Al-Azhar University Civil Engineering Research Magazine (CERM)
Vol. (41) No. (1) January, 2019

Mean Annual Runoff Modeling within the Budyko Framework for Climate Impact Studies: Case Study in the Eastern Nile River

Mohamed M. Fathi[1], Ayman G. Awadallah[1], Ahmed M. Abdelbaki[1], Mohammed Haggag[2]

[1] Civil Engineering Dept., Faculty of Engineering, Fayoum University, Fayoum, Egypt
[2] Hydraulics and Irrigation Dept., Faculty of Engineering, Cairo University, Giza, Egypt
Ayman G. Awadallah

ملخص البحث

يوفر مفهوم التوازن المائي إطاراً يهدف إلى زيادة معرفتنا بخصائص أحواض الأنهار تحت الدراسة. ويتلخص هذا الإطار في دراسة الدورة الهيدرولوجية. ويعتبر التبخر الفعلي أحد أهم مكونات إطار التوازن المائي ويمكن تقديره مباشرة باستخدام جهاز وعاء التبخر وبالرغم من دقة هذا الجهاز إلا انها تعد عملية صعبة ومكلفة جداً. وبالتالي سيتم استخدام منهجية بوديكو لتقدير التبخر الفعلي من خلال دراسة التوازن بين الطاقة ومدى توفر المياه.

في هذه الورقة البحثية، نقوم أولاً بتطبيق ثمانية معادلات خاصة بمنهجية بوديكو على رافدي النيل الأزرق وعطبرة لتقدير التوازن السنوي المائي. لم تُظهر أي معادلة من الثمانية نتائج مرضية لأي من الرافدين. وبعد ذلك تم تطبيق أحدث معادلة بمنهجية بوديكو ((Du et al. (2016) على مقياس زمني أدق (شهري)، ووُجد أن النموذج الشهري يحاكي البيانات المرصودة بشكل أفضل من النموذج السنوي وفقاً لمعايير الأداء المختارة. كما أن النموذج الشهري باستخدام معادلة ((Du et al. (2016) يعتبر مناسباً لاستخدامه في دراسات التغيرات المناخية التي تستطيع تقدير كميات الأمطار والبخر الفعلية على وجه مقبول، ولكنها تفشل في تقدير الجريان السطحي بدقة عالية.

Abstract

The water balance concept provides a framework that aims to increase our knowledge of the catchment characteristics. This framework is summarized in studying the hydrological cycle of the catchment. Actual evapotranspiration which is one of the main important components within the water balance framework, can be estimated directly using the weighting lysimeter in a difficult and expensive way. Consequently, the Budyko framework is used to estimate the actual evapotranspiration by investigating the balance process between the energy and the water availability.

In this paper, we first investigate eight Budyko framework equations on the Nile tributary catchments of Blue-Nile and Atbara Rivers to estimate the annual water balance. None of the eight versions of the Budyko produces satisfactory results. Then, the Du et al. (2016) the latest Budyko framework equation is applied on a finer timescale (monthly timescale). The monthly model found to fit the observed data better than the annual model according to six performance criteria. The monthly Du at al. (2016) model is well suited to be used in climate impact studies which reproduce monthly precipitation and evapotranspiration with a relatively good accuracy but fail to reproduce the resulting runoff with the same accuracy.

Key words: Water balance, Budyko Framework, Blue-Nile, Atbara, Nile River.

1. Introduction

One of the main objects of hydrology is estimating the water balance at the catchment scale (Wang and Zhou, 2016). The concept of water balance is summarized in studying the hydrological cycle of a catchment over a specific period of time. The water balance is an equality between the input resources represented in precipitation, and outputs represented in a combination between evapotranspiration, infiltration and runoff (Beniston, 2002). The water balance framework can be represented as:

$$P = ET + R + \Delta S \qquad \text{Equ. 1}$$

in which P is precipitation (mm), ET is actual evapotranspiration (mm), R is surface runoff (mm), and ΔS is the change in catchment water storage (mm) that may take positive or negative values (Zhang et al., 2011; Wu et al., 2017).

Precipitation is the most important component in the hydrological cycle. The world efforts are devoted to collecting the precipitation data because of its important role in the hydrological cycle. Actual evapotranspiration is another main important component in the hydrological cycle especially in arid catchments. Evapotranspiration is a combination between transpiration from the crops and evaporation from the soil surface from anywhere like lakes, rivers and bare land surfaces (Ward and Trimble, 2002). In fact, actual evapotranspiration is not easy to measure. This led to develop many techniques to estimate it (Rana and Katerji, 2000).

Surface runoff is the process by which the excess water of precipitation flows over the soil surface. A large portion of the runoff flows downslopes into several receiving water bodies like streams, lakes, rivers or oceans (Ward and Trimble, 2002). The change in catchment water storage is the last component of the water balance equation. Water balance studies have various ways to deal with it. Some studies assumed that the change in water storage can be neglected over long time scales without defining a specific period of time to neglect it (Istanbulluoglu et al., 2012); or with a specific period (5-10 years) (Zhang et al., 2001). Xu et al. (2013) and Zhang et al. (2008) assumed neglecting the change in water storage for annual models. However, due to the poor performance for some catchments, Zhang et al. (2008) concluded that more complicated models, taking into consideration the change in water storage, are required for better performance to be achieved using annual models. Chen et al. (2013) found that the effects of the change in water storage cannot be neglected for the annual models and concluded that the inter-annual models are more sensitive to the change in water storage.

The water balance framework has these four main components represented in: precipitation, evapotranspiration, surface runoff and change in water storage. The precipitation and surface runoff are the only two components that can be easily measured; while the evapotranspiration and the change in water storage components are difficult to measure (Alimohammadi, 2012). The change in water storage is difficult to measure because of its complexity with many other variables involved such as soil properties, soil texture, porosity, land slopes, topography, vegetation, air temperature and precipitation. Furthermore, modeling large catchments makes measurements very difficult and very costly. This is the main reason for some researchers to neglect the change in water storage to estimate or simulate the evapotranspiration. consequently, many techniques, equations or models were developed to take the change in water storage into consideration, and avoid measuring it.

In this research, we investigate the use of the Budyko framework equations applied on the Eastern Nile catchments of Blue-Nile and Atbara. The Budyko equations estimate the actual evapotranspiration using as input the precipitation and potential evapotranspiration. The paper is structured as follows: after the current introduction, the next section presents the research methodology. Section three describes the study area and the used data. Section four presents the obtained results and discusses the advantages and limitations of the used equations. Finally, section five summarizes the research conclusions and recommendations for future work.

2. Theory and method

2.1. Budyko framework

The Budyko framework is an effective tool used for estimating the actual evapotranspiration by investigating the balance process between the energy and the water availability (Alimohammadi, 2012; McMahon et al., 2013). The Budyko framework investigates the climatic condition using the aridity index (ϕ) term which is a dimensionless ratio of the potential evapotranspiration (PET) divided by the precipitation (P) (Greve et al., 2015).

The first start of this approach is much older than Budyko (1948). Schreiber (1904) developed the first non-linear relationship in the form of (the first equation in

Table **2**). A few years later, Ol'dekop (1911) introduced a hyperbolic tangent formula (the second equation in

Table **2**). Then, Budyko (1948) made a large number of studies over rivers in Europe and the former Soviet Union and the plotted data were found to be between Schreiber (1904) and Ol'dekop (1911) curves; then he proposed a newer formula as a geometric mean of the two formulas (the third equation in

Table **2**) (Arora, 2002). Budyko (1974) discovered that the main factors determining the long-term evapotranspiration on the catchment scale were the potential evapotranspiration (available energy) and precipitation (available water) (Yang et al., 2008).

As a continuation of the studies forming the Budyko framework, Turc (1954) proposed another formula with a different shape in the form of (Equ. 2) as a function of the aridity index (ϕ) (Arora, 2002). This equation was applied on about 254 catchments around the world (Lebecherel et al., 2013). The formula of (Equ. 2) was applied on 4 catchments in Malawi by Pike (1964) who modified the equation constant from 0.9 to 1.0 which gave better results (the fourth equation

Table **2**) (Choudhury, 1999).

$$ET = P * \frac{1}{\sqrt{0.9 + \left(\frac{1}{\phi}\right)^2}} \qquad \text{Equ. 2}$$

2.2. Analytical solutions for the Budyko framework

Bagrov (1953) made the first trial to derive an analytical equation to simulate the Budyko curve by introducing the first derivative in the form of (Equ. 3), where n indicates the catchment characteristics (Yang et al., 2007, 2008). This formula presents the ability to be integrated into Schreiber formula for n = 1; and into Ol'dekop formula for n = 2; but it cannot be integrated for other values of n (Lebecherel et al., 2013).

$$\frac{dE}{dP} = 1 - \left(\frac{E}{E_o}\right)^n \qquad \text{Equ. 3}$$

Table 2: Some of developed equations within the Budyko framework

N.	Equation	Parameter	Reference	
1	$\frac{ET}{P} = 1 - e^{-\phi}$	-- ---	(Schreiber, 1904)	First generation of the Budyko framework
2	$\frac{ET}{P} = \phi \, tanh\left(\frac{1}{\phi}\right)$	-- ---	(Ol'dekop, 1911)	
3	$\frac{ET}{P} = \left[\phi \, tanh\left(\frac{1}{\phi}\right) \, (1 - e^{-\phi})\right]^{1/2}$	-- ---	(Budyko, 1948)	
4	$\frac{ET}{P} = [1 + (\phi)^{-2}]^{-1/2}$	-- ---	(Turc, 1954; Pike, 1964)	
5	$\frac{ET}{P} = [1 + (\phi)^{-n}]^{-1/n}$	n	(Turc, 1954; Mezentsev, 1955; Pike, 1964; Choudhury, 1999; Yang et al., 2008)	Second generation of the Budyko framework
6	$\frac{ET}{P} = 1 + \phi - [1 + (\phi)^{\omega *}]^{1/\omega *}$	$\omega *$	(Fu, 1981; Zhang et al., 2004; Yang et al., 2007)	
7	$\frac{ET}{P} = \frac{1 + w\,\phi}{1 + w\,\phi + \phi^{-1}}$	w	(Zhang et al., 2001)	
8	$\frac{ET}{P} = 1 + \phi - [1 + (\phi)^{\omega} + \lambda]^{1/\omega}$	λ , ω	(Du et al., 2016)	

Mezentsev (1955) revisited Bagrov (1953) work and rewrote his formula in the form of (Equ. 4) (Lebecherel et al., 2013). This formula can be integrated analytically assuming that $m=(n+1)/n$ to obtain (the fifth equation in

Table **2**) (Yang et al., 2007). This equation is recognized with the first equation within the second generation of Budyko framework introducing one or more adjustable parameters. This adjustable parameter can be calibrated to individual catchments using observed data. Mezentsev (1955) calibrated this equation over 35 catchments over Siberia and suggested to use the value of 2.3 for the parameter n, which is close to Turc parameter (Lebecherel et al., 2013).

$$\frac{dE}{dP} = \left[1 - \left(\frac{E}{E_o}\right)^n\right]^m \qquad \text{Equ. 4}$$

Another equation was developed in China (the sixth equation in (

Table 2) by Fu (1981). This paper was published in Chinese only (Zhang et al., 2004). Zhang et al., (2004) revisited the Fu (1981) work to study the catchment characteristics (climate, vegetation, topography, soil properties, etc.) and their effects on water balance on the catchment scale. The equation's parameter w represents the catchment characteristics and its effect on the evapotranspiration (Yang et al., 2007).

Zhang et al. (2001) tried to develop a new generic formula (the seventh equation in

Table 2) to estimate the actual evapotranspiration taking into account the vegetation change effects on the catchment scale. The results of these studies have a clear conclusion that the plant available water capacity is the main factor causing greater evapotranspiration from the forests than from crops (Turner, 1991; Nepstad et al., 1994; Hodnett et al., 1995).

All the previously mentioned studies and equations of the Budyko framework were initially applied on the steady state conditions which can be summarized in two points: long term time scales and closed basins (Moussa and Lhomme, 2016). Du *et al.* (2016) who introduced an equation (improving Fu (1981) equation) with two adjustable parameters ω and λ, to overcome the two weak points, as ω reflects the catchment characteristics and λ extends the steady state condition (refer to the last equation in

Table 2). The developed equation with ω and λ enables the Budyko framework to apply the water balance process under unsteady state conditions (unclosed basins and finer time scales than the annual) (Du et al., 2016).

2.3. Performance criteria (efficiency criteria)

The efficiency criteria which are used in this study are presented in this section. These mathematical criteria are used for comparing models by computing some kind of distance between the simulated and observed values (Waseem et al., 2017). There are several performance criteria to evaluate the hydrological models, but six of the most known criteria are used in this study which are Coefficient of Determination (R^2), Root Mean Square Error ($RMSE$), Nash-Sutcliffe Efficiency (NSE), Bias, Mean Absolute Percentage Error ($MAPE$) and Kling-Gupta Efficiency (KGE) (summarized in the following table).

Table 3: Efficiency criteria used in the current study

Criteria	Formula
Coefficient of Determination R^2	$R^2 = \left(\frac{\sum_{i=1}^{n}(O_i - \bar{O})(S_i - \bar{S})}{\sqrt{\sum_{i=1}^{n}(O_i - \bar{O})^2}\sqrt{\sum_{i=1}^{n}(S_i - \bar{S})^2}} \right)^2$
Nash-Sutcliffe Efficiency (NSE)	$NSE = 1 - \frac{\sum_{i=1}^{n}(O_i - S_i)^2}{\sum_{i=1}^{n}(O_i - \bar{O})^2}$

Root Mean Squared Error (RMSE)	$RMSE = \sqrt{\sum_{i=1}^{n}(O_i - S_i)^2}$
Kling–Gupta efficiency (KGE)	$KGE = 1 - \sqrt{(r-1)^2 + (\alpha-1)^2 + (\beta-1)^2}$
Bias	$Bias = \frac{\sum_{i=1}^{n}(O_i - S_i)}{n}$
Mean Absolute Percent Error (MAPE)	$MAPE = 100 * \frac{\sum_{i=1}^{n}\frac{\lvert O_i - S_i \rvert}{\lvert O_i \rvert}}{n}$

where O_i, S_i are the observed and the simulated values respectively; r is the linear correlation coefficient between O and S (which is also the square root of R^2); β is the ratio between the mean of the simulated and observed values; and α is the ratio between the standard deviation of the simulated and observed values.

3. Study area and data sources

3.1. Study area

Ethiopia is the richest Nile Basin country in water resources (Yacob, 2002). Ethiopia highland plateau through the Blue-Nile and Atbara tributaries (

Figure 1) is considered the main source of the Nile River water with 72% of the Main Nile River water (the percentages for Blue-Nile and Atbara rivers are 59% and 13%, respectively) (Melesse, 2011). Ethiopia is subjected to a strong seasonality of precipitation, with most of the precipitation falling from June to September (only four months) (Melesse, 2011) with a peak point in July or August.

The Blue-Nile River flows from Lake Tana near Bahir Dar then travels to the Ethiopia-Sudan border where it flows in Sudan lands till joining the White-Nile at Khartoum. The Blue-Nile basin (8-16^{o}N, 32-40^{o}E) covers an area of approximately 310,000 Km^2, with elevations varying between 362 m and 4260 m (

Figure 1). The mean annual aridity index *(ϕ)* is 1.35. Consequently, the basin can be classified as a sub-humid basin (Arora, 2002). The mean annual precipitation over the catchment is 1100 mm/year observed over the period from 1901 to 2001.

Atbara River flows also from Ethiopia highlands. Its sources are not far from those of the Blue-Nile River, but it does not flow from a lake such as Lake Tana in the Blue Nile. The Atbara basin (12-19^{o}N, 34-40^{o}E) covers an area of approximately 231,000 Km^2, with elevations varying between 342 m and 4505 m (

Figure 1). Atbara River is also subjected to a strong seasonal precipitation. During the dry season, the flow in the river tributaries is greatly reduced and may dry up completely. The mean annual aridity index *(ϕ)* is *3.4*. Consequently, the basin can be classified as a semi-arid basin (Arora, 2002). The mean annual precipitation over the catchment is 530 mm/year observed over the period from 1901 to 2001.

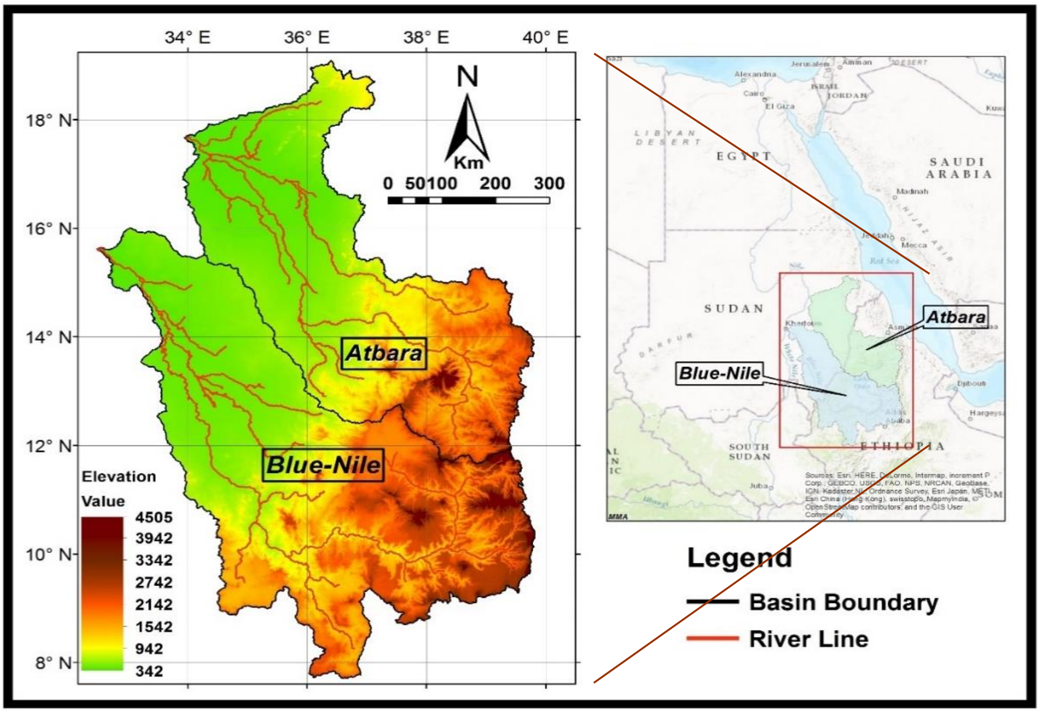


Figure 1: Location of study area

3.2. Data sources

The required data for the Budyko framework are the monthly precipitation, potential evapotranspiration and runoff data for each basin. The monthly runoff data are recorded in Sudan by the gauging stations on Atbara and Blue-Nile Rivers. They were naturalized, by the consulting firm Deltares through an Eastern Nile Technical Regional Office (ENTRO) Regional Office project to be used for the Nile basin model using Ribasim. The naturalization consists mainly of re-adding the amount of water evaporated from dams and abstracted for irrigation and other uses. Precipitation and potential evapotranspiration data were extracted from the global gridded database sources; the monthly precipitation data from the Global Precipitation Climatology Centre (GPCC, 2018) and the monthly potential evapotranspiration data from the Climatic Research Unit (CRU, 2018). The annual runoff, precipitation and potential evapotranspiration are obtained by summing up monthly data.

The Global Precipitation Climatology Centre (GPCC) is operated in support of the World Meteorological Organization (WMO), and its objective is to provide high-quality global precipitation database over land (Schamm et al., 2014) for the period from 1901 to 2013 at different spatial resolutions (0.5°, 1° and 2.5°) on the basis of in-situ measurements (Ziese et al., 2013). The variation in precipitation according to space-time directions requires high density of data. In this research, we used the monthly 0.5° gridded precipitation database, version 7 (GPCC, 2018).

The Climatic Research Unit (CRU) is widely recognized as one of the world's leading institutions concerned with the study of natural and anthropogenic climate changes. One of the main aims of this unit is to provide a time series of the past climate history database for the period from 1901 to 2015 (CRU, 2018). Time-series datasets are month-by-month variation in climate over the last century or so, as produced by the CRU at the University of East Anglia. These are calculated on high-resolution (0.5x0.5 degree) grids, which are based on an archive of monthly mean temperatures provided by more than 4000 weather stations distributed around the world (Jones and Harris, 2008). The CRU database (version 3.23) provides the potential evapotranspiration, and covers all land areas excluding Antarctica at 0.5^{o} resolution.

4. Results and discussion

4.1. Comparison between Budyko framework equations (Annual time scale)

The water balance framework presented in Equ. 1 was applied on the Blue-Nile and Atbara watersheds. On the Annual time scale, if the change in water storage can be neglected, then the water balance equation can be expressed as follows;

$$P = ET + R \qquad \text{Equ. 5}$$

The actual evapotranspiration is calculated based on Budyko framework equations (

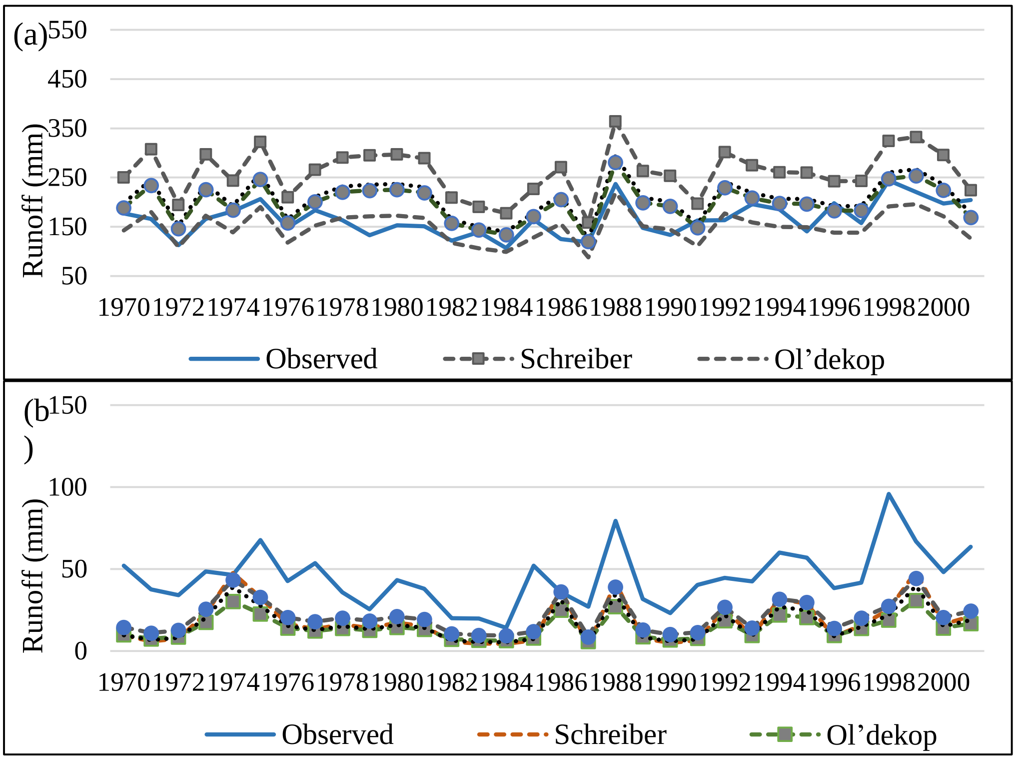
Table **2**).

4.1.1. First generation of Budyko framework equations

The first generation of Budyko framework (the first four equations in

Table **2**) does not have any calibrated parameters. The equations are defined only as a function of the aridity index *f(Ø)*. From the first generation of Budyko framework, the actual evapotranspiration can be estimated directly from any of the equations based on the aridity index; then the annual runoff values can be estimated according to Equ. 5.

The results of the comparison between the first generation of Budyko framework equations are summarized in **Error! Reference source not found.** and



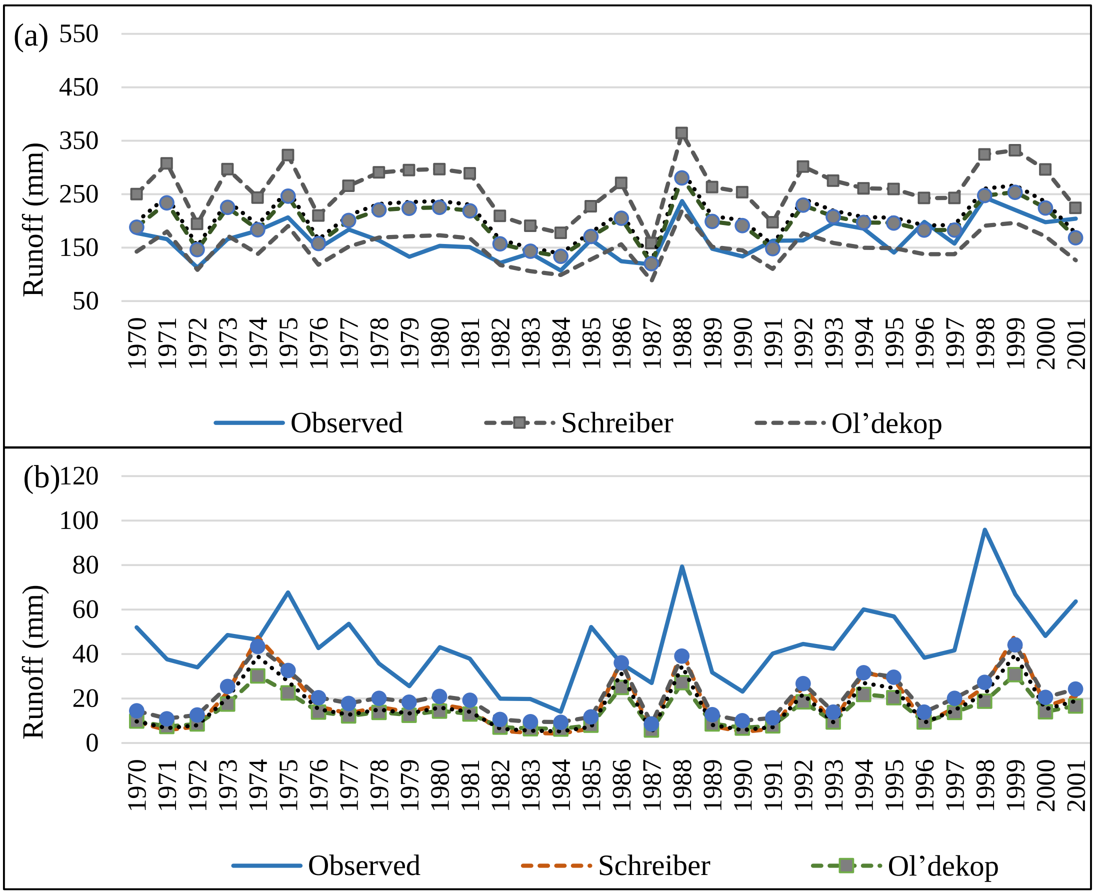


Figure 2. The four equations gave extremely poor results as indicated by the negative values of NSE for both the Blue-Nile and Atbara Rivers except one small positive value approaching zero. The main reason behind these results is of course the lack of adjustable parameters.

Table 4: Performance of the first generation of Budyko on the Blue-Nile and Atbara Rivers

Criteria	Basin	Schreiber	Ol'dekop	Budyko	Turc-Pike
R^2	Blue-Nile	0.38	0.37	0.37	0.37
	Atbara	0.36	0.37	0.36	0.37
RMSE (mm)	Blue-Nile	122.8	**33.0**	68.0	58.2
	Atbara	29.7	34.2	31.8	27.3
NSE	Blue-Nile	-12.31	**0.04**	-3.09	-1.99
	Atbara	-3.03	-4.34	-3.62	**-2.40**
Bias (mm)	Blue-Nile	113	**-6.6**	55	44
	Atbara	-27	-32	-29	**-25**
MAPE	Blue-Nile	67 %	**15.4 %**	34 %	28 %
	Atbara	57 %	66 %	61 %	**50 %**
KGE	Blue-Nile	-0.06	**0.58**	0.32	0.40
	Atbara	0.28	0.05	0.18	**0.25**

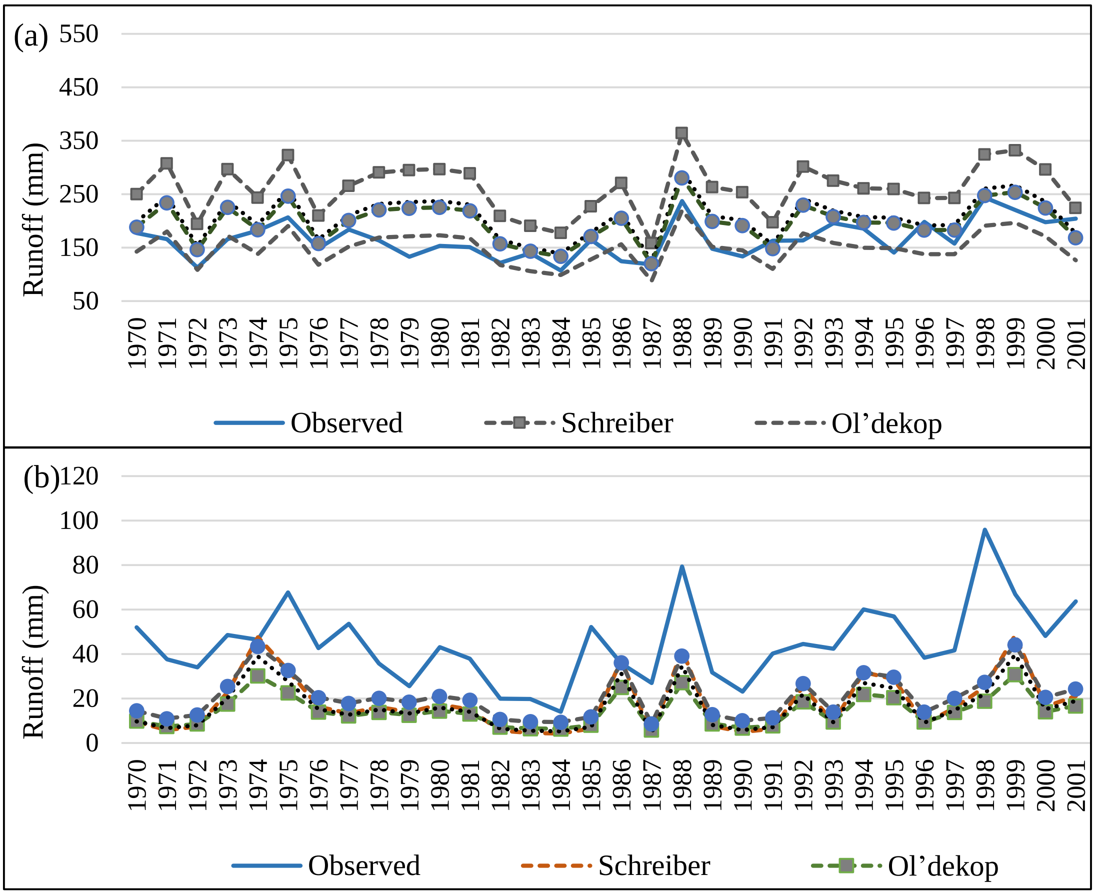


Figure 2: Simulated values for the first generation Budyko framework equations and observed annual runoff during 1970-2002; (a) Blue-Nile River (b) Atbara

For the Blue-Nile River, the Ol'dekop equation gave the closest simulated values to the observed values with NSE = 0.04, R^2 = 0.37, RMSE = 33.0 mm and KGE = 0.58; followed by Turc-Pike equation then Budyko and finally Schreiber equation with NSE values equal to -1.99, -3.09 and -12.3, respectively. For Atbara River, Turc-Pike equation was the best equation to simulate the runoff with NSE = -2.40, R^2 = 0.37, RMSE = 27.3 mm and KGE = 0.25, followed by Schreiber equation then Budyko and finally Ol'dekop equation with NSE values equal to -3.03, -3.62 and -4.34, respectively.

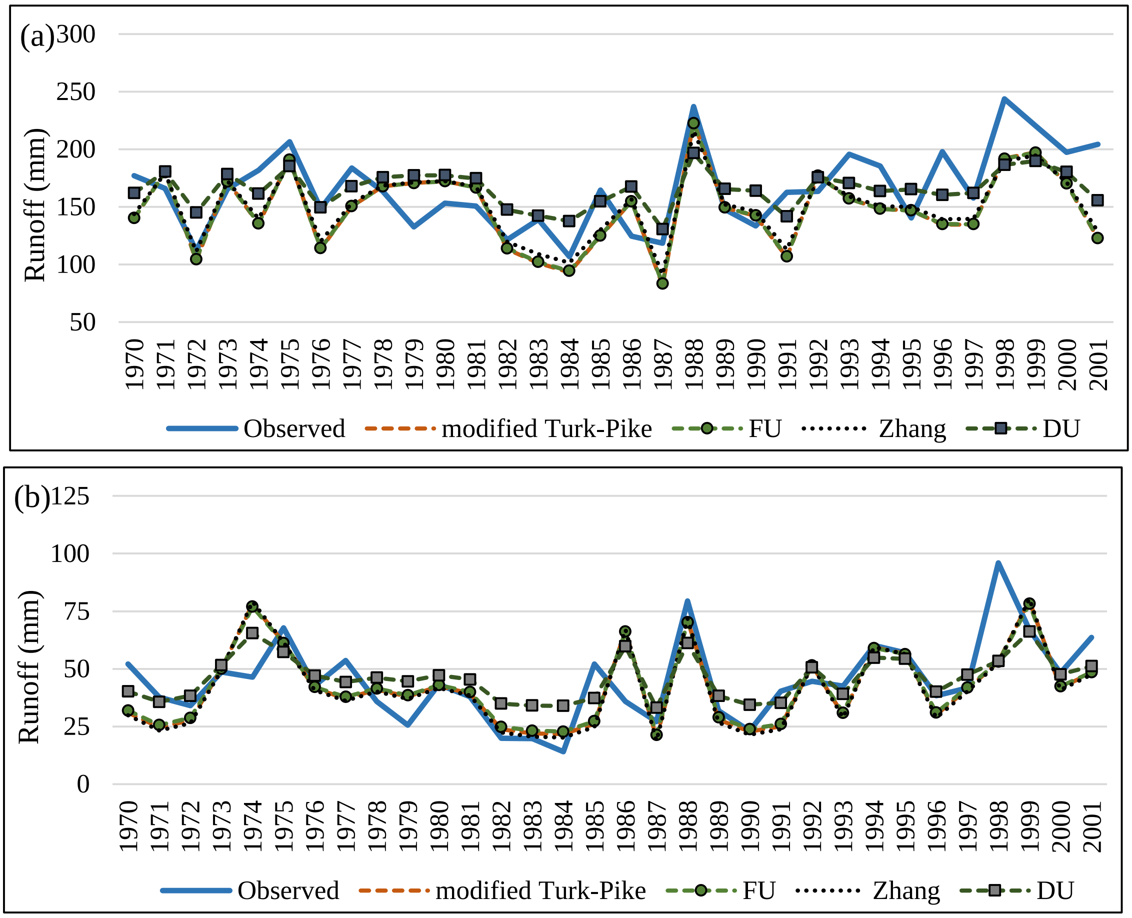
4.1.2. Second generation of Budyko framework equations

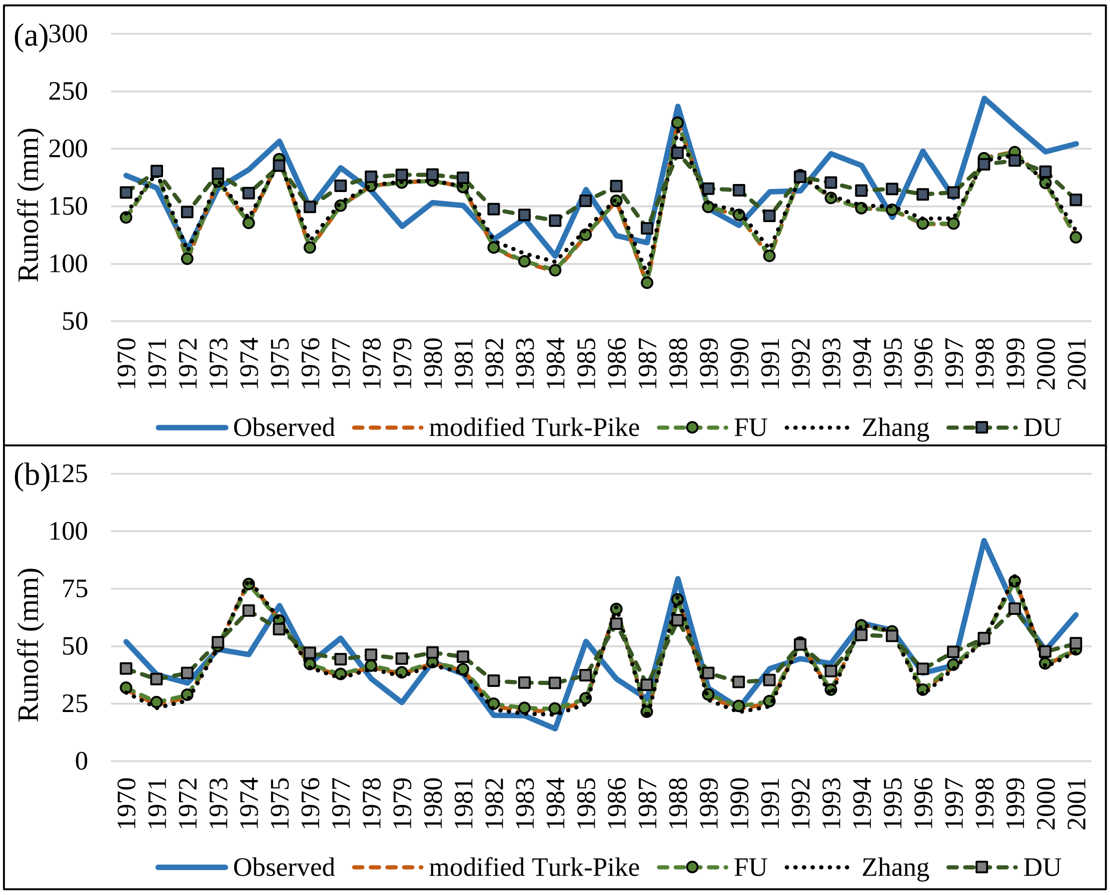
The main difference between the first and second generations of Budyko framework equations is that the second generation equations have adjustable (calibrated) parameters. All equations can be defined in terms of the aridity index in addition to one or more adjustable parameters as *f(Ø, C)*. The adjustable parameters are calibrated based on minimizing the RMSE for the period of 1901-1969 representing the first 70% of the available data. The calibrated parameters of the studied equations are summarized in **Table 5**. The remaining 30% of the data are used to validate the equations. After the calibration process, the actual evapotranspiration can be estimated based on each equation; then the annual runoff values can be estimated from Equ. 5. One of the main reasons for the difference between the values of adjustable parameters for the Blue-Nile and Atbara, is the difference in basin classification between sub-humid and semi-arid, respectively; besides other catchment characteristics such as vegetation, forest cover, soil properties or topography.

Table 5: Adjustable parameters of the second generation of Budyko framework equations for the Blue-Nile and Atbara basins

Equation	Modified Turc-Pike	FU	Zhang	Du	
Parameter	n	ω	w	ω	λ
Blue-Nile	2.43	3.14	2.36	1.29	-0.77
Atbara	1.58	2.33	0.69	1.25	-0.85

The results of the comparison between the second generation of Budyko framework equations are summarized in Table 6

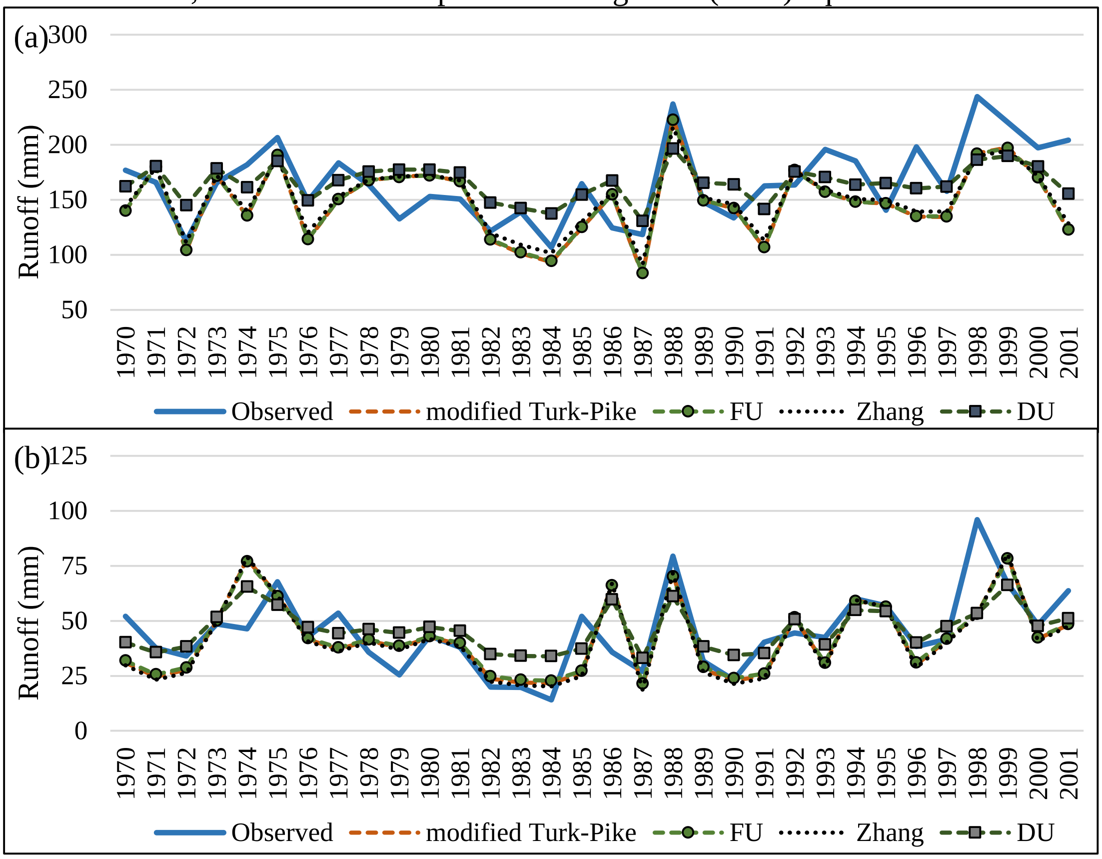




(a)
Runoff (mm)
300
250
200
150
100
50
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
Observed
modified Turk-Pike
FU
Zhang
DU
(b)
Runoff (mm)
125
100
75
50
25
0
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
Observed
modified Turk-Pike
FU
Zhang
DU

and Figure 3.

The results show better performance compared to the first generation equations indicated by an increase in NSE and R^2 values and a decrease in RMSE, bias and MAPE values. All the validation NSE values are positive for both the Blue-Nile and Atbara basins. For the Blue-Nile River, the equation that gave the best performance criteria is that of Du et al. (2016), with NSE = 0.39, R^2 = 0.43, RMSE = 27.0 mm and MAPE = 14.6 %, for the validation portion. Zhang et al. (2001) equation has the second



rank followed by Fu (1981) equation and finally the modified Turc-Pike equation with NSE values equal to 0.19, 0.07 and 0.06, respectively, for the validation portion. For Atbara River, also Du et al. (2016) equation gave the closest simulated values to the observed values, with NSE = 0.43, R^2 = 0.45, RMSE = 13.2 mm and MAPE = 28 %, for the validation portion. Its parameters are ω= 1,25 and λ= -0.85. Fu (1981) equation has the second rank followed by modified Turc-Pike and finally Zhang et al. (2001) equation with NSE values equal to 0.34, 0.31 and 0.26, respectively, for the validation portion.

Although, the second generation of Budyko framework gave better performance criteria than the first generation, these results are still not satisfactory for both basins. One of the first suggestions to improve the results is to use finer timescale models for the simulation, as described in the following section.

Table 6: Performance of the second generation of the Budyko framework equations on the Blue-Nile and Atbara Rivers

	Efficiency criteria	Basin	Modified Turc-Pike	FU	Zhang	Du
Validation	R^2	Blue-Nile	0.42	0.42	0.42	<u>0.43</u>
		Atbara	0.43	0.43	0.43	<u>0.45</u>

	RMSE (mm)	Blue-Nile	33.5	33.2	31.1	27.0
		Atbara	14.5	14.2	15.0	13.2
	NSE	Blue-Nile	0.06	0.07	0.19	0.39
		Atbara	0.31	0.34	0.26	0.43
	Bias (mm)	Blue-Nile	-17.8	-17.6	-15.1	-0.4
		Atbara	-3.0	-2.6	-3.7	1.4
	MAPE	Blue-Nile	17 %	16 %	15 %	14.6 %
		Atbara	24 %	23 %	24 %	28 %
	KGE	Blue-Nile	0.63	0.63	0.60	0.36
		Atbara	0.65	0.64	0.64	0.44

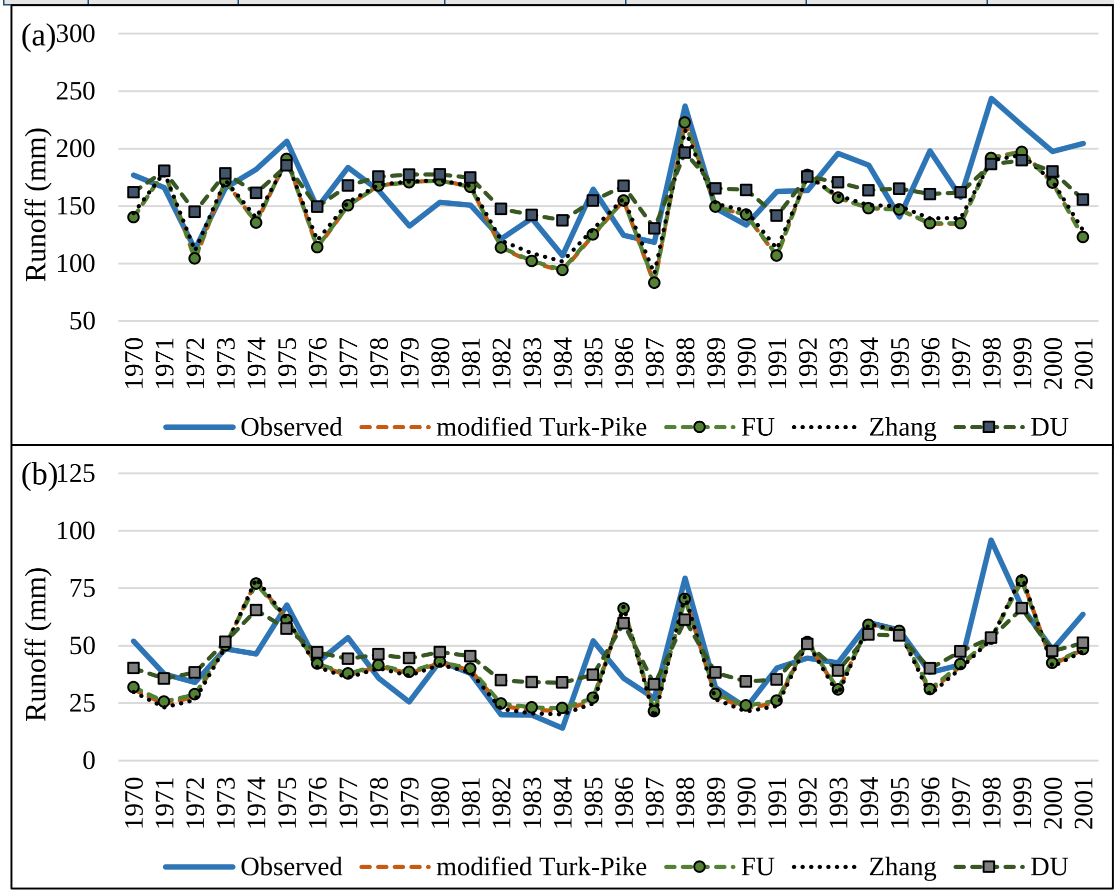


Figure 3: Simulated values for the second generation Budyko framework equation and observed annual runoff during 1970-2001; (a) Blue-Nile River (b) Atbara River.

4.2. Application of Du et al. (2016) equation on the Blue-Nile basin on the monthly time scale

To improve on the previously obtained unsatisfactory performance criteria, the Budyko framework is applied on a finer timescale (monthly timescale). The steady state condition is violated and the change in water storage cannot be neglected. The water balance framework presented in Equ. 1 will be applied on the Blue-Nile River. All Budyko framework equations

Table **2** cannot be used on the unsteady state condition except the Du et al. (2016) equation which could take into consideration the effects of the change in water storage over a finer timescale than the annual.

The calibration process is applied to estimate the equation's two adjustable parameters based on minimizing the monthly RMSE on the period of 1901-1969 representing the first 70% of the available data. The remaining 30% of the data are used to validate the calibrated parameters. The calibrated parameters are ω= 1.17 and λ= -0.85, with monthly results NSE = 0.49, R^2 = 0.51, RMSE = 13.21 mm and bias = -1.8 mm, for the calibration portion. The annual results are obtained by summing up the monthly values, with NSE = -0.11, R^2 = 0.37, RMSE = 34.12 mm and bias = -21.86 mm, for the calibration portion. The annual results show poor simulation for the Blue-Nile River compared to the monthly results. Consequently, a second suggestion is investigated based on minimizing the annual RMSE. The calibrated parameters are ω= 1.80 and λ= -0.79, with monthly results NSE = 0.33, R^2 = 0. 52, RMSE = 15.12 mm and bias = 0.12 mm, for the calibration portion; and the annual results are NSE = 0.41, R^2 = 0.41, RMSE = 24.88 mm and bias = 1.43 mm, for the calibration portion. The results of the second suggestion are much better.

The monthly and annual results are summarized in Table 7,

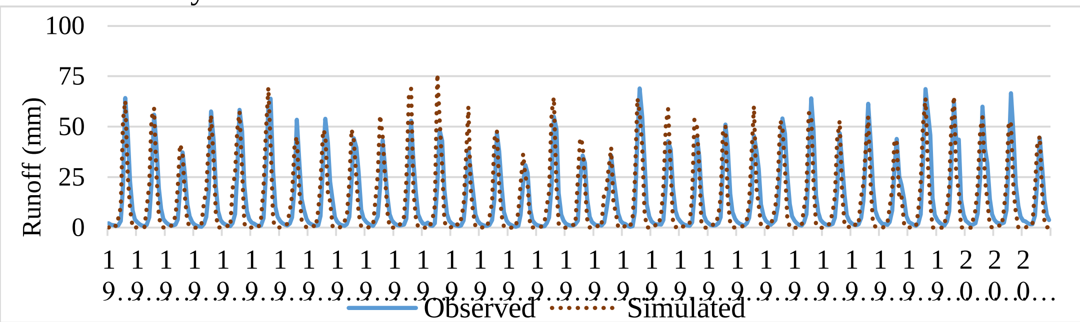


Figure 4 and

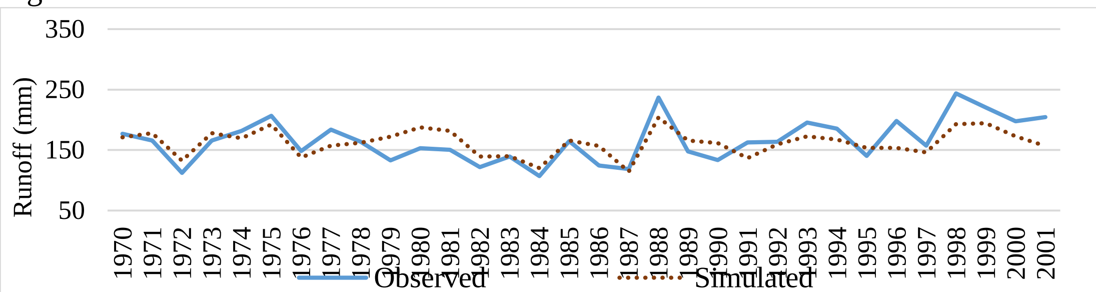


Figure 5. One can notice that the finer timescale model is fitting observed values better than the annual models. The annual NSE, R^2 and KGE values are increased from 0.39, 0.43 and 0.36 respectively for the annual model to 0.49, 0.51 and 0.52 for the monthly model for the validation portion of data.

The monthly Du at al. (2016) model is well suited to be used in climate impact studies which reproduce monthly precipitation and evapotranspiration with a relatively good accuracy but fail to reproduce the resulting runoff with the same accuracy.

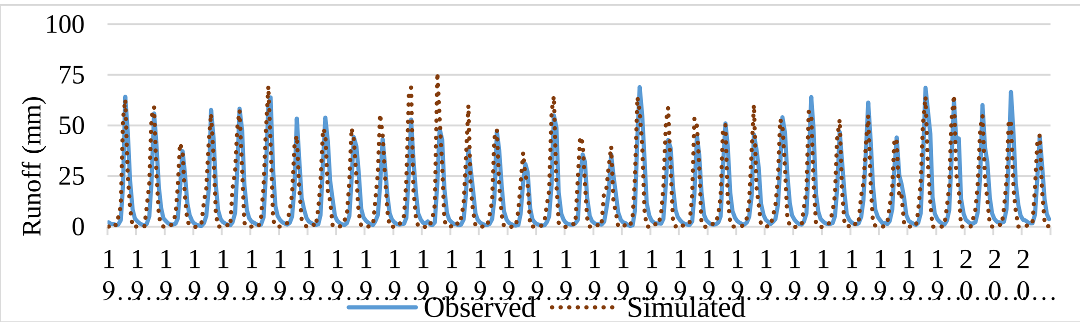


Figure 4: monthly results of Du et al. (2016) equation simulated (monthly model) and observed runoff values for the Blue-Nile River during 1970-2002 (validation portion).

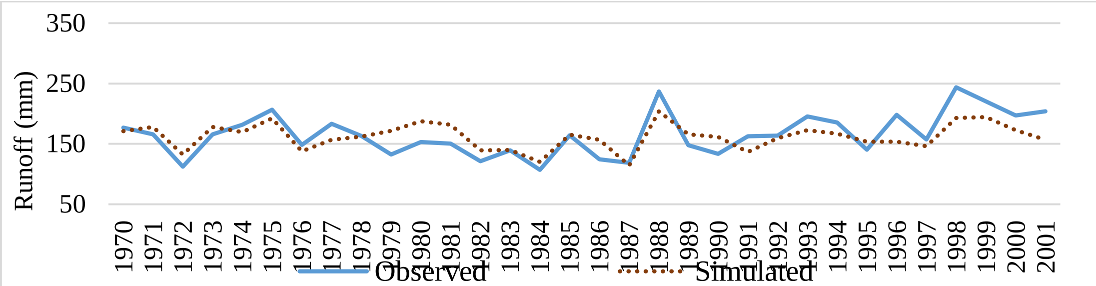


Figure 5: Annual results of Du et al. (2016) equation simulated (monthly model) and observed runoff values for the Blue-Nile River during 1970-2001 (validation portion).

Table 7: Performance of Du et al. (2016) equation on the Blue-Nile basin (monthly model)

Efficiency criteria		R^2	RMSE (mm)	NSE	Bias (mm)	MAPE	KGE
Monthly	Calibration	0.52	15.12	0.33	0.12	128 %	0.68
	Validation	0.62	11.81	0.52	-0.28	112 %	0.76
Annual	Calibration	0.41	24.88	0.41	1.43	11.4 %	0.53
	Validation	0.51	24.56	0.49	-3.41	12.4 %	0.52

5. Conclusion and recommendations

This study focused on evaluating the performance of the Budyko framework as a water balance approach when applied on the Blue-Nile and Atbara basins. Eight Budyko framework equations are applied on the Blue-Nile and Atbara Rivers to estimate the annual water balance. Du et al. (2016) equation is the best one among the eight to simulate runoff with annual validated NSE equals to 0.39 and 0.43 for the Blue-Nile and Atbara rivers respectively.

Because of the unsatisfactory results of the annual Budyko model results, the Budyko framework is applied on a finer timescale (monthly timescale). Du et al. (2016) equation is the only equation among the eight which can be applied on the steady and unsteady state conditions (the annual as well as the monthly time scales). The monthly model gives better performance criteria than the annual model. The annual validated NSE for the Blue-Nile River is increased from 0.39 for the annual model to 0.49. The monthly validated R^2, NSE and KGE are 0.62, 0.52 and 0.76 respectively.

The obtained results could be used for climate change impact assessments. Future work could focus on developing hybrid models that integrate Kaman filters on time series analyses with the Budyko framework.

References

Alimohammadi, N.: Modeling annual water palance in the seasonal Budyko framework, the University of Central Florida., 2012.

Arora, V. K.: The use of the aridity index to assess climate change effect on annual runoff, Journal of hydrology, 265, 164–177, 2002.

Bagrov, N.: On multi-year average of evapotranspiration from land surface, Met. Gidrol, 10, 21–30, 1953.

Beniston, M.: Climatic change: implications for the hydrological cycle and for water

management, Springer Science., 2002.
Budyko, M. I.: Evaporation under natural conditions, Israel Program for Scientific Translations, Jerusalem, 1948.
Budyko, M. I.: Climate and Life, 508 pp., Academic Press, New York, 1974.
Chen, X., Alimohammadi, N. and Wang, D.: Modeling interannual variability of seasonal evaporation and storage change based on the extended Budyko framework, Water Resources Research, 49(March), 6067–6078, doi:10.1002/wrcr.20493, 2013.
Choudhury, B. J.: Evaluation of an empirical equation for annual evaporation using field observations and results from a biophysical model, Journal of Hydrology, 216(1–2), 99–110, doi:10.1016/S0022-1694(98)00293-5, 1999.
CRU: Climatic Research Unit, [online] Available from: http://www.cru.uea.ac.uk/about-cru (Accessed 20 March 2018), 2018.
Du, C., Sun, F., Yu, J., Liu, X. and Chen, Y.: New interpretation of the role of water balance in an extended Budyko hypothesis in arid regions, Hydrology and Earth System Sciences, 20(1), 393–409, doi:10.5194/hess-20-393-2016, 2016.
Fu, B.: On the calculation of evaporation from land surface in mountainous areas, Sci. Atmos. Sin., 5(1), 23–31, 1981.
GPCC: Global Precipitation Climatology Centre, [online] Available from: https://www.esrl.noaa.gov/psd/data/gridded/data.gpcc.html (Accessed 20 March 2018), 2018.
Greve, P., Gudmundsson, L., Orlowsky, B. and Seneviratne, S. I.: Introducing a probabilistic Budyko framework, Geophysical Research Letters, 42(7), 2261–2269, 2015.
Hodnett, M., Da Silva, L. P., Da Rocha, H. and Senna, R. C.: Seasonal soil water storage changes beneath central Amazonian rainforest and pasture, Journal of hydrology, 170, 233–254, 1995.
Istanbulluoglu, E., Wang, T., Wright, O. M. and Lenters, J. D.: Interpretation of hydrologic trends from a water balance perspective : The role of groundwater storage in the Budyko hypothesis, Water Resources Research, 48(January), 1–22.
Jones, P. and Harris, I.: Climatic Research Unit (CRU) time-series datasets of variations in climate with variations in other phenomena. NCAS British Atmospheric Data Centre, University of East Anglia, 2008.
Lebecherel, L., Andr, V. and Perrin, C.: On regionalizing the Turc-Mezentsev water balance formula, , 49(November), 7508–7517, doi:10.1002/2013WR013575, 2013.
McMahon, T. A., Peel, M. C., Lowe, L., Srikanthan, R. and McVicar, T. R.: Estimating actual , potential , reference crop and pan evaporation using standard meteorological data : a pragmatic synthesis, Hydrology and Earth System Sciences, 17(4), 1331–1363.
Melesse, A.: Nile River Basin., 2011.
Mezentsev, V.: More on the calculation of average total evaporation, Meteorol. Gidrol, 5, 24–26, 1955.
Moussa, R. and Lhomme, J.-P.: The Budyko functions under non-steady-state conditions, Hydrology and Earth System Sciences, 20(12), 2016.
Nepstad, D. C., Carvalho, D., R, C., Davidson, E. A., Jipp, P. H., Lefebvre, P. A., Negreiros, G. H., da Silva, E. D., Stone, T. A., Trumbore, S. E. and Vieira, S.: The role of deep roots in the hydrological and carbon cycles of Amazonian forests and pastures, Nature, 372(6507), 1994.
Ol'dekop, E. M.: On evaporation from the surface of river basins, Trans. Met. Obs. lur-evskogo, Univ. Tartu 4 in Russian, 1911.
Pike, J. G.: The estimation of annual run-off from meteorological data in a tropical climate, Journal of Hydrology, 2(2), 116–123, doi:10.1016/0022-1694(64)90022-8, 1964.

Rana, G. and Katerji, N.: Measurement and estimation of actual evapotranspiration in the field under Mediterranean climate : a review, , 13, 125–153, 2000.

Schamm, K., Ziese, M., Becker, A., Finger, P., Meyer-Christoffer, A., Schneider, U., Schröder, M. and Stender, P.: Global gridded precipitation over land: A description of the new GPCC First Guess Daily product, Earth System Science Data, 6(1), 49–60, doi:10.5194/essd-6-49-2014, 2014.

Schreiber, P.: Über die Beziehungen zwischen dem Niederschlag und der Wasserführung der Flüsse in Mitteleuropa, Z. Meteorol, 21(10), 441--452, 1904.

Turc, L.: The water balance of soils. Relation between precipitation, evaporation and flow, Ann. Agron., 5, 491–569, 1954.

Turner, K. M.: Annual evapotranspiration of NATWE vegetation in a Mediterranean-type climate, JAWRA Journal of the American Water Resources Association, 27(1), 1991.

Wang, X. S. and Zhou, Y.: Shift of annual water balance in the Budyko space for catchments with groundwater-dependent evapotranspiration, Hydrology and Earth System Sciences, 20(9), 3673–3690, doi:10.5194/hess-20-3673-2016, 2016.

Ward, A. and Trimble, S.: Environment hydrology, Lewis publishers., 2002.

Waseem, M., Mani, N., Andiego, G. and Usman, M.: A review of criteria of fit for hydrological models, International Research Journal of Engineering and Technology, 4(11), 2017.

Wu, J., Miao, C., Wang, Y., Duan, Q. and Zhang, X.: Contribution analysis of the long-term changes in seasonal runoff on the Loess Plateau, China, using eight Budyko-based methods, Journal of Hydrology, 545(January 2017), 263–275.

Xu, X., Liu, W., Scanlon, B. R., Zhang, L. and Pan, M.: Local and global factors controlling water-energy balances within the Budyko framework, Geophysical Research Letters, 40(23), 6123–6129.

Yacob, Y.: Equitable utilization in the Blue Nile River sub-basin., 2002.

Yang, D., Sun, F., Liu, Z., Cong, Z., Ni, G. and Lei, Z.: Analyzing spatial and temporal variability of annual water-energy balance in nonhumid regions of China using the Budyko hypothesis, Water Resources Research, 43, 1–12.

Yang, H., Yang, D., Lei, Z. and Sun, F.: New analytical derivation of the mean annual water-energy balance equation, Water Resources Research, 44(3).

Zhang, L., Dawes, W. R. and Walker, G. R.: Response of mean annual ET to vegetation changes at catchment scale, Water Resources Research, 37(3), 701–708.

Zhang, L., Hickel, K. and Dawes, W. R.: A rational function approach for estimating mean annual evapotranspiration, Water Resources Research, 40(2), 1–14.

Zhang, L., Potter, N., Hickel, K., Zhang, Y. and Shao, Q.: Water balance modeling over variable time scales based on the Budyko framework - Model development and testing, Journal of Hydrology, 360(1–4), 117–131.

Zhang, Y., Guan, D., Jin, C., Wang, A., Wu, J. and Yuan, F.: Analysis of impacts of climate variability and human activity on streamflow for a river basin in northeast China, Journal of Hydrology, 410(3–4), 239–247.

Ziese, M., Schneider, U., Meyer-Christoffer, A., Finger, P., Schamm, K., Becker, A. and Rudolf, B.: Gridded Analysis Products provided by the Global Precipitation Climatology Centre (GPCC), and new Products getting operational 2013, EGU General Assembly Conference Abstracts, 15, 4123.