

# An improved model for multi-constellations GNSS single frequency precise point positioning

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#### الملخص العربى

## ABSTRACT

We develop a new single frequency PPP model, which uses constrained between- satellitesingle-difference ionosphere-free (BSSD IF) quasi-phase with quad GNSS single frequency observations. To overcome of the limitations of poor satellite geometry, especially in urban areas, multi-constellation GNSS observations such as Galileo and BeiDou observations are added to the GPS and GLONASS observations to enhance the satellite geometry and increase measurement redundancy. Furthermore, to completely remove the receiver code biases, loosely-coupled satellite single difference technique is applied on the multi-constellation GNSS single frequency observations. In addition, to overcome the limitation of the ionospheric impact, which represents the main source of error in single frequency PPP, ionosphere-free GNSS code and phase observations (known as a quasi-phase observations model) is employed. To overcome the rank deficiency problem due to the existence of ambiguity parameters, additional observations are added to constrain the ambiguity parameters working as a priori observations which are selected as half of code and phase difference. The developed model is compared with the traditional undifferenced with ionospheric error corrected by the final Global Ionospheric Model (GIM). It is shown that using of BSSDIF-PPP technique enhances the positioning accuracy by 35%, 30%, 34% 30% and 29% compared with the GIM based PPP model for the GPS only, GPS/GLONASS, GPS/Galileo, GPS/BeiDou and GNSS combinations, respectively after one hour of GNSS data processing. However, after six hours of GNSS data processing comparable positioning accuracy can be obtained from all developed single frequency models.

**KEY WORDS:** Single frequency PPP, BSSD, Quasi-Phase, GPS, GNSS, GLONASS, Galileo, Beidou.

## **1. INTRODUCTION**

Commonly, precise point positioning (PPP) uses dual-frequency GPS/GLONASS observations and can obtain positioning accuracy comparable to that of differential positioning in both static and kinematic modes. However, PPP is featured with a relatively long convergence time to achieve sub-decimeter positioning accuracy, which is mainly attributed to the un-calibrated receiver biases and poor satellite geometry. In addition, dual-frequency GNSS systems are expensive and may not provide a cost-effective solution in many instances. The use of low-cost single-frequency GNSS receivers, on the other hand, is limited by the effect of ionospheric delay. As a result, to obtain a cost effective precise PPP solution, a new single frequency PPP model should be developed taking into consideration the current limitations of PPP techniques.

The significance of using multi-constellation GNSS is mainly noticed in challenging environment, such as urban areas, where the signals are either partially blocked by urban obstacles or contaminated by multipath interference. In addition, measurements from multiple GNSS constellations not only improve the satellite geometry, but also increase the redundancy, which in turn improve the positioning accuracy and convergence time. However, the additional GNSS observations introduce additional biases such as intersystem biases, which can be treated as additional unknowns in the estimation filter. The drawback of this strategy is that the number of unknowns will be increased by one for each GNSS system added. The minimum number of satellites for the basic combined GNSS positioning solution will be (3+j), where j is the number of systems used representing the additional receiver clock unknowns to the filter. For GNSS applications in dense areas, increasing the number of unknowns might be critical in obtaining a good solution. In addition, further un-calibrated GNSS biases such as receiver and satellites differential code biases will be added, which, unless properly handled, degrade the positioning accuracy. The contribution of the additional observations from a particular constellation to the existing GPS observations is mainly based on the number of satellites from that constellation and the satellite geometry enhancement. A minimum of two visible satellites is required from a particular constellation to contribute to the positioning solution considering the additional receiver clock unknown term added for each constellation. Due to the relatively large number of visible GLONASS satellites at present, the additional GLONASS observations improve the PPP positioning accuracy and convergence time (Choy et al. 2013 and Abd Rabbou and El-Rabbany 2015). On the other hand, the contribution of adding Galileo observations to those of GPS can be considered marginal due to the limited number of Galileo satellites (e.g., Píriz et al. 2008; Montenbruck et al. 2011; Steigenberger et al. 2011; Abd Rabbou and El-Rabbany 2015).

This research aims to develop a single frequency PPP model, which combines the observations of all current GNSS constellations, including GPS, GLONASS, Galileo and Beidou. The developed model uses constrained between satellite single difference quasiphase GNSS observations. The developed model is compared with the traditional undifferenced and BSSD models with ionospheric error corrected by the final Global Ionospheric model (GIM) and the undifferenced quasi-phase GNSS observation model. The final precise products of the International GNSS Service multi-GNSS experiment (IGS-MEGX) network are used to account for the GNSS satellite orbit and clock errors (Montenbruck et al., 2014). The ionospheric delay is largely corrected through the IGS global ionosphere maps (GIM) model (Schaer et al., 1998). The hydrostatic and wet

components of the tropospheric zenith path delay are modelled through the UNB3 model. All remaining errors and biases are accounted for using existing models as shown in Kouba (2009).

#### 2. SINGLE FREQUNECY BSSD IF PPP MODEL

The basic GNSS observation equations for single-frequency pseudorange and carrierphase observations of a particular constellation can be written as:

$$P = \rho + cdt_r - cdt^s + T + I + c(d_r - d^s) + d_m + e \qquad (1)$$

$$\Phi = \rho + cdt_r - cdt^s + T - I + c(\delta_r - \delta^s) + \delta_m + \lambda(N + \varphi_r - \varphi^s) + \varepsilon$$
<sup>(2)</sup>

where P is GNSS pseudorange measurement in meters;  $\phi$  is the GNSS carrier phase measurement in meters;  $\rho$  is the true geometric range in meters from the antenna phase center of the receiver at reception time to the antenna phase center of the satellite at transmission time;  $dt_r$  and  $dt^s$  are the clock errors for receiver and satellite, respectively;  $d_r$  and  $d^s$  are frequency-dependent code hardware delay for receiver and satellite, respectively in seconds;  $\delta_r$  and  $\delta^s$  are frequency-dependent carrier phase hardware delay for receiver and satellite, respectively in seconds; e,  $\varepsilon$  are relevant system noise and unmodeled residual errors in meters; N is the integer ambiguity parameters in cycles;  $\varphi_r$  and  $\varphi^s$  are the initial phase biases at the receiver and the satellite, respectively in cycles;  $\lambda$  is the wavelengths of the carrier frequency in meters; c is the speed of light in vacuum in meter/second; T is the tropospheric delay component in meters; I is the ionospheric delay component in meters;  $d_m$  and  $\delta_m$  are the multipath interference component for code and phase, respectively in meters.

Current GNSS receivers take the GPS time system as a reference, which introduces an inter-system bias (ISB) when combining the observations of GPS and other GNSS system. In addition, the IGS-MGEX satellite clock corrections, which are used in this research, are referred to the GPS time and include the ionosphere-free linear combinations of the satellite code hardware delays of the various GNSS systems (Steigenberger et al. 2014). As such, using Equations 1 and 2 and considering the multi-GNSS observations, including GPS and the other GNSS systems (Subscripted by J), the mathematical model for single frequency GNSS PPP can be written as (Abd Rabbou and El-Rabbany, 2015):

$$P_{G} = \rho_{G} + c \left( dt_{r} + d_{G} \right) - c d\overline{t}_{G}^{s}$$

$$- c \left( B_{G}^{s} \right) + T_{G} + I_{G} + e_{G}$$

$$P_{J} = \rho_{J} + c \left( dt_{r} + d_{G} \right) - c d\overline{t}_{J}^{s} - c \left( B_{J}^{s} \right)$$

$$+ c \left[ ISB_{J} \right] + T_{J} + I_{J} + e_{J}$$

$$\Phi_{G} = \rho_{G} + c \left( dt_{r} + d_{G} \right) - c d\overline{t}_{G}^{s} - c \left( B_{G}^{s} \right) + T_{G} - I_{G}$$

$$+ \left[ \lambda \overline{N}_{G} + c \left( \delta_{G} - d_{G} \right) - c \left( \delta_{G}^{s} - d_{G}^{s} \right) \right] + \varepsilon_{G}$$

$$\Phi_{J} = \rho_{J} + c \left( dt_{r} + d_{G} \right) - c d\overline{t}_{J}^{s} - c \left( B_{J}^{s} \right) + T_{J} - I_{J}$$

$$+ \left[ \lambda \overline{N}_{I} + c \left( \delta_{I} - d_{I} \right) - c \left( \delta_{I}^{s} - d_{I}^{s} \right) \right] + \varepsilon_{I}$$

$$(6)$$

Where  $d\bar{t}^s$  is the satellite clock error lumped with the ionosphere-free differential code bias, which can be obtained from the IGS-MGEX; *B* is a bias term representing the combined effect of differential code bias of the satellite; *ISB* is the inter-system bias. In

our single-frequency GNSS model (Equation 3 to 6), the GPS receiver hardware delay  $d_G$ is lumped to the receiver clock error and the combined receiver clock bias is considered as a single unknown in our estimation filter.

Considering the opposite ionospheric impact in both code and phase measurements, the ionospheric error can be effectively removed by taking the average of the code and phase observations which is known as the quasi-phase observation (Yunck, 1993). The mathematical equations of the single frequency ionosphere-free observation can be written as

$$\bar{\Phi}_{G} = \frac{\Phi_{G} + P_{G}}{2} = \rho_{G} + c(dt_{r} + d_{G}) - cd\bar{t}_{G}^{s} - c(B_{G}^{s}) \qquad \bar{\Phi}_{J} = \frac{\Phi_{J} + P_{J}}{2} = \rho_{J} + c(dt_{r} + d_{J}) - cd\bar{t}_{J}^{s} - c(B_{J}^{s}) + T_{J} + c[ISB_{J}] + T_{G} + \frac{[\lambda \overline{N}_{G} + c(\delta_{G} - d_{G}) - c(\delta_{G}^{s} - d_{G}^{s})]}{2} + \bar{\varepsilon}_{G} \qquad (7) \qquad + \frac{[\lambda \overline{N}_{J} + c(\delta_{J} - d_{J}) - c(\delta_{J}^{s} - d_{J}^{s})]}{2} + \bar{\varepsilon}_{J}$$
(8)

where  $\hat{\Phi}_{G}$  and  $\hat{\Phi}_{J}$  are the quasi-phase observations for GPS and other GNSS observations. According to Equations (7) and (8), we can note that the ionospheric effect is totally removed. In addition, due to the small phase measurements noise which can be neglected, the noise characteristics of the quasi-phase  $\hat{\varepsilon}_{G}$  and  $\hat{\varepsilon}_{J}$ , are mainly contributed by half of the code measurements noise,  $e_G$  and  $e_I$ , respectively.

However, the mathematical model for the ionosphere-free described in Equations (8) and (9) presents a singularity model. To overcome the rank deficiency problem, additional observations should be added to constrain the ambiguity parameters working as a priori observations. Commonly, the code observations are used as a priori observations (Andrei et al, 2009 and Choy, 2013). In current research, the a priori observation is selected as the half of code and phase difference as follows,

$$\hat{\Phi}_{G} = \frac{\Phi_{G} - P_{G}}{2} = \frac{\left[\lambda \overline{N}_{G} + c(\delta_{G} - d_{G}) - c(\delta_{G}^{s} - d_{G}^{s})\right]}{2} - I_{G} + \hat{\varepsilon}_{G}$$
(9)  
$$\hat{\Phi}_{J} = \frac{\Phi_{J} - P_{J}}{2} = \frac{\left[\lambda \overline{N}_{J} + c(\delta_{J} - d_{J}) - c(\delta_{J}^{s} - d_{J}^{s})\right]}{2} - I_{J} + \hat{\varepsilon}_{J}$$
(10)

(10)

To completely remove the receiver biases and ionospheric error from the single-frequency observations, the ionosphere-free code and phase combinations described in Equations 7 to 10 can be applied on the satellite single difference observations. The mathematical for BSSD quasi-phase observation can be written as follows

$$\overline{\Phi}_{gnss}^{ij} = \frac{\Phi_{gnss}^{ij} + P_{gnss}^{ij}}{2} = \Delta \rho_{gnss}^{ij} - cd\overline{t}_{gnss}^{ij} - c(B_{gnss}^{ij})$$

$$+ \Delta T_{gnss}^{ij} + \frac{\Delta [\lambda \overline{N} - c(\delta_J^s - d_J^s)]_{gnss}^{ij}}{2} + \overline{\varepsilon}_{gnss}^{ij}$$
(11)

The a priori observation is selected as the half of the BSSD code and phase difference as follows

$$\hat{\Phi}_{gnss}^{ij} = \frac{\Phi_{gnss}^{ij} - P_{gnss}^{ij}}{2} = \frac{\Delta [\lambda \overline{N} - c(\delta_J^s - d_J^s)]_{gnss}^{ij}}{2}$$
$$-\Delta I_{gnss}^{ij} + \hat{\varepsilon}_{gnss}^{ij} \tag{12}$$

where the IGS GIM model is used for correcting the ionospheric term.

## **3. GNSS DATA PROCESSING**

The extended Kalman filter (EKF) is employed to estimate the unknown parameters, as detailed in Jekeli (2001). For the BSSD IF single frequency PPP techniques, the estimation state vector consists of the three GNSS receiver coordinates, namely latitude, longitude, and altitude, the wet tropospheric unknown and the float ambiguity parameters. The complete state vector for the BSSD IF model can be written as:

 $\delta x = [\delta \phi, \delta \lambda, \delta h, \Delta T_w, A_{1i}, \dots A_{ni}]$ (13)

where  $\delta\phi$ ,  $\delta\lambda$  and  $\delta h$  are the positioning errors in latitude, longitude and altitude  $T_w$  is the wet tropospheric component ; *A* is the float ambiguity term as described in Equations 11 and 12. For the standard single frequency GNSS PPP model, the GNSS observations are assumed to be uncorrelated and followed the Gaussian distribution with zero mean. As a result, the variance-covariance matrix takes the form of a diagonal matrix with a 100 times ratio between the GNSS code and phase observation precision. The GPS and GLONASS code and phase observation precision is set to be 0.5 and 0.005 m, respectively. According to Steigenberger et al, (2015), the clock and orbital products for Galileo and BeiDou are less accurate compared with GPS clock and orbital products. As a result, the Galileo and BeiDou code and phase observations are weighted by <sup>1</sup>/<sub>4</sub> with a precision taken as 1 and 0.01 m, respectively. For the ionosphere-free model, the quasiphase observables are assumed linearly correlated with the ambiguity-constrained observables and the precision of both observables are taken as <sup>1</sup>/<sub>2</sub> of the code precision (Choy et al., 2013).

To verify the performance of our single frequency GNSS PPP models, data sets from eleven globally distributed IGS-MGEX stations are processed. The datasets collected at the selected stations on seven consecutive days, i.e. April 1–7, 2014, are used for numerical analysis. The selected stations are occupied by different types of GNSS receivers. Single-frequency observations from GPS L1, GLONASS G1, Galileo E1and BeiDou B1 signals are adopted in this study. The BeiDou and Galileo antenna offsets recommended by the MGEX project are used to correct the PCOs of BeiDou and Galileo satellites (Rizos et al., 2013). Six-hour position solutions are analysed to represent the PPP performance in a short observation time. For the seven-day datasets, each day is divided into four sessions. Each session is processed separately so that a total of 308 sets of results are obtained to derive a statistical estimate on the positioning accuracy.

## 4. GNSS DATA ANALYSIS AND RESULTS

To evaluate the performance of different GNSS combinations namely GPS only, GPS/GLONASS, GPS/Galileo, GPS/BieDou and GPS/GLONASS/Galielo/BeiDou (GNSS), the positioning results with time for BRST at April 1, 2014, are shown herein as an example. Figures 1 shows the positioning errors with time for the various GNSS

constellation combinations at stations BRST. All PPP solutions are referenced to the GNSS station coordinates published by Center for Orbit Determination in Europe (CODE, 2015). It can be seen that the major contribution to the PPP solution enhancement is due to the additional GLONASS observations. This is due to the good availability of GLONASS compared with the other constellation, which significantly affects the overall satellite geometry. On the other hand, because of their limited number of visible satellites, the addition of Galileo and BeiDou systems has a marginal effect on the positioning accuracy, in comparison with the GPS-only solution. In contrast, comparable results are obtained with the GPS/GLONASS and the all-constellation GNSS solutions. It can be also seen that the ionosphere-free single frequency PPP model gives significant positioning accuracy enhancement compared with the undifferenced GIM based technique.



Figure 1. The positioning errors using the Single frequency GNSS PPP model for the different GNSS combinations



Figure 2. The positioning errors using the BSSD IF Single frequency GNSS PPP model for the different GNSS combinations

Table 1 summarize the 3D average positioning errors for the developed single frequency PPP models after 2h of data processing. For GPS only, it can be seen that the 3D positioning accuracy is enhanced by 13 cm compared with the GIM based technique when the BSSD IF- PPP is used. Comparable results are obtained for both the GPS/Galileo and GPS/BeiDou combinations. For the GPS/GLONASS combination solution, the 3D positioning mean is enhanced by 6 cm when the BSSD IF-PPP technique is used compared with the GIM based model.

Table 1. the 3D average positioning errors in meter for the four single-frequency PPP developed models after 2 hours of data processing

GNSS Combination	GPS	GPS/GLONASS	GPS/Galileo	GPS/BiDou	GNSS
Standard PPP	0.5	0.27	0.45	0.39	0.22
BSSD-IF-PPP	0.3	0.17	0.3	0.26	0.21

Figure 3 shows the mean of positioning errors after one hours of data processing. It can be seen that using the BSSD IF-PPP technique enhanced the positioning accuracy by 25%, 20%, 24% 20% and 19% compared with the GIM based PPP model for the GPS only, GPS/GLONASS, GPS/Galileo, GPS/BeiDou and GNSS combinations, respectively.



Figure. 3. The mean positioning accuracy after one hour for the different GNSS combinations

### 5. CONCLUSION

We developed PPP models, which combines single frequency observations of multiconstellation GNSS systems, including GPS, GLONASS, Galileo and BeiDou. The between-satellite-single-difference ionosphere-free (BSSDIF) model is developed in comparison with the standard single frequency PPP model. The IGS-MGEX final precise products were used to account for the orbital and clock errors, respectively. The contribution of the additional GNSS observations to the PPP solution was assessed through comparison with the traditional GPS-only counterpart. It was shown that the contribution of the additional GLONASS and BeiDou observations with good satellite availability is significant. It was also shown that the using of IF-PPP model significantly enhanced the positioning accuracy compared with the GIM based PPP. After two hours of data processing, the 3D positioning accuracy was enhanced by 13 cm compared with the GIM based technique when the BSSDIF- PPP is used and comparable results are obtained from both the GPS/Galileo and GPS/BeiDou combinations. For the GPS/GLONASS combination solution, the 3D positioning mean was enhanced by 6 cm when the BSSD IF-PPP technique is used compared with the GIM based model.

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