figure 10*** were plotted by dividing the head (h) by orifice diameter (d) ***Table 1*** to ***Table 4.*** The diameter was constant for each single plot. From these plots, apparent was the following:

- ***At orifice area = 78.54 mm²: Figure 7*** shows that the C_d is inversely proportional to (h/d). As (h/d) increases from 35 to 50, the discharge coefficient C_d decreases about (7.42% for circular orifice, 6.97% for equilateral rectangular orifice, 6.40% for square orifice, and 7.07% for rectangular orifice).
- ***At orifice area = 176.71 mm²: Figure 8*** shows that the C_d is inversely proportional to (h/d). As (h/d) increases from 23.33 to 33.33, the discharge coefficient C_d decreases about (3.28% for circular orifice, 5.35% for equilateral rectangular orifice, 5.66% for square orifice, and 5.20% for rectangular orifice).
- ***At orifice area = 314.16 mm²: Figure 9*** shows that the C_d is inversely proportional to (h/d). As (h/d) increases from 17 to 25, the discharge coefficient C_d decreases about (2.41% for circular orifice, 2.60% for equilateral rectangular orifice, 1.95% for square orifice, and 1.80% for rectangular orifice).

- ***At orifice area = 490.87 mm²: Figure 10*** shows that the C_d is inversely proportional to (h/d). As (h/d) increases from 14 to 20, the discharge coefficient C_d decreases about (1.02% for circular orifice, 1.71% for equilateral rectangular orifice, 1.21% for square orifice, and 1.23% for rectangular orifice).

Table 1: Ratio "h/d" vs. "C_d" (at A = 78.54 mm^2)

Discharge Coefficients for all tested shapes at : Orifice Area = 78.54 mm^2 (Diameter of Equivalent Circle = 10 mm)					
Head (cm)	h/d	C_d			
		Circular Orifice	Equilateral Triangular Orifice	Square Orifice	Rectangular Orifice
35	35	0.539	0.531	0.516	0.509
40	40	0.524	0.515	0.503	0.499
45	45	0.515	0.510	0.499	0.489
50	50	0.499	0.494	0.483	0.473

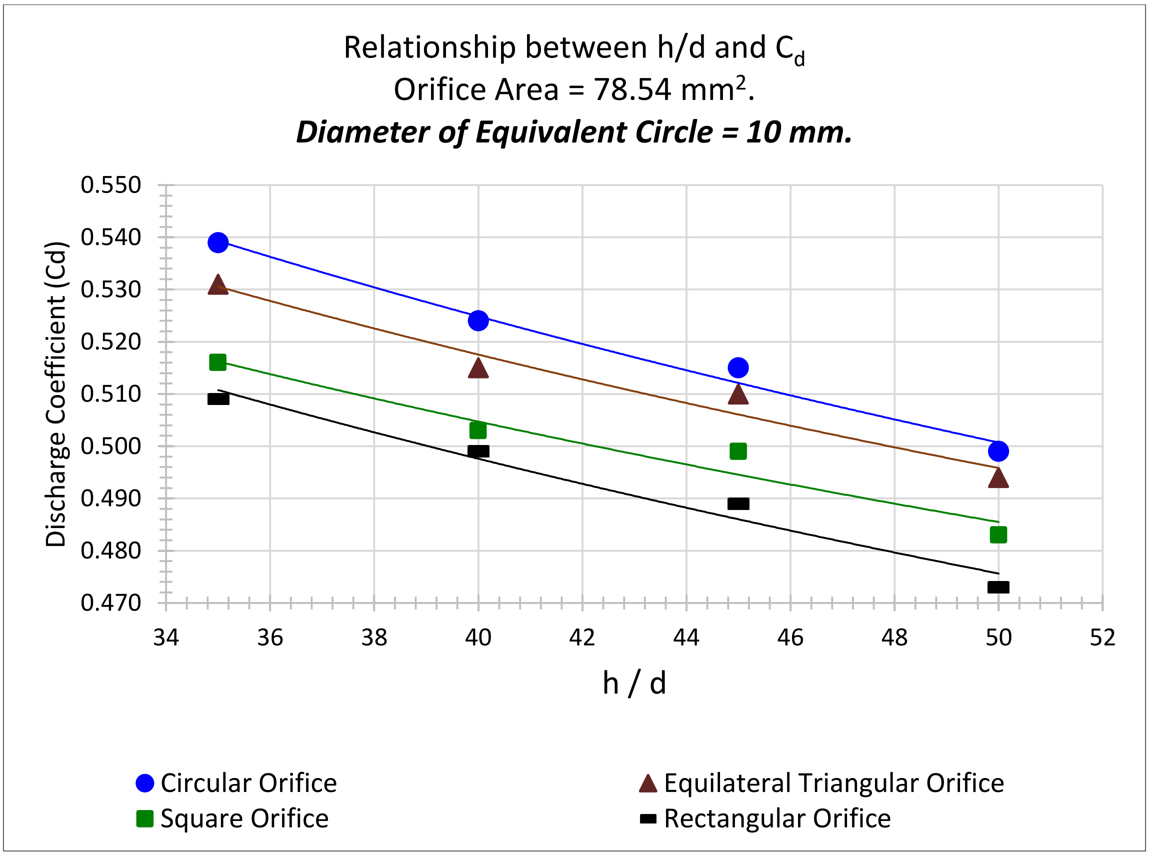

Figure 7: Relationship between "h/d" and "C_d" (at A = 78.54 mm^2)

Table 2: Ratio "h/d" vs. "C_d" (at A = 176.71 mm^2)

Discharge Coefficients for all tested shapes at : Orifice Area = 176.71 mm^2 (Diameter of Equivalent Circle = 15 mm)					
Head (cm)	h/d	C_d			
		Circular Orifice	Equilateral Triangular Orifice	Square Orifice	Rectangular Orifice
35	23.33	0.549	0.542	0.530	0.519
40	26.67	0.544	0.533	0.519	0.508
45	30	0.533	0.521	0.509	0.500
50	33.33	0.531	0.513	0.500	0.492

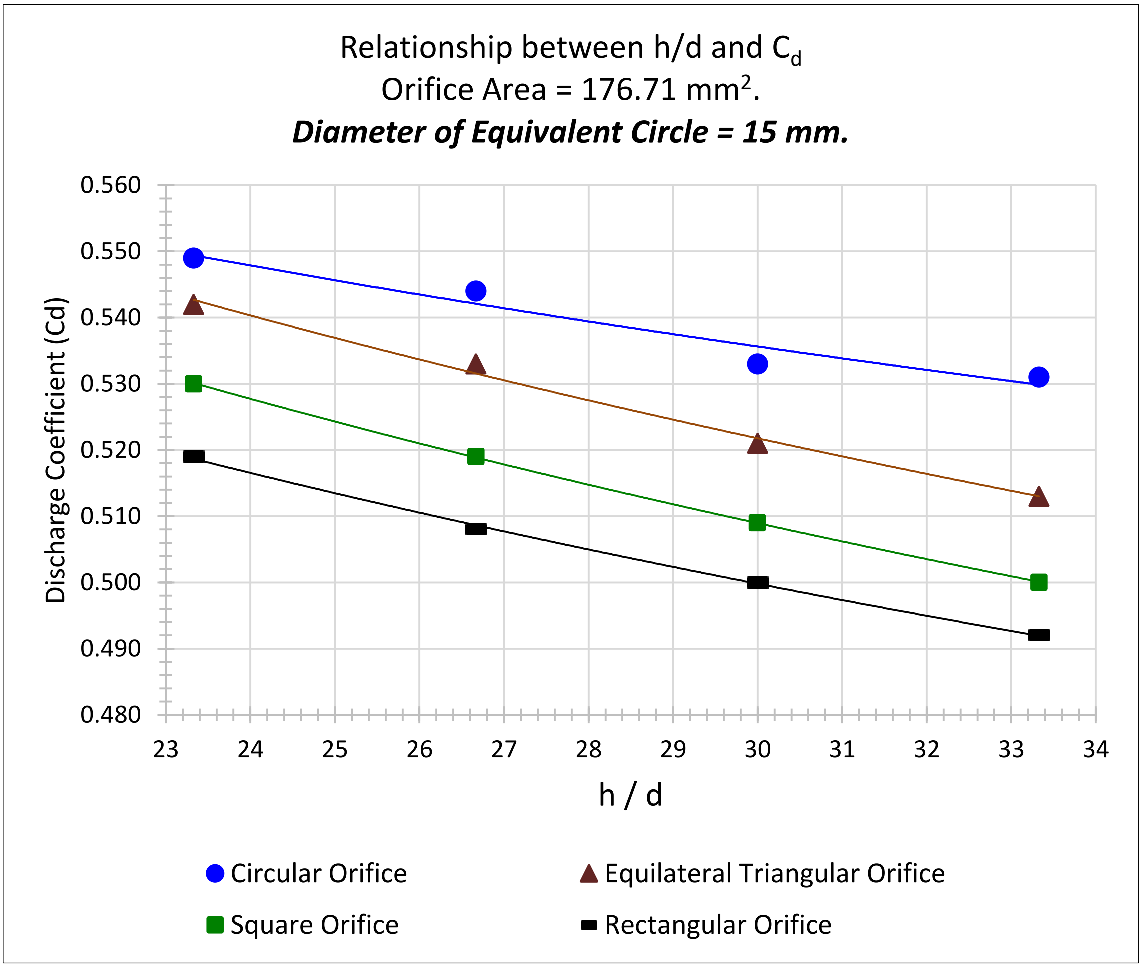

Figure 8: Relationship between "h/d" and "C_d" (at A = 176.71 mm^2)

Table 3: Ratio "h/d" vs. "C_d" (at A = 314.16 mm^2)

Discharge Coefficients for all tested shapes at : Orifice Area = 314.16 mm^2 (Diameter of Equivalent Circle = 20 mm)					
Head (cm)	h/d	C_d			
		Circular Orifice	Equilateral Triangular Orifice	Square Orifice	Rectangular Orifice
35	17.5	0.579	0.578	0.565	0.557
40	20	0.576	0.575	0.562	0.553
45	23	0.572	0.570	0.559	0.550
50	25	0.565	0.563	0.554	0.547

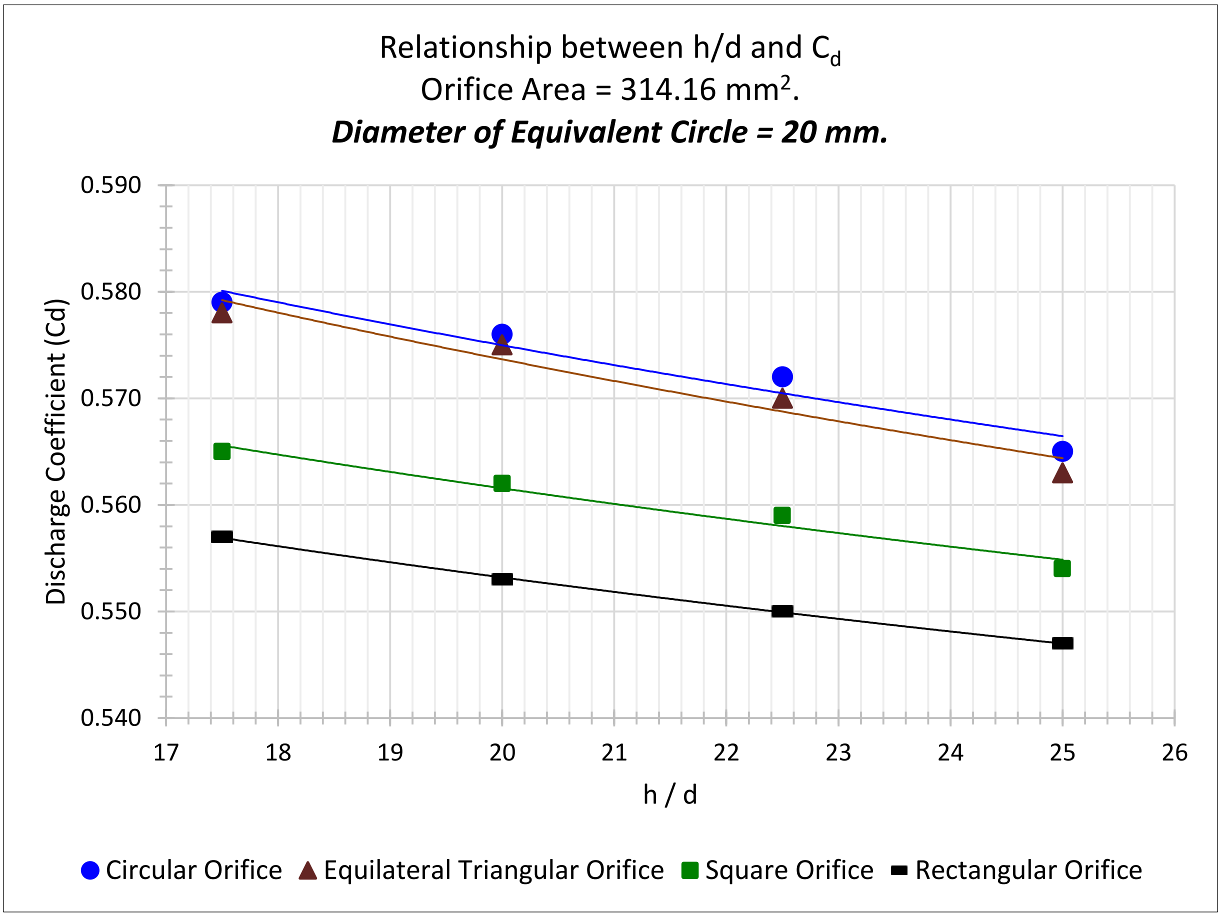

Figure 9: Relationship between "h/d" and "C_d" (at A = 314.16 mm^2)

Table 4: Ratio "h/d" vs. "C_d" (at A = 490.87 mm^2)

Discharge Coefficients for all tested shapes at : Orifice Area = 490.87 mm^2 (Diameter of Equivalent Circle = 25 mm)					
Head (cm)	h/d	C_d			
		Circular Orifice	Equilateral Triangular Orifice	Square Orifice	Rectangular Orifice
35	14	0.596	0.589	0.580	0.572
40	16	0.595	0.586	0.579	0.571
45	18	0.592	0.583	0.576	0.568
50	20	0.590	0.580	0.574	0.567

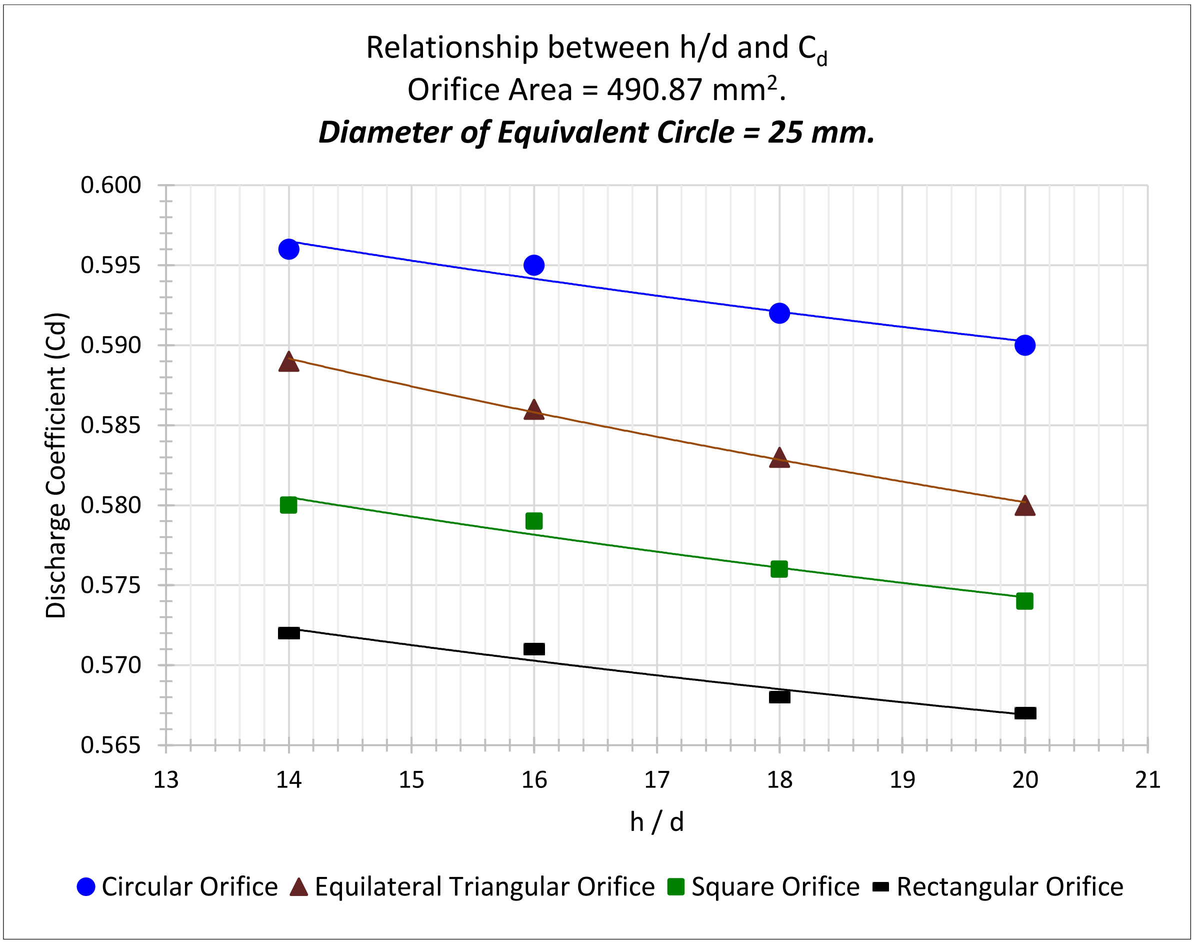

Figure 10: Relationship between "h/d" and "C_d" (at A = 490.87 mm^2)

11. CONCLUSIONS

- It is clear from the analysis that the rate of decreasing the discharge coefficient with the increase of (h/d) decreases gradually as the orifice area increases or in other words as the velocity of the flow decreases.
- The circular orifice shape was found to give the higher values of the discharge coefficient (c_d) as compared to all tested shapes. On the other hand, the rectangular orifice shape was found to give the lowest values of the discharge coefficient (c_d).

12. RECOMMENDATIONS

Some recommendations were obtained from the above outcomes:

- More deep investigations on the effect of pressure head above the orifice centerline on the discharge coefficients (C_d) are recommended to be carried out to extend the presented results.
- Future studies are recommended to use non-conventional orifice shapes with the same parameters tested here to determine if the discharge coefficients (C_d) vary in the same pattern or not.

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