Al-Azhar University Civil Engineering Research Magazine (CERM)
Vol. (41) No. (2) April, 2019

Application of Reliability Analysis for the Determination of Middle Ordinate (M) at Horizontal Curves of Two-lane Highways

Elshotairy, O.M.[1], Hashim, I.H.[2], and Abu-Elmaaty, A.E.[3]

1. Demonstrator, Faculty of Engineering, Menoufia University, Egypt.
2. Professor, Faculty of Engineering, Menoufia University, Egypt.
3. Associate Professor, Faculty of Engineering, Menoufia University, Egypt.

الملخص العربي

معايير التصميم الهندسي الموجودة حاليا قائمه علي استخدام معايير تصميم حتميه لعناصر الطريق والتي تتجاهل عدم التاكد من ارتياط العديد من معايير التصميم. هذا الأسلوب ياخذ في الإعتبار متوسط خصائص السائق وقدرات المركبه للحصول علي تصميم مرضي لغالبيه مستخدمي الطريق. ومع ذلك فان التصميم وفقاً لهذا الأسلوب قد يكون غير اقتصادي بمجرد إحتمال وجود القيم الحرجه لجميع عناصر التصميم في وقت واحد. ويعتمد الأسلوب الجديد علي حساب عدم التأكد وتقييم المخاطر المرتبطه بملامح تصميم معينه. هذا البحث يناقش تطبيق التحليل المبني علي الموثوقيه وتقييم المخاطرعلي التخطيط الأفقي للطرق. هذه البحث اضاف مجموعة منحنيات تم خلالها المقارنة بين قيم بعد العائق عن منتصف الحارة الداخلية للمنحني في حالة القيم المحددة المحسوبة بالكود الامريكي للطرق عند سرعات محددة والقيم المحسوبة لنفس المتغير في حالة النهج موضوع البحث عند نفس السرعات. اشارت النتائج الي انه دائما في القيم المحسوبة من النهج المقترح تكون اقل من تلك الموجودة بالكود وبالتالي استخدام تلك المنحنيات يمكن مصممي الطرق من تقليل تكاليف انشاء الطرق خاصة في المناطق الجبلية.

Abstract

Existing geometric design guides provide deterministic design criteria for highway elements that ignore the uncertainty associated with many design parameters. Alternatively, several recent studies have advocated probabilistic geometric design where reliability analysis can be used to account for the uncertainty in the design parameters and to provide a risk measure of the degree of deviation from design standards. In reliability analysis, the risk is represented by the probability of non-compliance (P_{nc}) defined as the probability that the demand exceeds the supply. This study provides calibrated design charts for the middle ordinate (M), defined as the lateral distance between side obstruction and centerline of the inner traffic lane, at different probability of noncompliance levels and compare the calibrated values with AASHTO values. Non-compliance occurs whenever the available sight distance (ASD; supply) falls short of the stopping sight distance (SSD; demand). The inputs of SSD are random variables with appropriate probability distributions assumed for each input, and the input of ASD are deterministic values. The results show that the values of M calibrated from new approach are generally lower than those calculated from the AASHTO design guide. The charts can help the highway designers in reducing the cost of the construction especially in a mountainous areas.

Key words: Reliability analysis, Risk based approach, Geometric design.

1. Introduction

Existing highways geometric design guides provide a deterministic approach for design requirements using conservative percentile values of design inputs to account for the uncertainty associated with these inputs. The deterministic approach has two main shortcomings: First, many design parameters, such as perception and brake reaction time

(PRT) and operating speed, are stochastic in nature. The values used for design are typically selected at conservative percentile values extracted from their respective distributions among the general population of road users. The safety margin of the design output in this approach is unknown and no clear value is known to be targeted. Second, in some situation, the designers may need to deviate from the design standards due to some constraints (e.g. restricted right of way, nature of the landside). Existing geometric design guides provide little knowledge on the safety implications of deviating from standard requirements, and, in the deterministic approach, the slight violation to standards is considered as an unacceptable. Reliability theory can be used to develop factors of safety that incorporate the uncertainty of the supply and demand variables. The resulting factor of safety is termed the probability of noncompliance (p_{nc}), which is associated with a measure of probability that the demand will exceed the supply (Richl and Sayed, 2006). This paper discussed the importance of using reliability analysis to account for the uncertainty in design inputs and proposes one important application of reliability analysis in highway geometric design. As an example of geometric design code calibration, this paper provides calibrated design charts for the middle ordinate (M), defined as the lateral distance between side obstruction and centerline of the inner traffic lane as shown in Fig.1, at different probability of noncompliance. Such design charts may help designers to estimate the safety implications of their design decisions.

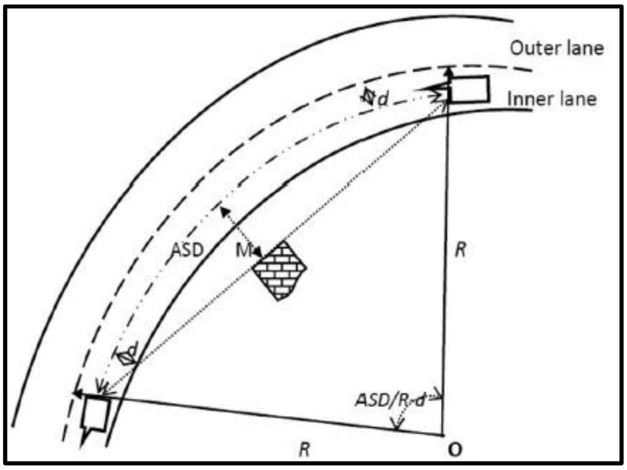

Fig. 1: Middle ordinate in two lane two way highway

2. Literature Review

Several applications of reliability theory in road transportation engineering have been reported in the literature. Moyer and Berry (1940) were the first persons introduced probabilistic methods into the area of highway design. A new method to determine the safe speed at which vehicles should be traveling on highway curves was developed by them. A ball-bard indicator was used by the authors to establish an acceptable "safe speed" on horizontal curves. The authors identified the percentile values for the operating speed at various design speeds which was the starting point upon which other studies have based their results on. The operating speed was considered to be a random variable and they recommended using the 85th percentile as the operating speed, for a design speed ≤ 30 mph and the 90th percentile for 35 mph. Faghri and Demetsky (1988) adopted a probabilistic approach to assess limitation in sight distance at road-railway grade crossings and the probability of collision. Navin (1990) was the first to use the term probability of noncompliance to refer to the probability of a design that does not meet the standard requirement. He provided an important theoretical discussion of using reliability-based design of typical highway elements. Easa (2000) applied the mean value first order second moment reliability method (MVFOSM) in order to evaluate sight distance at intersections. Sarhan and Hassan (2008) used Monte Carlo simulation to calculate the probability of three-dimensional sight distance limitation in the design of horizontal curves overlapping with flat grade, crest curve, and sag curves. The probability of sight distance limitation in their study was called probability of hazard. Ismail and Sayed (2009) proposed that the safety level associated with standard design outputs should be consistent and close to a prespecified target level. They introduced a general framework for calibrating standard design models as well as set of methods aiming at determining a target value for design safety. Ismail and Sayed (2012) presented a methodology for re-dimensioning cross sections located at highway segments with restricted sight distance to minimize the overall risk of the design. Ibrahim et al. (2012) presented a methodology for selecting appropriate combination of highway cross-section elements with restricted sight distance. The optimization method aimed at (1) minimizing the risk associated with restricted sight distance, (2) balancing the risk across the two carriageways of the highway, and (3) reducing the expected collision frequency. Dhahir and Hassan (2015) used reliability analysis to estimate the probability of failure POF on a specific horizontal curve. Authors represented each of the driver and vehicle characteristics by one value that is normally a conservative one, although these characteristics vary for individual drivers and vehicles and are better represented by a statistical distribution. Essa et al. (2016) demonstrates the application of multi-mode reliability analysis to the design of horizontal curves. The process is demonstrated by a case study of Sea-to-Sky Highway located between Vancouver and Whistler, in southern British Columbia, Canada. Two non-compliance modes were considered: insufficient sight distance and vehicle skidding. The results show the importance of accounting for several non-compliance modes in the reliability model. Rajbongshi and Kalita (2018) evaluated stopping sight distance in horizontal curves, considering the variability of all input parameters of sight distance. It is observed by authors that the 98th percentile sight distance value is much lower than the sight distance corresponding to 98th percentile speed. The distribution of sight distance parameter is also studied and found to follow a lognormal distribution. Finally, the authors also give a chart illustrate the variation of SSD with reliability.

3. Reliability

Reliability analysis assesses the system's ability to accommodate the demand of a specific design element against its capacity (Sarhan and Hassan, 2008). The basic reliability problem is a component problem with two random variables, supply and demand. The performance function in the plane represented by these two variables leads to failure or non-compliance when the demand exceeds the supply.

A generalized model representing the performance function is shown in Eq. (1)

$$\mathrm{g}(\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, \mathrm{X}_4, ..\mathrm{X}_n) = \mathrm{S}(\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, \mathrm{X}_4, ..\mathrm{X}_n) - \mathrm{D}(\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, \mathrm{X}_4, ..\mathrm{X}_n) \quad (1)$$

where:-

g = performance function (otherwise referred to as limit state function), S and D denote supply and demand, respectively, with non-compliance occurring when g < 0.0 as shown in Fig. 2, and

X_i= a combination or supply and demand variables explaining the reliability problem.

The outcomes of the reliability analysis are the reliability index β shown in Eq. (2) and the probability of non-compliance, p_{nc} as shown in Eq. (3)

$$\beta = \frac{\mu_g}{\sigma_g} \quad (2)$$

where:-

μ_g and σ_g are the mean and standard deviation of the performance function respectively,

$$P_{nc} = P(g < 0) = \int \dots \int f_x(x_1, x_2, \dots x_n) d_{x_1} d_{x_2} \dots d_{x_n} \quad (3)$$

where:-

f_x = the joint probability density function (PDF) for x_1, x_2, ... $x_n$and the integration is carried out over the failure or "non-compliance" domain (g < 0).

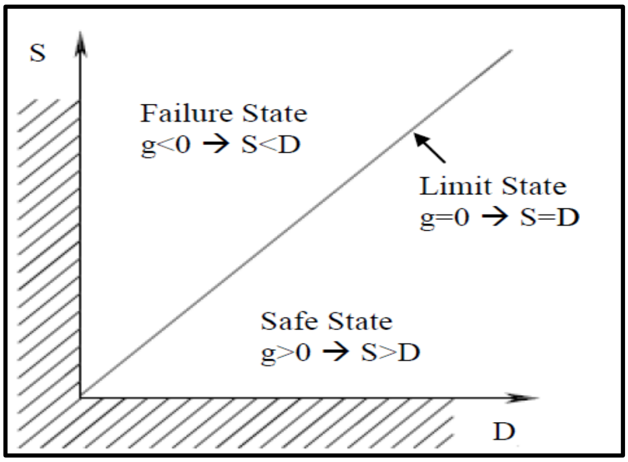

Fig. 2: Definition and failure limit state of a reliability analysis model (Ditlevsen and Madsen, 2007)

In most cases, there is no analytical method to get an exact solution of Eq. (4), and therefore, many reliability methods are used to get an approximate solution of the probability of noncompliance. The methods include the MVFOSM, first order reliability method (FORM), second order reliability method (SORM), and sampling (e.g., Monte Carlo sampling) (Haukaas, 2011). In this study, the Monte Carlo sampling method is selected for the analysis. This requires computer capabilities to simply generate series of numbers (in rows) for each random variable (in columns). As shown in Fig. 4, each column should follow a predetermined distribution with specific characteristics. A physical relationship then uses the set of numbers generated in each row to calculate the intended function. The calculated series of numbers would finally describe the probable distribution of the expected output (Sarhan and Hassan, 2008). Monte-Carlo simulation methods are ideally used when the limit state function is associated with difficulties such as when the limit state function is not differentiable or when there is more than one design point at which non-compliance occurs (Faber, 2006).

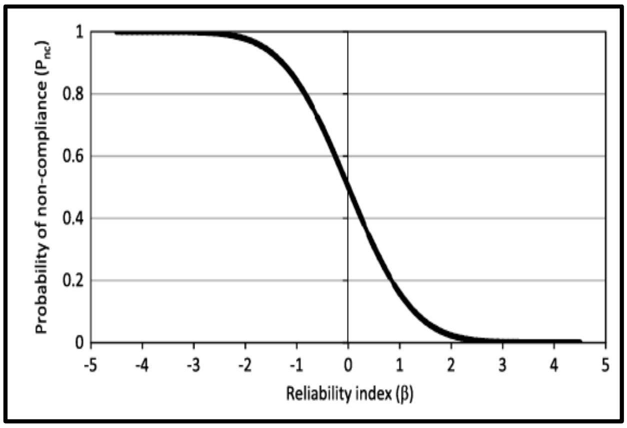

Fig. 3: Relationship between reliability index and probability of noncompliance (Hussein et al., 2014)

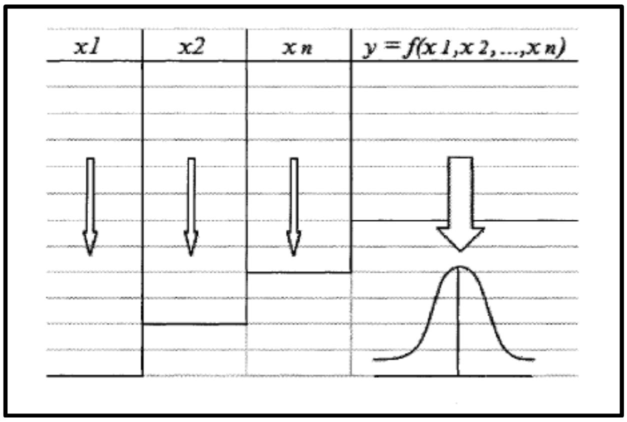

Fig. 4: Simplification of Monte Carlo Simulation Technique (Sarhan and Hassan, 2008).

4. Limit State Function

Design requirements necessitate that the length of a highway ahead that is visible to a driver should be adequate to recognize an object in the driver's path and stop before hitting this object. Accordingly, the stopping sight distance is the main focus of the design of horizontal curves. For the present application, the limit state function is defined in terms of:

$$g = \mathrm{ASD} - \mathrm{SSD} \tag{4}$$

And non-compliance occurs when ASD is less than SSD ($g < 0$). ASD is the portion of the road currently available to the driver.

The Stopping Sight Distance (SSD) is the total distance a vehicle travels from the time the driver sees an obstruction on the road ahead and comes to a complete and safe stop. It consists of the brake reaction distance and the braking distance. The former being the distance traveled from the moment the driver sees an obstruction on the road ahead to the moment before the brakes are applied. The braking distance is the distance the vehicle travels until it comes to a complete stop.

The SSD (i.e., the demand variable) is computed as follows

$$\mathrm{SSD} = 0.278\mathrm{VT} + \frac{\mathrm{V}^2}{254((\frac{a}{9.81}) \pm g)} \tag{5}$$

where:-

V = the operating speed (km/h),
T = the perception reaction time (s),

a = the deceleration rate (m/s^2), and

g = the longitudinal grade (%).

The ASD is determined and calculated according to Eq. (7) using 2.5 s perception and reaction time and 3.4 m/s^2deceleration rate with any selected design speed and SSD is probabilistic value. In the proposed probabilistic design, the limit state function is alternatively defined as the difference between the available and required sight distance.

The middle ordinate (M) can be calculated as the following equation:

$$M = R(1 - \cos\frac{ASD}{2R}) \tag{6}$$

where:-

R = horizontal curve radius (m).

5. Data Distribution

Three design parameters are required as design inputs to the limit state function presented in Eq. (6); speed (V), Perception and reaction time, and braking deceleration (a). Table 1 provides a summary of the design input distributions that are discussed below:

- Braking deceleration (a): Based on AASHTO (2011), the distribution of the driver deceleration was assumed to be a normal distribution, with a mean 4.2 m=s2 and a variance of 0.6 m=s2 as estimated from Fambro et al. (1997);
- Perception and reaction time (T): The distribution of perception and reaction time is based on a study conducted by Lerner (1995).

The same study was used as reference for perception and reaction time distribution for the National Cooperative Highway Research Program (NCHRP) (Fitzpatrick and Wooldridge, 2001). The mean value for perception-reaction time was 1.5 s with 0.4 s standard deviation and it was assumed to be log-normally distributed. Lerner also found that the longest PRT was 2.54 s and the second longest was 2.39 s while the 85th percentile was 1.9 s. This shows that the 2.5 s PRT value used for design in many design guides is conservative and may lead to relatively high stopping sight distances; and

- Operating speed (V): Assumed as deterministic values to facilitate the comparison between new approach and AASHTO values at the same design speed. A summary of these values is presented in Table 1.

Table 1: The probability distributions for the random input parameters

Parameter	Mean	Standard Deviation	Distribution	Reference
PRT	1.5 s	0.40 s	Lognormal	Lerner (1995)
A	4.2m /s^2	0.60 m/s^2	Normal	Fambro et al. (1997)

6. Analysis and Discussion of Results

The calibration process starts with a prespecified (target) p_{nc}. Limit state function (g) is defined in terms of all design inputs. The goal of the calibration process then is to find a value of a design parameter; the middle ordinate M in this case; such that the probability of noncompliance of the limit state function equals the prespecified p_{nc}. This could be easily obtained through an iterative process in which the value of M is changed until the resulting p_{nc} values equal the prespecified probability. The choice of a target p_{nc} is a paramount decision that the code developer is required to take. To satisfy the proposition that the design safety of standard design outputs should be consistent and close to some acceptable level, a penalty function can be used to quantify the difference between each p_{nc} associated with a specific design output, and the target p_{nc}.

In the calibration PRT and the braking deceleration are considered as random variables with known distributions according to Table 1, whereas the operating speed is constant and equal to the design speed. Calibration was conducted for a range of values of design speeds (between 40 and 80 km/h) and curve radius (between 200 and 1000 m). The goal is to obtain middle ordinate M values resulting in the probability of noncompliance of the limit state function is equal to specified probability of noncompliance value for each combination of design speed and horizontal curve radius. The analysis was conducted for three different prespecified values for the probability of noncompliance (5, 10, and 15%). Figures 5, 6 & 7 show the calibration results compared to the AASHTO design values.

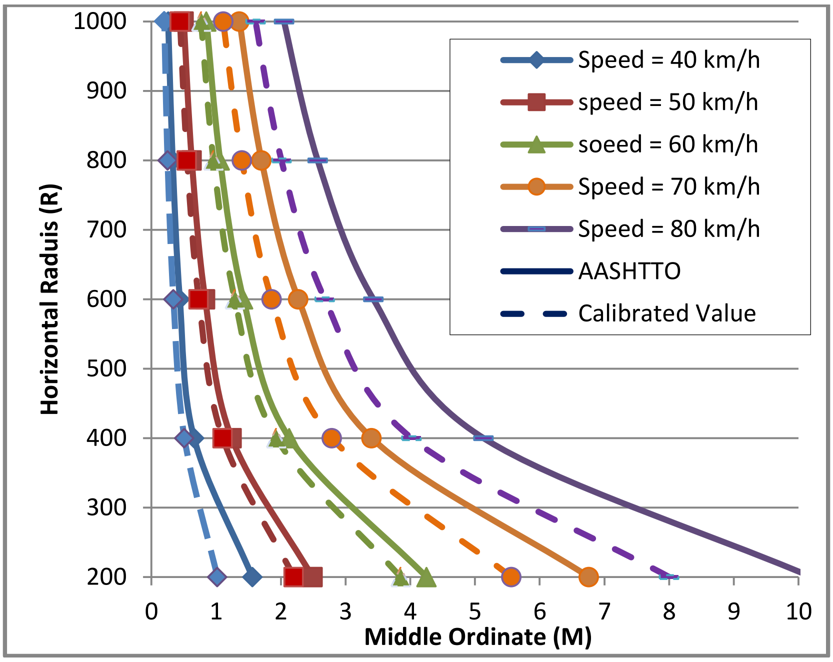

Fig.5: Calibrated middle ordinate M design chart using design speed (target p_{nc} = 5%)

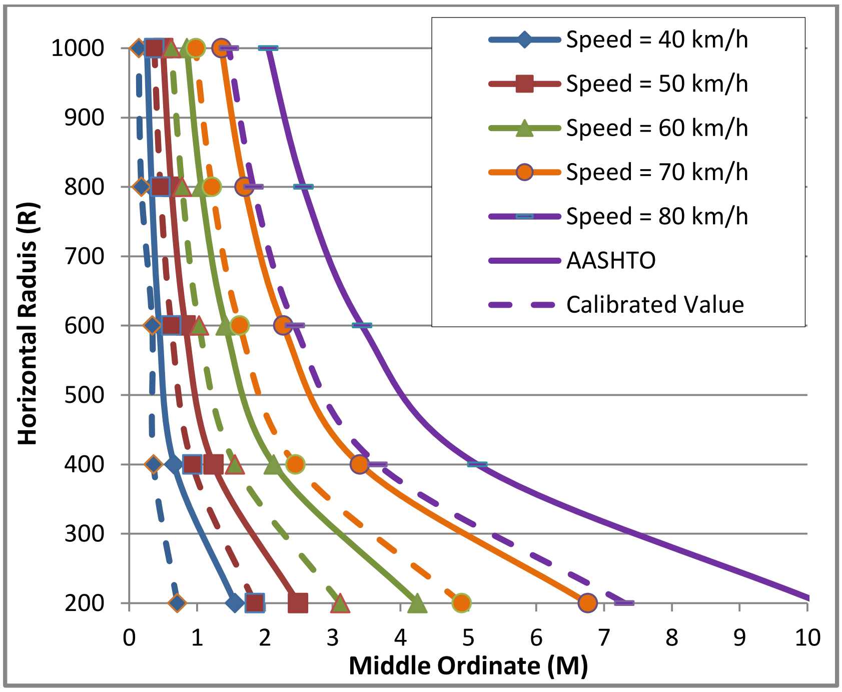

Fig.6: Calibrated middle ordinate M design chart using design speed (target p_{nc} = 10%)

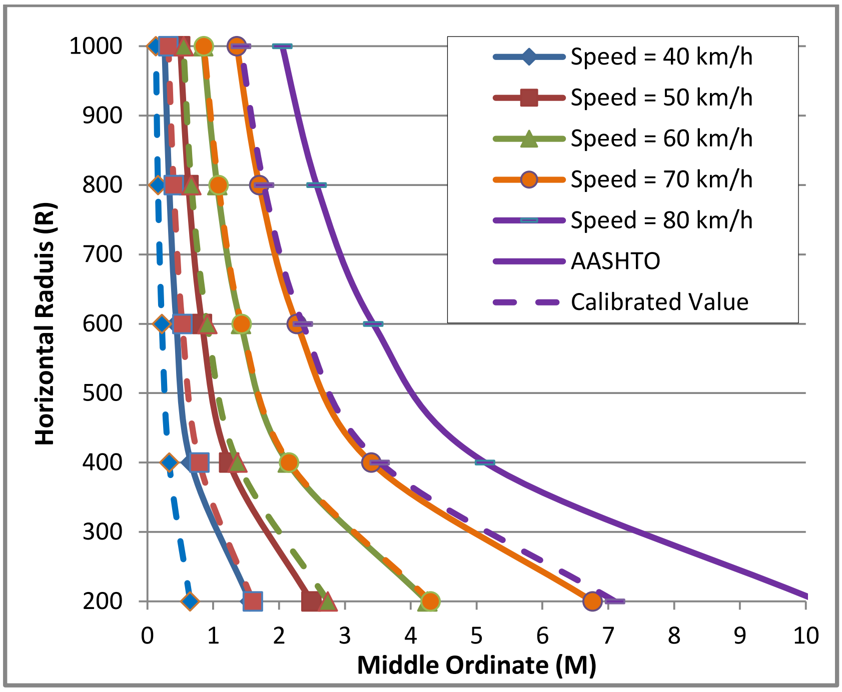

Fig.7: Calibrated middle ordinate M design chart using design speed (target p_{nc} = 15%)

Figs. (5, 6, and 7) show the calibrated middle ordinate M values and the corresponding AASHTO design values. The figures show that the calibrated values are generally lower than those derived from AASHTO for the three p_{nc} values. The difference between calibrated and AASHTO values increases as the radius decreases for the same design speed. For example, for a design speed of 80 km/h, the difference between the calibrated and AASHTO values for the middle ordinate M is 0.85 m for R =400 m and 0.4 m for R = 600 m at a probability of noncompliance of 5%. This difference increases for higher p_{nc} values. This finding can be very important especially for highways located in mountainous terrain. In this road side environment, most highway developments will have a constricted right-of-way. As a result, the designer will be faced with the dilemma of budget constraints and the need to approve of geometric designs that involve some violation or exception to standard requirements. The calibrated charts may offer designers an option to use lower middle ordinate values and also knowing the safety consequences of their decisions in terms of added risk (probability of noncompliance). Figs. (5, 6, and 7) also show that the difference between the calibrated M and the values obtained from AASHTO is directly proportional to the design speed for the same curve radius. For example, for a radius of R = 400 m, speed=60km/h and at a p_{nc} of 5%, the difference between calibrated M and the value obtained from AASHTO is 0.2 m for design and for the same radius for speed = 80 km/h is 0.4 m. This indicates that current design guides are conservative at high speeds and may be inconsistent in terms of the target risk level for different speeds.

7. Conclusions and Recommendations

Existing geometric design guides provide a deterministic approach for design requirements using conservative percentile values for uncertain design inputs to account for this uncertainty. Recently, researchers have advocated the use of reliability analysis to account for uncertainty in the geometric design process and to evaluate the risk associated with a particular design. In this approach, a risk measure (e.g., probability of noncompliance) is calculated representing that a specific design would not meet standard requirements. This paper presented an application of reliability analysis for the calibration of geometric design models to yield consistent and adequate safety levels. The main assumption is that the design safety level associated with standard design outputs should be consistent and close to a predefined level. An example was presented that provided calibrated design charts for the middle ordinate M, defined as the lateral distance between edge of edge of side obstruction and centerline of the adjacent traffic lane, at different probability of noncompliance levels. The calibration was conducted using both design and operating speeds considering different values for the target probability of noncompliance. Results showed that current design guides are conservative especially at high speeds and sharp curves. Significant reductions to current design requirements could be obtained at reasonable reliability (risk) levels. The calibrated charts can help the designer to assess the safety implications of different design alternatives and decrease the amount of cut and fill in a mountainous terrain. Several future directions can be derived from this study. Establishing more reliable distributions for the design inputs, especially operating speed, should receive more focus. Many parameters such as operating speed and braking deceleration were assumed to be normally distributed. This should be further investigated. Finally, more research is needed to identify suitable target probability of noncompliance values for calibration.

8. References:

AASHTO, (2011), A policy on geometric design of highways and streets, 6th Ed., Washington, DC.

Dhahir, B. and Hassan, Y., (2015), Using vehicle simulation in reliability analysis of horizontal curves based on vehicle stability and driver comfort, Proceeding of the Road and Safety Simulation Conference, October 6-8, Orlando, FL.

Ditlevsen, O. and Madsen, H. O. (2007), Structural Reliability Methods, Coastal, Maritime and Structural Engineering, Department of Mechanical Engineering. Denmark. pp. 13-34.

Easa, S. M., (2000), Reliability approach to intersection sight distance design, Transportation Research Record 1701, Transportation Research Board, Washington, DC, 42–52.

Essa, M., Sayed, T. and Hussein, M., (2016), Multi-mode reliability-based design of horizontal curves, Accident Analysis and Prevention 93 (2016) 124–134. 00014575.

Faghri, A., and Demetsky, M. J., (1988), Reliability and risk assessment in the prediction of hazards at rail-highway grade crossings, Transportation Research Record 1160, Transportation Research Board, Washington, DC, 45–51

Fambro, D., Fitzpatrick, K., and Koppa, R., (1997), Determination of stopping sight distance, NCHRP Rep. 400, Transportation Research Board, Washington, DC.

Fitzpatrick, K., and Wooldridge, M., (2001), Recent geometric design research for improved safety and operation, NCHRP Synthesis 299, Transportation Research Board, Washington DC.

Hussein, M., Sayed, T., Ismail, K. and Van Espen, A., (2014), Calibrating road design guides using risk-based reliability analysis, American Society of Civil Engineers, 10.1061/(ASCE)TE.1943-5436.0000694.

Ibrahim, S., and Sayed, T. (2012), Developing safety performance functions incorporating reliability based risk measures, Accid. Anal. Prev., 43(6), 2153–2159.

Ibrahim, S., Sayed, T., and Ismail, K., (2012), Methodology for safety optimization of highway cross-sections for horizontal curves with restricted sight distance, Accid. Anal. Prev., 49, 476–485.

Ismail, K., and Sayed, T., (2009), Risk-based framework for accommodating uncertainty in highway geometric design, Can. J. Civ. Eng., 36(5), 743–753.

Ismail, K., and Sayed, T., (2010), Risk-based highway design: case studies from British Columbia, Canada, Transportation Research Record 2195, Transportation Research Board, 3–13.

Ismail, K., and Sayed, T., (2012), Risk-optimal highway design: Methodology and case studies, Saf. Sci., 50(7), 1513–1521.

Lerner, N., (1995), Age and driver perception-reaction time for sight distance design requirements." ITE Compendium of Technical Papers, Institute of Transportation Engineers, Washington, DC, 624–628.

Navin, F. P. D., (1990), Safety factors for road design: Can they be estimated, Transportation Research Record 1280, Transportation Research Board, Washington, DC, 181–189.

Navin, F. P. D., (1991), Safe road design as limit state, Proc. Conf. Strategic Highway Research Program and Traffic Safety on Two Continents, Swedish National Road and Transport Research Institute (VTI) Rapport 372A, Part 2.

Rajbongshi, P. and Kalita, K., (2018), Reliability Based Geometric Design of Horizontal Circular Curves, The Institution of Engineers, India, 99(2):333–339.

Richl, L., and Sayed, T. (2005), Effect of speed prediction models and perceived radius on design consistency, Can. J. Civ. Eng., 32(2), 388–399.

Richl, L., and Sayed, T. (2006), Evaluating the safety risk of narrow medians using reliability analysis, J. Transp. Eng., 10.1061/(ASCE) 0733-947X(2006)132:5(366), 366–375.

Sarhan, M., and Hassan, Y., (2008), Three-dimensional, probabilistic highway design: Sight distance application, Transportation Research Record 2060, Transportation Research Board, Washington, DC, 10–18.

Transportation Research Board., (1998), Managing speed: Review of current practice for setting and enforcing speed limits, TRB Special Rep. 254, Washington, DC.