

Nonlinear Finite Element Analysis of RC Flat Slabs Strengthened with Steel Shear Bolts

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ملخص البحث:

الهدف من هذه الدراسة هو تقديم نموذج لتحليل البلاطات الخرسانية المسطحة والمدعمة بمسامير القص من الصلب باستخدام طريقة العناصر المحددة ثلاثية الأبعاد اللاخطية. تم إستخدام برنامج ANSYS (الإصدار الثاني عشر). تم تمثيل حديد التسليح ومسامير القص من الصلب كعناصر خطية منفصلة داخل العناصر الطابوقية مع إفتراض وجود ترابط تام بين الخرسانة وبين حديد التسليح ومسامير القص من الصلب. هذا الاسلوب يعتبر أن المواد لاتتصرف خطيا تبعا إلي تشقق الخرسانة في مناطق الشد والتصرف اللاخطي للخرسانة تحت الضغط وتهشم الخرسانة وخضوع حديد التسليح. أيضا تهدف الدراسة إلي إيجاد الزيادة الحاصلة سعة القص للبلاطات المسطحة المدعمة بمسامير القص من الصلب. تم مقارنة النتائج من طريقة العناصر المحددة بنتائج الإختبارات المعملية وتم الحصول علي توافق جيد بين النتائج التحليلية بطريقة العناصر المحددة والنتائج.

Abstract:

This study aims to present a model suitable for analyzing reinforced concrete flat slabs strengthened in punching shear with steel shear bolts at column zone using finite element method. A nonlinear three dimensional finite element analysis has been used to conduct an analytical investigation on the overall behavior of reinforced concrete flat slabs. ANSYS computer program version 12 is utilized in the analysis. The concrete was idealized by using the 8-node isoparametric brick elements in ANSYS, while both flexural steel reinforcement and the steel shear bolts were modeled as link element by assuming perfect bond between the concrete and both of the flexural steel reinforcement and steel shear bolts. The numerical analysis includes material nonlinearity due to concrete cracking in tension, nonlinear stress strain relations of concrete in compression, crushing of concrete and yielding of both flexural steel reinforcement and steel shear bolts. The validity of the adopted models was verified through comparison with the available experimental data, and the agreement has proven to be good.

Keywords: Punching shear, Flat Slabs, Punching Shear RFT., Shear Bolts, Slab-Column Connections, Strengthening, Nonlinear finite element analysis.

1. INTRODUCTION

Flat slabs system is a beamless slabs and may have column capitals or drop panels to resist punching shear stresses. This research deals with flat slabs system without column capitals or drop panels, which make formwork very simple and widely used. This system is architecturally advantageous in that smaller overall story heights can be achieved due to the reduced floor structural depth required for beams, also the locations

of columns and walls are not restricted by the location of beams. The great disadvantage of flat slabs system is that they are highly susceptible to failure in punching shear at column connection zone. The punching shear failure is a brittle and non-ductile failure this failure may be occur in flat slabs due to changing building use, construction errors, or design mistakes, which may cause that punching strength of flat slab is insufficient. Due to reduce time and cost otherwise expensive experimental tests, we can develop a powerful and reliable analytical techniques, such as finite element method. The finite element method may better simulate the loading and support conditions of the actual experimental test. Adequate modeling of the actual behavior of reinforced concrete, and reinforcing steel including nonlinearity is required to get the accurate structure behavior and results of finite element analysis. Reinforced concrete exhibits nonlinearity because of cracking, inelastic material behavior, stiffening and softening phenomena, complexity of bond between reinforcement and concrete, and other factors (Chen and Saleeb, 1982). The derivation and implementation of various analytical finite element and material models studied in many researches to investigate the behavior of reinforced concrete slabs, also material modeling has been the subject of many researches. A lot of researchers studied different behavioral aspects of reinforced concrete flat slabs such as (Vidosa et al., 1988; Marzouk and Chen, 1993; Marzouk and Jiang, 1996; Jiang and Mirza, 1997; Reitman and Yankelevsky, 1997; Polak, 1998, 2005; Staller, 2000; Salim and Sebastian, 2002; Vainiunas et al., 2004; Murray et al., 2005; Deaton, 2005; Smadi and Belakhdar, 2007) and other. Many experimental studies are carried out on such reinforcements which are sometimes supported by theoretical investigations (Hawkins, 1974; Dilger and Ghali, 1981; Mokhtar, Ghali and Dilger, 1985; Elgabry and Ghali, 1990; Lim and Rangan, 1995; Marzouk and Jiang, 1996; El-Salakawy, Polak and Soliman, 2000; Alaaet al., 2000; Adetifa and Polak, 2005). These studies confirm that shear reinforcement in flat slabs is effective in improving ductility, and increasing punching shear capacity for flat slabs; if the proper amount of reinforcement, placement of shear bolts, spacing between shear bolts rows, and anchorage conditions are satisfied. Due to great results of shear studs as shear reinforcement in flat slabs, many researchers studied strengthening of flat slabs in punching shear using shear bolts manufactured from steel or FRP (El-Salakawy et al. 2003; Adetifa and Polak 2005; Bu and Polak 2009; M.A.Polak 2005; Hamed S.Askar 2015; M. Hamdy et al. 2018). M.Hamdy et al. studied the efficiency of using steel Shear Bolts to strengthen RC slabs in punching shear at interior column zone taking into consideration the column aspect ratio and the spacing between shear bolts peripheral rows. Analytical finite element analysis is used in current study to predict the behavior of flat slabs strengthened with steel shear bolts at column connection concentrically loaded up to failure. The validity and calibration of the adopted finite element model is verified through comparison of analytical results with the available experimental data obtained by M.Hamdy et al. 2018.

2. FINITE ELEMENT MODEL

ANSYS computer program is utilized for analyzing all tested slabs by (M.Hamdy et al. 2018). Structural components encountered throughout the current study, corresponding finite element representation and elements designation in ANSYS program will be represent below.

2.1. Element types

Concrete Element

Solid65, an eight-node solid element is used to model the concrete, which is special for 3-D modeling for solid concrete elements with or without reinforcing rebar. The

element allows the presence of three different reinforcing materials. The solid element has eight nodes with three degrees of freedom at each node translations in the nodal x, y, and z directions. The element is capable of plastic deformation, cracking in three orthogonal directions, and crushing. The geometry and node locations for this element type are shown in (Figure 1). (Figure 2) and (Figure 3) shows finite element modeling of flat slabs which connected by different aspect ratio of columns in ANSYS program.

Flexural Steel Reinforcement and Shear Bolts Elements

Link8, For the discrete model, Link8 is an element used to model the flexural and shear bolts steel reinforcement. Two nodes are required for this element. Each node has three degrees of freedom, translations in the nodal x, y, and z directions. The element is also capable of plastic deformation. The geometry and node locations for this element type is shown in (Figure 4). (Figure 5) and (Figure 6) shows modelling of bottom, top and column steel reinforcement with Shear Bolts.



Figure 1 : Solid65– 3-D reinforced concrete solid (ANSYS 12.0).



Figure 3 : Modeling of Flat Slab Which Connected with Rectangular Column (2:1).



Figure 5 : Geometrical Dimensions of Specimen With Shear Bolts.



Figure 2 : Modeling of Flat Slab Which Connected with Square Column (1:1).



Figure 4 : Link8 Element, ANSYS Manual.



Figure 6 : Details of Steel Reinforcement and Shear Bolts in Specimens.

Lead Plates and Supports

Solid45element used for steel plates at the supports for the column. This element has eight nodes with three degrees of freedom at each node translation in the nodal x, y, and z directions. The geometry and node locations for this element as shown in ($_{Figure 7}$).

2.2. Real Constants

Element real constants are properties that depend on the element type, such as crosssectional properties of a beam element. Not all element types require real constants, and different elements of the same type may have different real constant values and a single element type may reference several real constant sets.

Concrete Element

Real Constant set used for the Solid65 element. Values can be entered for Material Number, Volume Ratio, Orientation Angles, and Crushed Stiffness Factor (CSTF). The Material Number refers to the type of material for the reinforcement. The Volume Ratio refers to the ratio of steel to concrete in the element. The Orientation Angle is the direction of the reinforcement in the concrete element as shown in Figure (208). The Crushed Stiffness Factor (CSTF): A value of (0.002) is entered to simulate the negative stiffness of the stress strain curve of concrete.

Steel Reinforcement and Shear Bolts Elements

Real Constant set is defined for the Link8 element. Values for cross-sectional area and initial strain are entered. A value of zero is entered for the initial strain because there are no initial stresses in the reinforcement.

Lead Plates

No real constant set exists for the Solid 45 element.

2.3. Material Properties

Concrete Elements

There are multiple parts of the material model for the concrete element as can be found in (Table 1), this material model refers to the Solid 65 element, used for all the concrete elements, and it is defined as linear isotropic for the elastic zone of the concrete, and multilinear isotropic for the plastic zone of the concrete. The multilinear isotropic material uses the von Mises failure criterion along with the Willam and Warnke (1974) model to define the failure of the concrete. The modulus of elasticity (EX) is defined, the poison's ratio (PRXY), and the compressive uniaxial stress-strain relationship for the concrete model. Implementation of the Willam and Warnke (1974) material model in ANSYS requires that different constants. These 9 constants are:

- 1. Shear transfer coefficients for an open crack.
- 2. Shear transfer coefficient for a closed crack.
- 3. Uniaxial tensile cracking stress.
- 4. Uniaxial crushing stress (Positive).
- 5. Biaxial crushing stress (Positive).
- 6. Ambient hydrostatic stress state for use with constants 7 and 8.
- 7. Biaxial crushing stress (Positive) under the ambient hydrostatic stress state (const.6).
- 8. Stiffness multiplier for cracked tensile condition.

Typical shear transfer coefficients range from 0.0 to 1.0 with 0.0 representing a smooth crack (complete loss of shear transfer) and 1.0 representing a rough crack (no loss of shear transfer). The shear transfers coefficients for open and closed cracks are determined using the work of Kachlakev, et al. as a basis. Convergence problems occurred when the shear transfer coefficient for the open crack dropped below 0.2. No deviation of the response occurs with the change of the coefficient. Therefore, the coefficient for the open crack is set to 0.30.

The uniaxial cracking stress is based upon the modulus of rupture. This value is determined using,

$$f_r = 7.5 \sqrt{f_c'}$$

[1]

The uniaxial crushing stress in this model is based on the uniaxial unconfined compressive strength (f'c) and is denoted as (f_t) . It can be entered as -1 to turn off the crushing capability of the concrete element as suggested by past researchers (Kachlakev, et al.). Convergence problems have been repeated when the crushing capability is turned on. The biaxial crushing stress refers to the ultimate biaxial compressive strength (f'cb). The ambient hydrostatic stress state is denoted as (σh) . This stress state is defined as:

$$\sigma h = (\sigma xp + \sigma yp + \sigma zp)/3$$

[2]

Where $(\sigma xp, \sigma yp, and \sigma zp)$ are the principal stresses in the principal directions. The biaxial crushing stress under the ambient hydrostatic stress state refers to the ultimate compressive strength for a state of biaxial compression superimposed on the hydrostatic stress state (f1). The uniaxial crushing stress under the ambient hydro static stress state refers to the ultimate compressive strength for a state of uniaxial compression superimposed on the hydrostatic stress state (f2). The uniaxial crushing stress state (f2). The failure surface can be defined with a minimum of two constants, (f t) and (f 'c). The remainder of the variables in the concrete model is left to default based on these equations:

$$\begin{aligned} f_{cb}' &= 1.2 \ f_{c}' & [3] \\ f_{1} &= 1.45 \ f_{c}' & [4] \\ f_{2} &= 1.725 \ f_{c}' & [5] \end{aligned}$$

Table 1 : Material Properties for Concrete Element.

These stress states are only valid for stress states satisfying the condition: $\sigma h \le \sqrt{3} f_c'$

[6]

Linear Isotropic			Multilinear Isotropic		
EX	21053		Point	Stress	Strain
PRXY	0.18		Point1	6.00	0.000284
			Point2	11.90	0.00060
Concrete			Point3	15.90	0.00090
Coeff.of Open Shear	0.30		Point4	18.39	0.0012
Coeff.of Closed Shear	0.90		Points	19.64	0.0015
Uniaxial Cracking Stress(f _{str})	3.5		Point6	20	0.0018
$\underbrace{Uniaxial}_{Crushing} \operatorname{Stress} (f_{cu})$	25		Point7	25	0.0030

Flexural Steel Reinforcement and Steel Shear Bolts

The steel for the finite element models was assumed to be an elastic-perfectly plastic material and identical in tension and compression. (Figure 8) shows the stress-strain relationship used in this study.



Parameters needed to define the material models can be found in (Table 2). There are multiple parts of the material model for the Steel Reinforcement Element. This Material

Model refers to the Link8 element, used for all the longitudinal steel reinforcement in the beam and it is defined as linear isotropic for the elastic zone of the steel and bilinear isotropic to define the second part of the curve as a straight line. Bilinear isotropic material is also based on the von Mises failure criteria. The bilinear model requires the yield stress (f_y), as well as the hardening modulus of the steel to be defined. Also the modulus of elasticity (EX), and the Poisson's ratio (PRXY).

Material Model	Element Type	Material Properties				
		Linear Isotropic		Bilinear Isotropic		
2	Link8	EX	2e5	Yield Stress	490	
(Tension RFT)	LIIKO	PRXY	0.30	Tangent Modulus	6000	
		Linear Isotropic		Bilinear Isot	ropic	
3	Limb-9	EX	2e5	Yield Stress	440	
(Compression RFT)	LIIKO	PRXY	0.30	Tangent Modulus	6000	

 Table 2 : Material Properties for The Steel Reinforcement Element.

Lead Plate

This Material Model refers to the Solid45 element can be defined in (Table 3). The Solid45 element used for the steel plates at top of column (loading point). Therefore, this element is modeled as a linear isotropic element with a modulus of elasticity for the steel (Es), and poison's ratio (PRXY).

Table 3 : Material Properties for the Lead Plates and Supports Element.

Material Model Number	Element Type	Material Properties		
		Linear Isotropic		
4	Solid45	EX	2e5	
		PRXY	0.30	

2.4. Loads and Boundary Conditions

Displacement boundary conditions are needed to constrain the model to get a unique solution. To ensure that the model acts the same way as the experimental flat slab specimens, boundary conditions need to be applied where the supports and loadings exist. If the support is modeled in such a way that a hinge is created, a four lines of nodes is given constraint in the vertical, and horizontal directions, applied as constant values of 0. If the support is modeled as a roller, a single line of nodes is given constraint in the vertical direction only (Y direction), applied as constant values of 0. By doing this, the slab will be allowed to rotate at the support but there are four quadrant nodes constrained in the two main direction (X and Z) directions as shown in (Figure 9) The displacement, Δ , is concentrated in the middle point of column head as a displacement control choice.



Figure 9 : Loads and Boundary Conditions of Flat Slab Models.

3. VALIDATION OF THE ANALYTICAL FINITE ELEMENT RESULTS

3.1. Specimens Details

The test specimens by M.Hamdy et al 2018 were half-scale models assumed to be equivalent to a slab-column connection in continuous slab system. The dimensions of the specimens are boundaries representing the lines of contra-flexure (approximately 0.4 times the span) in addition to 10cm from every side for the supporting steel frame. The continuous system is a flat slab with span equal to 9.00m in both directions. Specimens' dimensions are 2000 x 2000 x 150 mm with clear span between supporting beams equal to 1800mm in both directions. Two column aspect ratios were chosen; 1:1, and 2:1 with column dimensions equal to 220 x 220 x 220 mm, and 300 x 150 mm respectively as shown in (

Figure 10) and (Figure 11).

The clear concrete cover used was 10 mm for bottom and top reinforcement mesh. All specimens were reinforced with bottom longitudinal steel bars $\emptyset 18$ @ 100 mm in both directions and $\emptyset 10$ @ 200 in both directions for top reinforcement. Columns longitudinal reinforcement were 4 $\emptyset 18$ and confined with $\emptyset 10$ @ 100 mm in transverse direction.

Three main parameters were taken into consideration: column rectangularity, arrangement of bolts around columns, and spacing between peripheral rows. Seven specimens divided into two main groups were tested; the first group deal with three specimens with column aspect ratio 1:1, and four specimens in the second group with column aspect ratio was 2:1.

In group A, first specimen (S0) casted and tested without strengthening as a control specimen and the other two specimens (SC3X8–d1, and SC3X8–d2) were strengthened with eight rows of shear bolts, each row had 3 bolts with constant distance from column face to first peripheral row equal to 0.5 d and distance between peripheral rows was equal to 0.5 d, and ³/₄ d for second and third specimens as shown in (

Table 4).

Also for group B, the first three specimens (R0, RC3X8–d1, and SC3X8–d2) are as before in-group A. Specimens reinforcement are shown in (

Figure 10) and (Figure 11).

Table 4 :	General	Description	of Test Specimens.
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Group	Specimen	Column Aspect Ratio	Column Dimensions (mm)	SO	Si
	S 0	1	220 x 220		
А	SC3X8-d1	1	220 x 220	0.5 d	0.5 d
	SC3X8-d2	1	220 x 220	0.5 d	3/4 d
В	R0	2	300 x 150		
	RC3X8-d1	2	300 x 150	0.5 d	0.5 d
	RC3X8-d2	2	300 x 150	0.5 d	3/4 d



3.2. Comparison between Experimental and Numerical Analysis Results

Table 5) describes and illustrates the maximum punching shear force, corresponding vertical deflection for experimental results (M.Hamdy et al. 2018) and finite element results which was extracted from the finite element analysis using (ANSYS) program. The percentages between experimental punching shear force $V_{(EXP)}$ and the maximum punching load from finite element program $V_{(ANSYS)}$ were also monitored. These ratios have also been calculated for vertical deflection in experimental test $\Delta_{(EXP)}$ and finite element program $\Delta_{(ANSYS)}$.

Table 5) illustrates the maximum punching shear force, and corresponding vertical displacement resulting from experimental tests and finite element analysis using ANSYS program. The percentages between the maximum punching load from finite element program (PFEM.) and experimental punching shear force (PEXP.) were also monitored. These ratios have also been calculated for vertical deflection in finite element program (Δ FEM.) and experimental test (Δ EXP.). From results shown in (

Table 5) shows that the difference in punching loads between experimental and numerical results which does not exceed $\pm 6\%$. But regarding for vertical deflection, the difference was $\pm 9\%$. These values are very satisfactory and reliable in the work of the various parametric studies in the next chapter by using finite element program (ANSYS).

Cracking Pattern

(Figure 12) to (Figure 19) show the crack pattern at the top and bottom of the flat slabs specimens in the FE model (crack and crushing) which describe the cracks in the model due to tensile stresses. In the flat slabs models reinforced with steel Shear Bolts, existence of flexural cracks is obtained in both of experimental and FE model. This means that, the behavior of specimens contained steel shear head sections improved from brittle punching failure to semi brittle (Punching Flexural) failure.

Specimen	Experimental Results		Finite element Results		Difference	
Designation	Vu	Δu	Vu	Δu	V _{FEM.} /	$\Delta_{\mathrm{FEM.}}$ /
	(kN)	(mm)	(kN)	(mm)	V _{EXP.}	$\Delta_{\mathrm{EXP.}}$
S 0	473.00	10.81	479.77	10.25	1.01	0.95
SC3x8-d1	702.40	16.95	669.71	15.65	0.96	0.93
SC3x8-d2	646.39	15.36	636.23	14.14	0.99	0.92
SC3x8-d3	616.10	12.48	579.19	11.70	0.94	0.94
R0	468.00	11.20	493.00	10.35	1.05	0.92
RC3x8-d1	673.70	14.60	668.22	14.55	0.99	0.99
RC3x8-d2	663.60	14.12	650.58	13.67	0.98	0.97
RC3x8-d3	628.80	14.68	619.94	14.00	0.99	0.95

 Table 5 : Summary For Experimental And Finite Element Results.





Figure 12 : The Crack Pattern at The Top and Bottom of Specimen (S0).





Figure 13 : The Crack Pattern and Failure Mode at The Bottom and Top of Specimen (SC3X8-d1).



Figure 14 : The Crack Pattern and Failure Mode at The Bottom and Top of Specimen (SC3X8-d2).



Figure 15 : The Crack Pattern and Failure Mode at The Bottom and Top of Specimen (SC3X8-d3).



Figure 16 : The Crack Pattern at The Top and Bottom of Specimen (R0).





Bottom Top Figure 17 : The Crack Pattern and Failure Mode at The Bottom and Top of Specimen (RC3X8-d1).



Figure 18 : The Crack Pattern and Failure Mode at The Bottom and Top of Specimen (RC3X8-d2).





Load Displacement Curve

(Figure 20) to (Figure 27) illustrate the comparison between load-mid span deflections in experimental and FE model using ANSYS program. The differences between experimental and FF in terms of concrete deformations were considered due to both the limitations of concrete to deform with cracks and the crushing technique in ANSYS program. In general, both of FE and experimental responses have the same trend.

As shown in the following figures, the use of shear bolts in strengthening increase the displacement by 14% to 50% which mean that specimens with shear bolts are more ductile than control specimens. Also the failure type changed from brittle failure to semi brittle failure or (flexural punching).



F.E. Results of Specimen (RC3X8-d2).



4. The SUMMARY AND CONCLUSION

- ANSYS computer program was used to develop a nonlinear analysis of three dimensional reinforced concrete flat slabs under concentric increasing loads to simulate the behavior of flat slabs strengthened with shear bolts.
- Efficiency of the proposed finite element model for the reinforced concrete flat slabs strengthened with shear bolts, was proved from the comparison of the finite element model results with experimental results by M.Hamdy et al. 2018.
- Nonlinear finite element method based on advanced 3D models is a powerful and relatively economical tool which can be effectively used to simulate the true behavior of reinforced concrete slabs even under complex conditions.
- The crack pattern given by finite element model almost similar to the experimental ones and the same trend of the load-displacement response.
- The difference in failure load obtained from the finite element model and the experimental failure load was $\pm 6\%$ and $\pm 9\%$ for the maximum displacement.
- Flat slab specimens which connected by a square column with aspect ratio (1:1) give almost same results with other specimens connected by a rectangle column with aspect ratio equal (2:1).
- The contribution of steel shear bolts in punching load was about 30%, 26%, and 23% for the specimens strengthened with spacing d/2, 3/4d, and (d) respectively.
- Using shear bolts to strengthen column-slab connection against punching shear phenomenon is simple and easy to install, and it effectively improves the capacity of the slab.
- The use of shear bolts increases the maximum deflection and consequently increases ductility, which improves the column-slab connection capacity.

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