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APPLICATION OF TOPOLOGY OPTIMIZATION ON DEEP BEAMS

AHMED ALAA ELDIN ABDELHAMED[1], FATHY SAAD[2], KHALED H. Riad[3]

1 M.Sc. Student, Structural Engineering Department, Ain Shams University
2 Professor, Structural Engineering Department, Ain Shams University
3 Associate Professor, Structural Engineering Department, Ain Shams University

ملخص البحث

يهدف هذا البحث الي استخدام كود مقترح من الكاتب على برنامج الماتلاب للوصول للطبولوجية الأمثل للمنشآت الخرسانية وخاصة الكمرات العميقة. يتناول البحث عدة نماذج لكمرات ذات نسب طول الى عمق مختلفة 1:2و 1:3 و 1:4. يتم تقسيم المجال التصميمي الذي يعبر عن ابعاد العينات الى عناصر شبكية صغيرة, بحيث تكون ابعاد العناصر 0.05م × 0.05م من ثم يتم الوصول للطبولوجيا المثلى عن طريق التخلي عن العناصر التى تحمل الاجهادات الاقل في المنشأ و تتم عملية تتابعية بالتخلي عن العناصر الغير كفئ للوصول للحجم المطلوب من العينة. في نهاية البحث يتم عمل مقارنة للنتائج مع الابحاث السابقة و ايضا يتم عمل مقارنة لبيان مدى التشابه مع نماذج الضاغط و الشداد المكافئ.

ABSTRACT

Topology optimization has gained a lot of attention and popularity over the last years as a result of the technological developments and the improvement in the optimization algorithms which led to the wide spread of the topology optimization to not only include academic interests but also to a growing number of structural engineers to benefit from this technique.

The Topology Optimization technique aims to reach the most efficient percentage of the design domain that has the ability to resist a certain load applied to it through the removal of inefficient elements. The technique itself has great resemblance to the famous Strut and Tie method which is a well-known design tool and is recommended by various design code.

This research focuses on using a proposed MATLAB code along with the analysis engine of the SAP2000 to find the optimized topologies of concrete deep beams subjected to a concentrated load at mid-span using the Bidirectional Evolutionary Structural Optimization algorithm (BESO).

INTRODUCTION

There has always been a general trend in the engineering society to find the most economic designs due to the limited resources, this trend aims to get rid of the inefficient part of the structures. One of the most efficient approaches is the Topology Optimization approach. The principals of the topology optimization technique started to emerge in the early 1900’s when Michell (1904) introduced the advantages of using low volume, truss-like structures and was later on continued by Rozvany’s (1977) grillage approach that resulted in the appearance of the topology optimization techniques with the landmark paper by Bendsøe, M and Kikuchi, N(1988) which introduced a material distribution method based on the use of artificial composite material with microscopic voids [1] using a homogenization method which was then developed to a simpler method called (SIMP) Solid Isotropic Material with Penalization which was introduced by Bendsøe and Zhou and Rozvany.

Another approach was developed by Xie and Steven [2]that is called the (ESO) Evolutionary Structural Optimization method which was continuously developed to solve many optimization problems. The (ESO) method is a simple method that is based on the gradual removal of inefficient material in a selected design domain to reach an optimum structure. An extension to this method was later developed which is called the (BESO) Bi-directional Evolutionary Structural Optimization method which permits the material inside a design domain to be added and removed simultaneously. The (BESO) method was initially developed by Yang et al.[3]where topology optimization was subjected to stiffness and displacement constraint. The (BESO) concept has later been applied by Querin et al [4],[5] to implement the stress constraint in the topology optimization procedure which ensures full stress design applying the von Mises stress criterion.

The early developments of the BESO approach was introduced by Querin et al [4]which used the von Mises stress as their removal and addition criterion through defining a rejection ratio RR and an inclusion ratio IR, which allowed the elements with low stress to be removed from the design domain and void elements near highly stressed regions to be converted back to solid elements in order to relieve the stress. The rejection and inclusion procedure were treated separately according to the following equations

$$\frac{\sigma^{VM}}{\sigma^{VM}_{max}} < RR_i \quad \rightarrow \quad \text{Element is removed}$$

$$\frac{\sigma'^{VM}}{\sigma^{VM}_{max}} > IR_i \quad \rightarrow \quad \text{Element is added}$$

Where σ'^{VM} being the historical stress level of the void elements.

All elements inside the design domain are ranked by their corresponding sensitivity numbers and elements with the lowest sensitivity numbers are removed and removed elements with the highest sensitivity numbers are reverted to solid elements in order to reach the targeted volume at current iteration according to the following equation
Vreq = Vk-1 (1 - ER).

Where Vk-1 is the volume of the previous iteration and ER is the Volume evolutionary ratio.

The Bidirectional Evolutionary Structural Optimization Algorithm

The Bidirectional Evolutionary Structural Optimization (BESO) method was developed to overcome the major drawbacks of the ESO method, since the ESO approach allows only the removal of elements with low stress ratios which in some cases produced inadequate final designs, so the main idea for the BESO approach is to permit the simultaneous addition and removal of elements in the continuum.in order to reach the optimum design.

The following steps summarizes the main steps for the BESO method:

1. Defining the domain, and discretizing the structure into a fine mesh of finite elements and assigning initial material property values to form an initial design (Solid/Void).

2. Performing finite element analysis for the structure and calculating elemental sensitivity numbers.

3. Averaging the elemental sensitivity numbers with its predecessors to obtain a historical sensitivity number

4. Determining the target volume Vreq for the next iteration.

5. Addition and Removal elements satisfying their respective criteria is conducted.

6. The rejection ratio is increased by the evolutionary rate when a steady state is reached.

7. Repetition of steps 2 through 7 until a prescribed optimum condition is reached (For example a certain volume of the structure is obtained V*).

It was noted that when using ESO/BESO approaches, massive fluctuation in the evolution history of the objective function is observed as seen in Figure 2 5 (a), this behavior was due to the definitive nature of the material property values (0 or 1) which affects the sensitivity numbers and results in difficulty in convergence [7]A stabilization scheme was introduced by Huang and Xie to solve this problem through averaging the elemental sensitivity numbers with its predecessors from previous iterations to form a historical elemental sensitivity number [8] which resulted in a more stable topology as well as objective function as seen in 1 The averaging scheme was given as:

$$\alpha_i = \frac{\alpha_i^k + \alpha_i^{k-1}}{2}$$

Where k is the current iteration number.

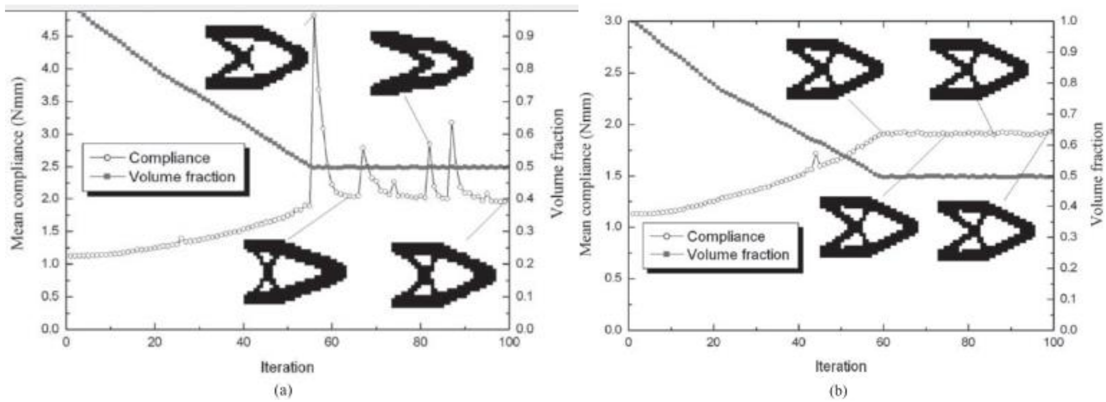


Figure 1 – Evolution history comparisons (a) Using sensitivity numbers without stabilization (b) Sensitivity numbers with stabilization [7]

Numerical instabilities and Filtering schemes

It is noted that most of the optimization approaches has been associated with different types of numerical instabilities as shown in the following sections:

1- Checker board pattern

When combining a discretized continuum of finite element along with the optimization algorithms a discontinuous solution may arise, meaning that some singular elements may be removed despite being surrounded with solid domain. This leads to the so-called Checkerboard pattern in the final topologies [9].

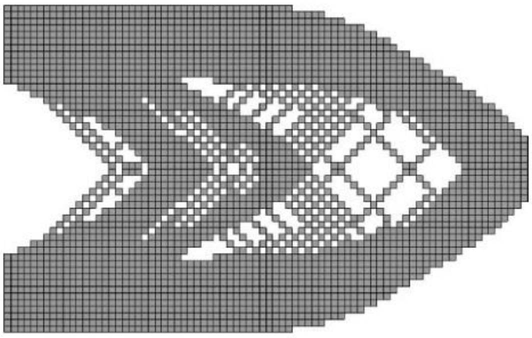
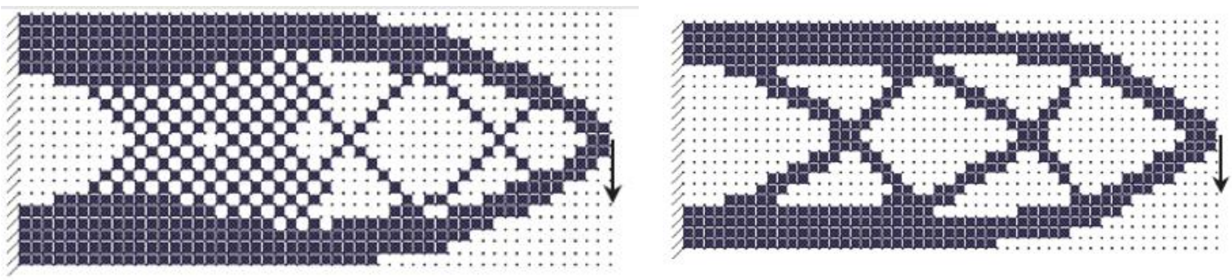


Figure 2 – Checker-Board pattern for a cantilever problem [7]

The existence of the Checker-Board pattern in the resulting topology may cause unnecessary stress concentration in certain regions of the structure as well as the difficulty in constructing the final topology into a real structure.

A solution to this problem was first introduced by Li et al to suppress the Checker-Board pattern formation in the ESO method [10]by using a simple smoothing technique that average sensitivity number of the neighboring elements and assign it to the considered element, this prevents elements near highly sensitivity numbered elements from being removed and forces elements near low sensitivity numbered elements to be removed resulting in a non-Checker-Board topology.



(a) ESO without Checkerboard solution

(b) ESO with Checkboard solution

Figure 3 – ESO Checkerboard solution [10]

However, this scheme didn’t solve the mesh dependency problem, so in the BESO approach a filtering scheme was introduced to overcome both checkboard and mesh dependency.

2. Mesh Dependency filtering scheme

A filtering scheme was introduced into the BESO approach in order to overcome both mesh-dependency and checker-board pattern. First a nodal sensitivity number is defined by averaging elemental sensitivity numbers that a certain node is connected to as follows[7]:

$$\alpha_j^n = \sum_{i=1}^{M} w_i \alpha_i$$

Where M and wi are the total number of elements connected to the jth joint, and the ith element weight factor respectively.

$$w_i = \frac{1}{M-1}\left(1 - \frac{r_{ij}}{\sum_{i=1}^{M} r_{ij}}\right)$$

Where rij denotes the distance between the jth node and the ith element's center.

Then the above nodal sensitivity number will be projected to the studied domain through a radial effectiveness filter rmin which length does not change with mesh size, the rmin filter determines the nodes which lies inside a circular sub-domain Ω_i that will affect the sensitivity number of the ith element as seen in the following figure.

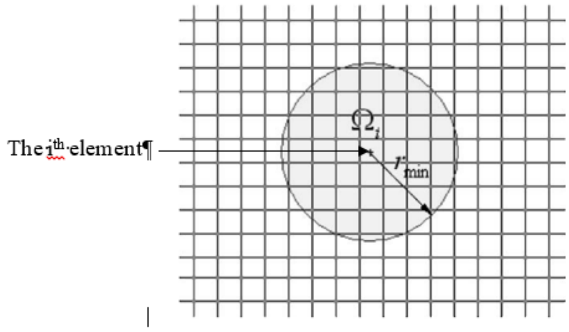


Figure 4 – The circular sub-domain for ith element [7]

For the filter to take effect the value of rmin should be large enough to include more than the ith element and it is recommended to be taken about 1 to 3 times the size of the studied element.[8]

TOPOLOGY OPTIMIZATION APPLICATION TO DEEP BEAMS

A MATLAB code proposed by the author was applied to perform the topology optimization procedures to deep beams with different depth to span ratios. Various L/d ratios were taken into consideration 2:1, 3:1 and 4:1. SAP2000 which is a well-known structural analysis software was used to obtain the stresses of the shells defining the design domain. The obtained topologies as well as the stress levels are presented as follows:

1. Deep beams with length to depth ratio 2:1

The deep beam is assumed to have a length of 4 meters and depth of 2 meter. The shell elements discretizing the design domain sizes are 0.05m x 0.05m and have a thickness of 0.25 m. The load applied is 1200 kN distributed over 5 points, each having 240 kN. The compressive strength for the used concrete fcu = 42 MPa, Young's modulus of concrete E = 28515 MPa and Poisson's ratio ν = 0.30. The element removal evolutionary ratio Cer = 3%, and maximum admission ratio Carmax = 2%. The volume fraction required to be obtained at the final topology Vfrac = 25%. The sensitivity radius rsen = 2 shells, and power norm p = 4.

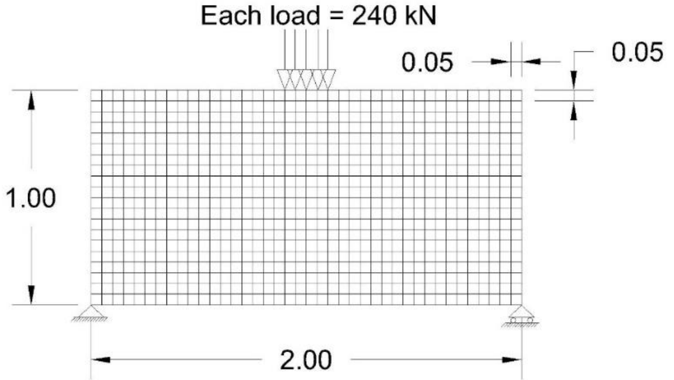


Figure 5 – Simply supported deep beam with L/D = 2.

The topology optimization algorithm was applied to the given deep beam and iterative procedures were carried out to remove the shell elements with the least stress levels until a target volume is reached.

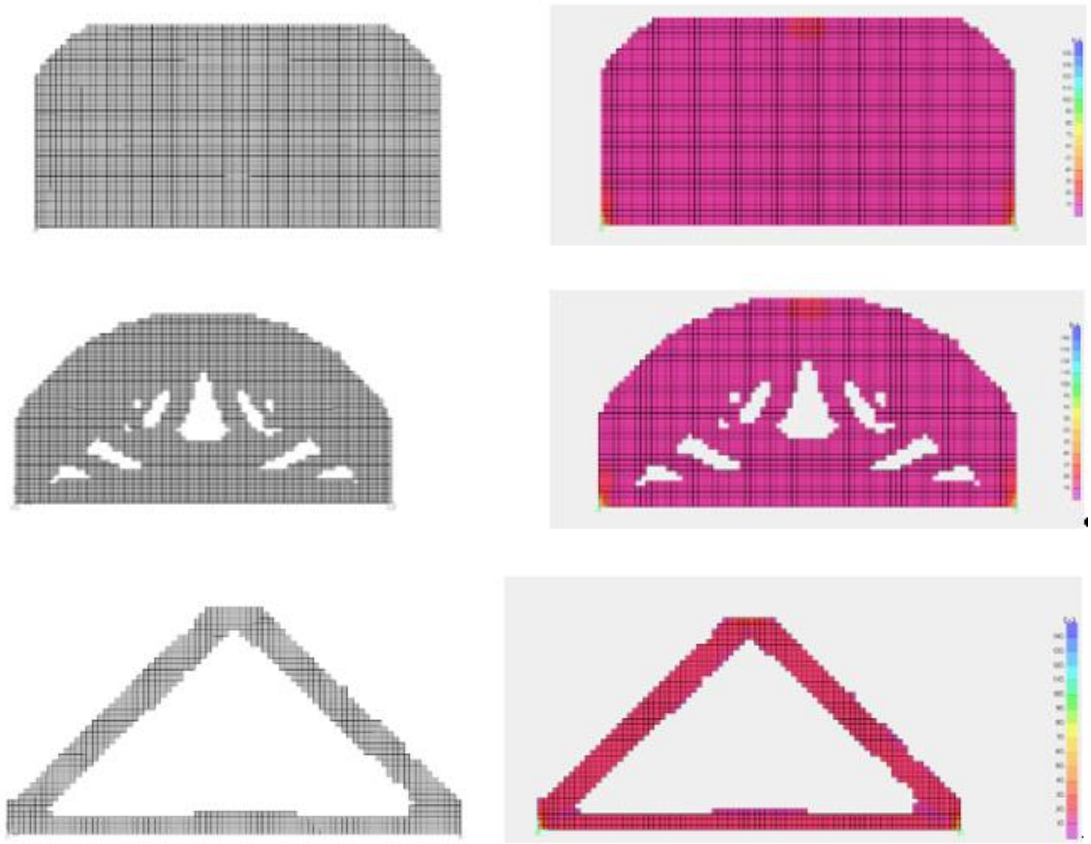


Figure 6 – Iterative procedure until target volume is reached

The target volume is reached at the 49th iteration as shown in the previous figure.

It can be noted that there is a great agreement between the reached topology and the anticipated Strut and Tie model for this shape.

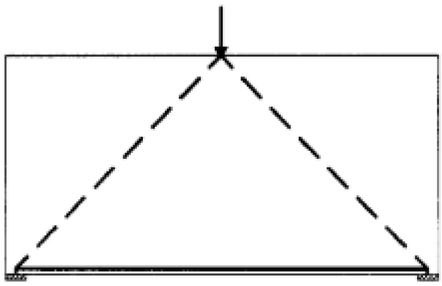


Figure 7 – Corresponding strut-and-tie model for L/D = 2 [11]

2. Deep beams with length to depth ratio 3:1

The deep beam is assumed to have a length of 3 meters and depth of 1 meter. The shell elements discretizing the design domain sizes are 0.05m x 0.05m and have a thickness of 0.25 m. The load applied is 1200 kN distributed over 5 points, each having 240 kN. The compressive strength for the used concrete fcu = 42 MPa, Young's modulus of concrete E = 28515 MPa and Poisson's ratio ν = 0.30. The element removal evolutionary ratio Cer = 2%, and maximum admission ratio Carmax = 1%. The volume fraction required to be obtained at the final topology Vfrac = 45%. The sensitivity radius rsen = 2 shells, and power norm p = 3.

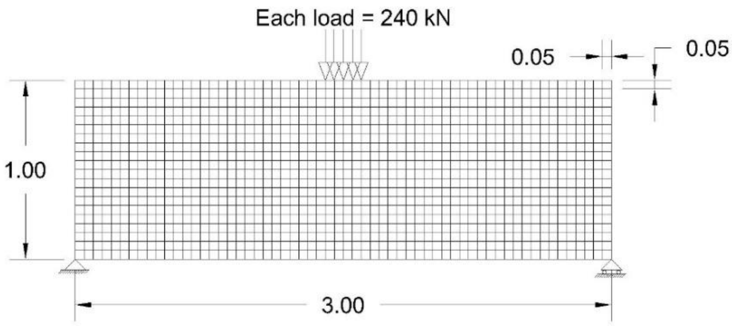


Figure 8 – Simply supported deep beam with L/D = 2.

The topology optimization algorithm was applied to the given deep beam and iterative procedures were carried out to remove the shell elements with the least stress levels until a target volume is reached.

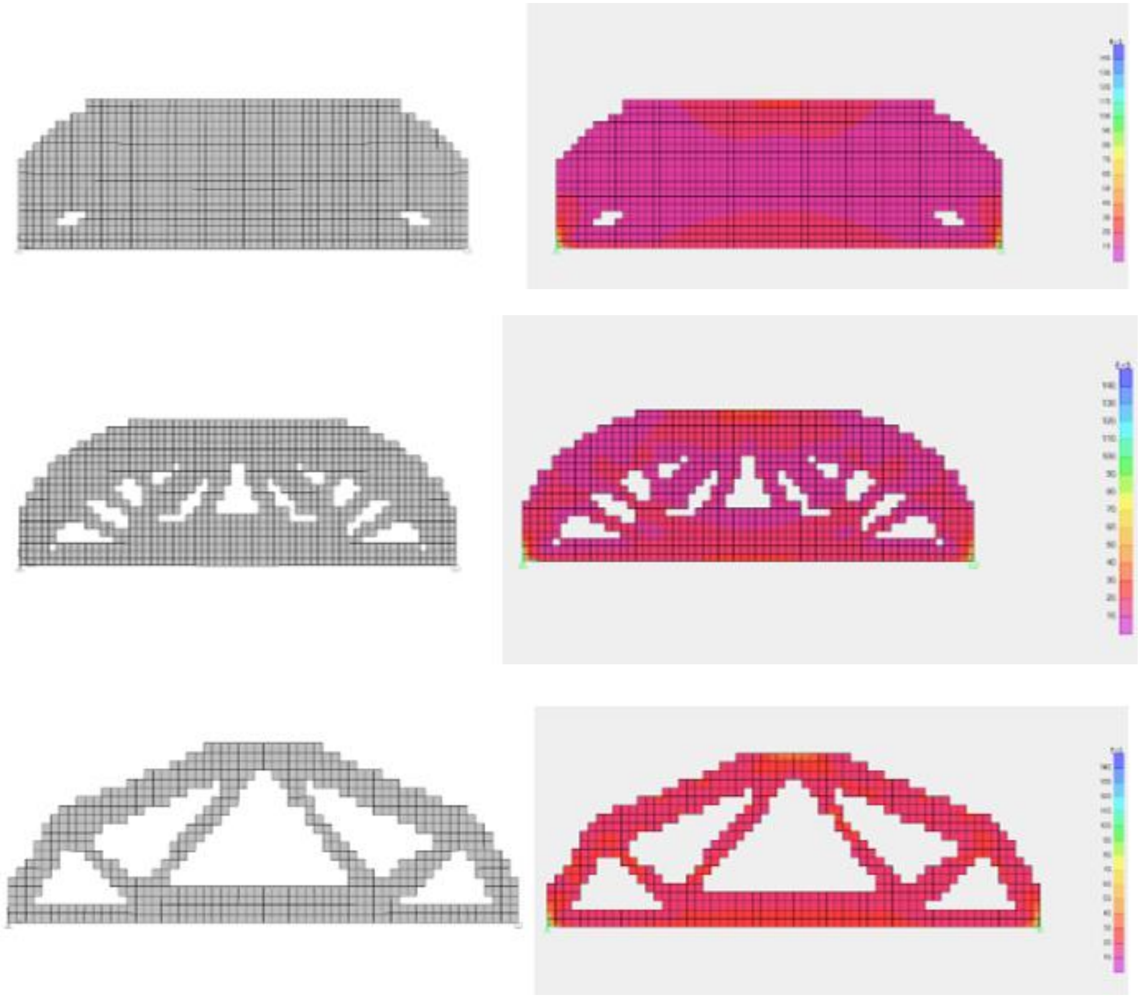


Figure 9 – Iterative procedure until target volume is reached

The target volume is reached at the 45th iteration as shown in the previous figure.

It can be noted that there is a great agreement between the reached topology and the anticipated Strut and Tie model for this shape.

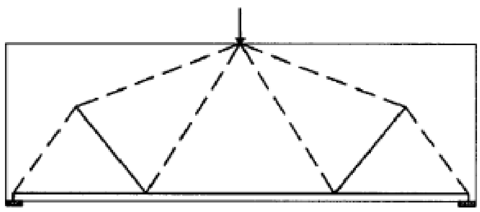


Figure 10 –Corresponding strut-and-tie model for L/D = 3 [11]

3. Deep beams with length to depth ratio 4:1

The deep beam is assumed to have a length of 4 meters and depth of 1 meter. The shell elements discretizing the design domain sizes are 0.05m x 0.05m and have a thickness of 0.25 m. The load applied is 1200 kN distributed over 5 points, each having 240 kN. The compressive strength for the used concrete fcu = 42 MPa, Young's modulus of concrete E = 28515 MPa and Poisson's ratio ν = 0.30. The element removal evolutionary ratio Cer

= 2%, and maximum admission ratio Carmax = 1%. The volume fraction required to be obtained at the final topology Vfrac = 60%. The sensitivity radius rsen = 2 shells, and power norm p = 4.

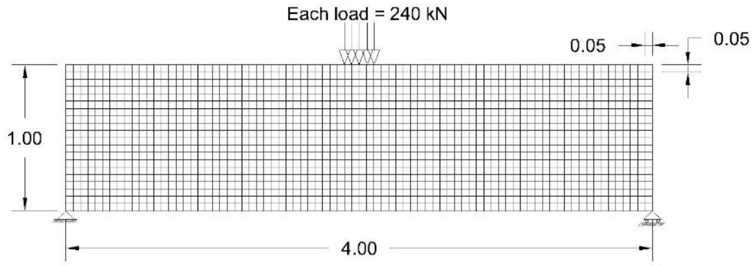


Figure 11 – Simply supported deep beam with L/D = 4.

The topology optimization algorithm was applied to the given deep beam and iterative procedures were carried out to remove the shell elements with the least stress levels until a target volume is reached.

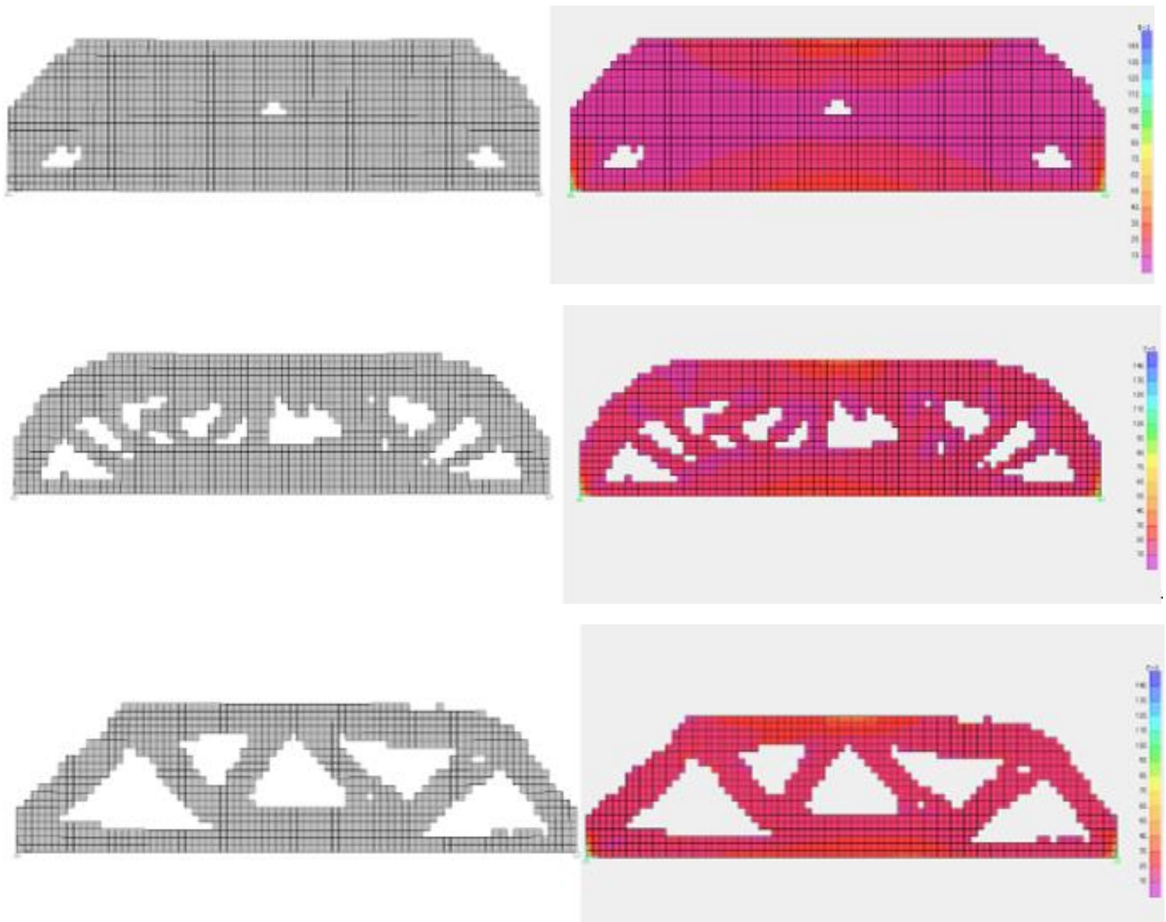


Figure 12 – Topology, stress levels and volume at different iterations for 4:1 ratio.

It can be noted that there is a great agreement between the reached topology and the anticipated Strut and Tie model for this shape.

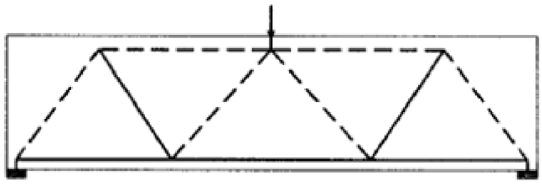


Figure 13 –Corresponding strut-and-tie model for L/D = 4 [11]

CONCLUSION

A MATLAB code was developed to perform topology optimization procedure on Deep beams using SAP2000. The main conclusions can be summarized as:

- Topology Optimization is a reliable technique that can be used in the analysis and optimizing of various structures.
- There is a great agreement between the topologies and the Strut-and-Tie method which is a well-known reliable design method in most of the design codes.
- The Bidirectional evolutionary structural optimization can be used to predict the optimized Strut-and-Tie model for complicated structures.
- The proposed MATLAB code allows the use of SAP2000 which is a widely used structural analysis program to define the design domain and loading conditions, and perform Topology optimization on the defined domain.
- The Proposed code can be used to predict the failure load through extrapolation.

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