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STUDYING THE EFFECT OF STIFFNESS VALUES OF DIFFERENT ELEMENTS OF CONCRETE STRUCTURES ON ITS STRUCTURAL BEHAVIOR

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ملخص البحث

الهدف الأساسي من هذا البحث هو دراسة تأثير تغير الجساءة في العناصر الخرسانية المسلحة على السلوك الإنشائي لهذه العناصر. جساءة العناصر الخرسانية المختلفة تعتمد بشكل كبير على نسبة الشروخ في هذه العناصر والتي تعتمد على نوعية وقيمة الأحمال التي تتعرض لها هذه العناصر. لدراسة هذا الموضوع، تم عمل نموذج ثلاثي الأبعاد لكمرة من الخرسانة المسلحة باستخدام طريقة العناصر المحددة اللاخطية. هذا النموذج يأخذ في الاعتبار تأثير الشروخ في الخرسانة، كما أنه يحتوي على التفاصيل الخاصة بالحديد الطولي وحديد الكانات. بعد التأكد من صلاحية النموذج بمقارنة النتائج بتجارب معملية سابقة، تم استخدام النموذج بتحميله بأحمال رأسية ومحورية بنسب مختلفة ودراسة السلوك الإنشائي للكمرة في كل حالة وكذلك دراسة تأثير الشروخ على الجساءة المحورية. بالإضافة إلى ذلك، تم عمل نموذج كمرة مكونة من بحرين وتم تحليلها تحت تأثير حمل محوري يؤثر في المنتصف بين البحرين لدراسة كيفية توزيع الحمل على بحري الكمرة، ومدى اعتماد هذا التوزيع على اختلاف نسبة الشروخ بين بحري الكمرة. الدراسة أوضحت أهمية الجساءة والتي تعتمد بشكل كبير على نسبة الشروخ في السلوك الإنشائي للعناصر الخرسانية المختلفة.

ABSTRACT

The main objective of this research is to study the effect of the change in stiffness of reinforced concrete elements on the structural behavior of these elements. The stiffnesses of the different types of concrete elements depends on the extent of cracking in these elements, which depends in turn on the type and value of the loads to which these elements are subjected. To study this subject, a three-dimensional model of a reinforced concrete beam was constructed using non-linear finite elements. This model considered the impact of cracks in concrete and included the details of longitudinal and transvers reinforcement. After verifying the model by comparing the results to previous laboratory experiments, the model was analyzed under vertical and axial loads of different proportions to study the structural behavior of the beam and the effect of cracking on axial stiffness in each case. Additionally, a model of a 2-span beam was analyzed to study the distribution of an axial load applied on the middle support on the two different spans of the beam, and the dependence of this distribution on the extent of cracking of the two spans. The study showed the importance of stiffness, which depends on the extent of cracking, on the structural behavior of different concrete elements.

INTRODUCTION

The effective stiffness of different elements of a structure is a very important factor in shaping the structural behavior of the structure. In reinforced concrete structures, the evaluation of the relative stiffness of different parts of the structure is not straight forward, since in most cases the concrete sections are cracked and behave in a non-linear manner. The fact that cracking is depended on the state of stress and strain of each component complicates the matter further. The state of stress and strain is dependent on the location and type of the component and the value of loads imposed on the structure. Several researchers have studied the variation of stiffness of reinforced concrete structures. **Ashour et. al.**[1] observed that the rate of decay of the beam effective moment of inertia was lower for beams with fibers than that of beams with no fibers. It was also observed that as the concrete compressive strength and steel fiber content increased, the flexural rigidity increased significantly. **Castel et. al.**[2] proposed a new formula for calculating the effective tensile active section $A_{ct.ef}$ of reinforced concrete beams to estimate of the deflection of structural members. It was noted that the experimental results were closer to the proposed approach than **CEB-FIP model code**[3]. **Issa et. al.**[4] found that the effective moment of inertia of concrete beams was influenced by the loading type and the reinforcement ratio. **Akmaluddin**[5] proposed an equation to determine the crack moment of inertia for lightweight concrete. He proved that crack moment of inertia increased with the increased tensile reinforcement ratio. **Bashara et. al.**[6] estimated the value of the stiffness for deep continuous reinforced concrete beams. The results showed that, the value of stiffness was affected by both f_{cu} as well as higher shear span-to depth ratio. **Vu et. al.**[7] proposed an equation to calculate the moment of inertia of the conventionally reinforced concrete coupling beam (CCB) and the diagonally reinforced concrete coupling beam (DCB) The study showed that stiffness ratios of the conventionally reinforced concrete coupling beam (CCB) and the diagonally reinforced concrete coupling beam (DCB) increased with an increase in transverse reinforcement ratio (ρ_v), diagonal reinforcement ratio (ρ_{sd}) and longitudinal reinforcement ratio(ρ_s). **Hu et. al.**[8] developed comprehensive design equations for the effective flexural and shear stiffness of Concrete filled steel tubular (CFST) rectangular sections. Available test results were used to verify the accuracy of the proposed equations.

Most researchers, however, didn’t study the effect of the axial stiffness and the ratio of the axial and transverse loads. Both factors being considerably different for different components of the structure. To study this issue a non-linear finite element analysis of a reinforced concrete beam subjected to transverse and axial loading with varying ratios was performed. After the model was constructed and verified using previous experimental results, the model was used to study the change of behavior due to the imposed varying stress conditions.

THE FINITE ELEMENT MODEL

A 3D finite element model for a simple beam (hinged-roller) was constructed using ANSYS[9] finite element package. The details of the model shown in Fig. 1 were taken from previous experimental work performed by Abd-Alkhalik[10]. The beam was loaded by two vertical loads near mid span.

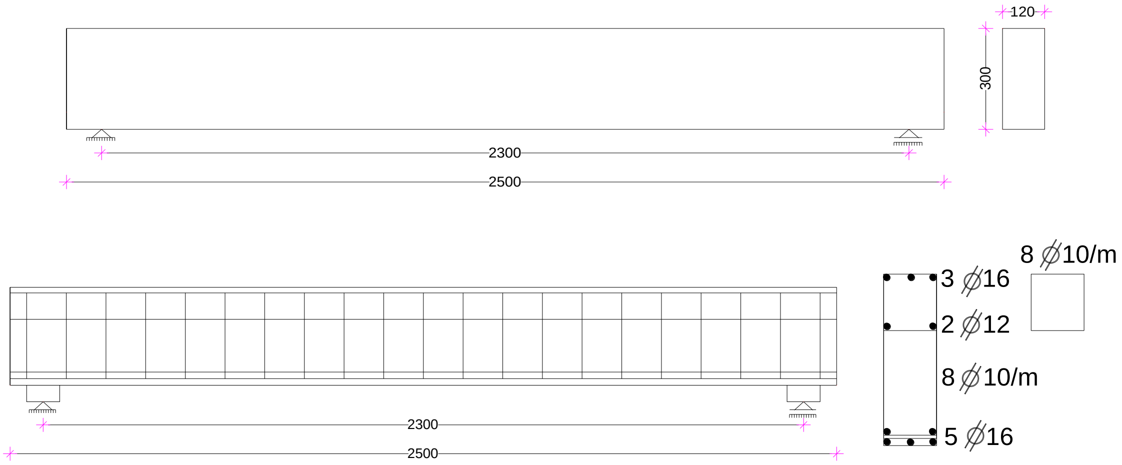

Fig. 1–Details of the experimental model by Abd-Alkhalik[10]

The finite element model was constructed using SOLID65 3D solid elements to represent concrete and LINK180 line truss elements to represent reinforcement as shown in **Fig. 2**. The concrete and steel stress-strain non-linear curves are shown in **Fig. 3**. The smeared crack approach was used to represent cracking in concrete. Elements with strong material properties were introduced at the concentrated loads and supports to distribute loads on a large area and prevent premature numerical failures.

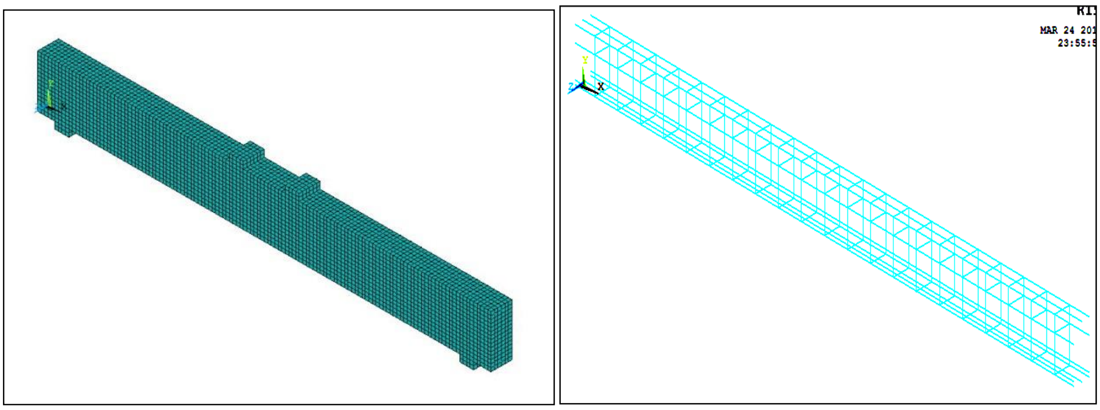

Fig. 2– 3D Finite Element Model

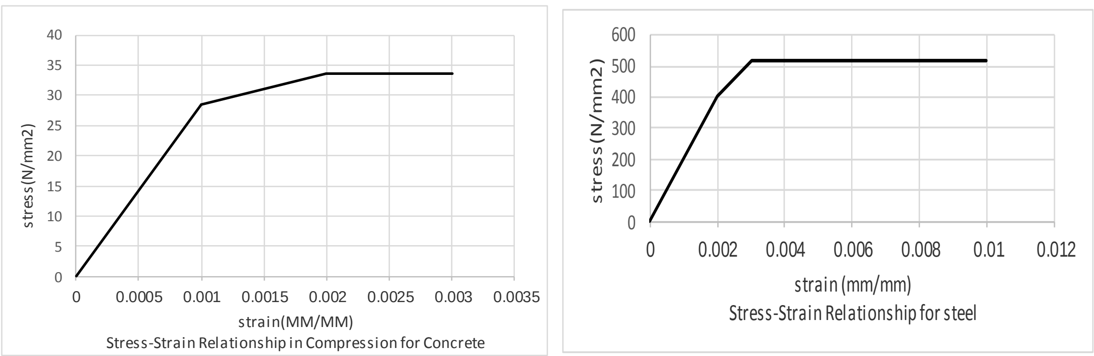

Fig. 3 – Concrete and Reinforcement Stress-Strain Curves

Due to symmetry about the X-Y plane and symmetry about the Y-Z plane, only one quarter of the model was used in the analysis to reduce computational time. The symmetry condition was modeled using special restraints as shown in Fig. 4.

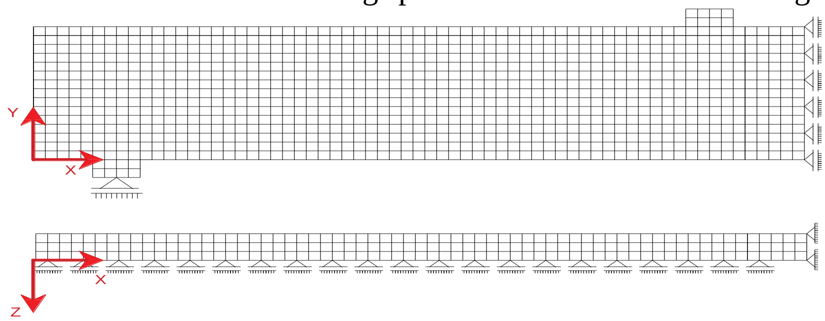

Fig. 4– Model Restraints to Simulate Symmetry

The vertical load was applied at the locations shown in **Fig. 5.** The load was applied in load steps to facilitate convergence of the non-linear solution. At each load step, the solution is repeated for several iterations until the error is reduced below a certain tolerance, after which the program proceeds to the next step. This continues until the failure occurs.

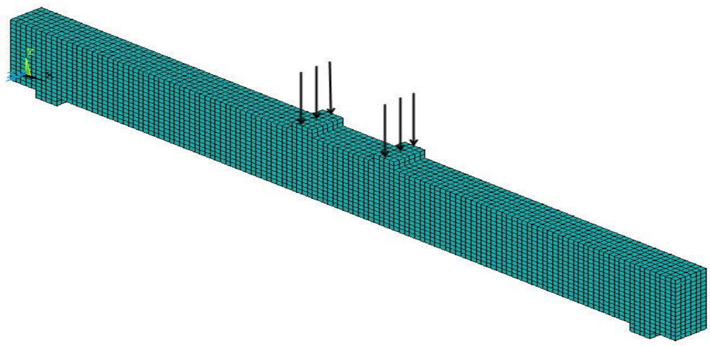

Fig. 5 – Location of Vertical Load

EXPERIMENTAL VERIFICATION

To obtain confidence in the results of the finite element model, that model was verified using the experimental work carried by Abd-Alkhalik[10]. The results obtained by the 3D finite element model were compared to the experimental results.

Failure Load and Deflection

Fig. 6 shows the load vs mid-span deflection for the experimental specimen and the finite element model. The failure vertical load of the specimen was 239KN compared to 213 KN in the experimental study. The load deflection relation showed similar behavior, but the maximum deflection was much higher in the experimental specimens. This difference, however, should not be of concern since this difference mostly occurred in the post-failure phase.

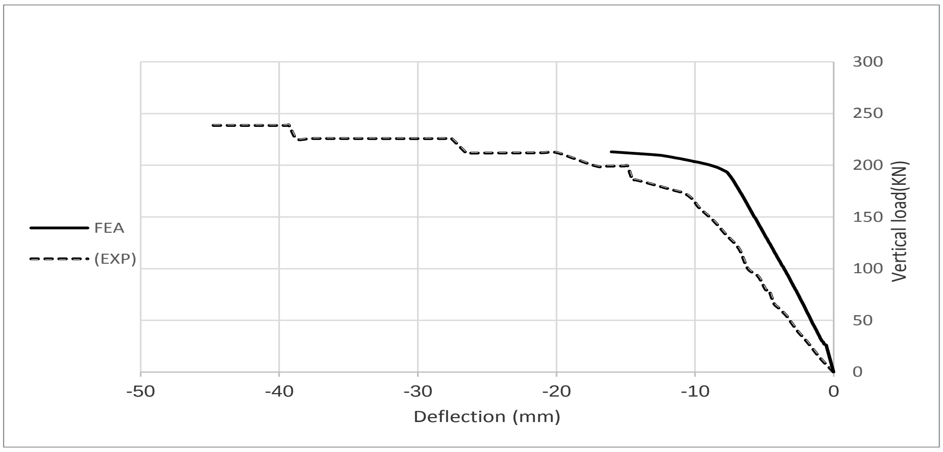

Fig. 6- Load –Deflection at Mid-Span Relationship in (EXP) & (FEA)

Concrete and Reinforcement Strain

Fig. 7 shows the relation between the load and the concrete strain at top of the beam measured at mid-span for both the experimental and finite element models. Similar behavior for both cases were noticed.

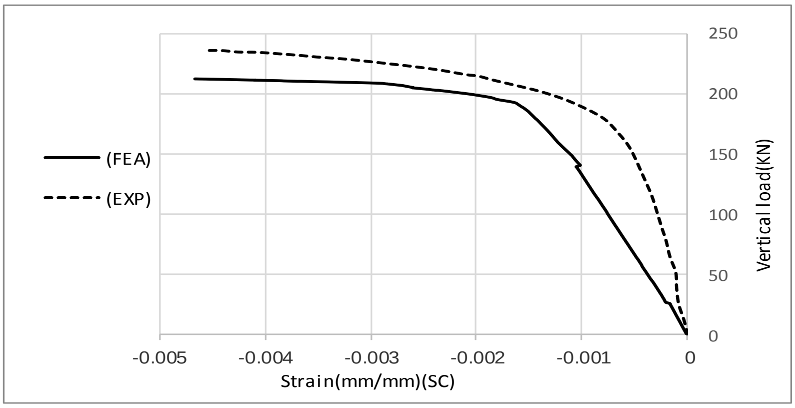

Fig. 7 - Load –Compressive Concrete Strain at Mid-Span Section

Fig. 8 shows the relation between the load and bottom longitudinal steel strains at mid-span for the experimental and finite element models. The experimental results and finite element results showed the same behavior and the values were very near.

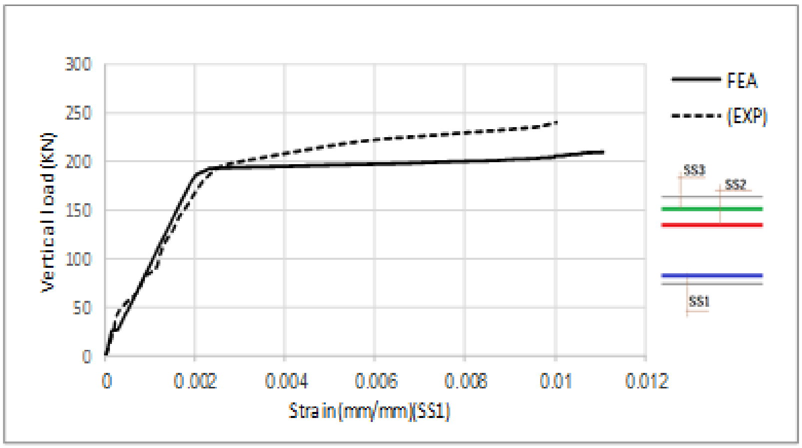

Fig. 8 - Load-Longitudinal Tensile Strain Relationship For Bottom Steel Reinforcement (SS1) in (EXP) & (FEA)

Cracking at Failure

Fig. 9 shows a comparison of the cracking in both the finite element and the experimental model near the failure load. As shown in the figure, cracking was similar for both models.

The comparison between the finite element results and the experimental results showed similar behavior giving confidence in the results of the finite element model.

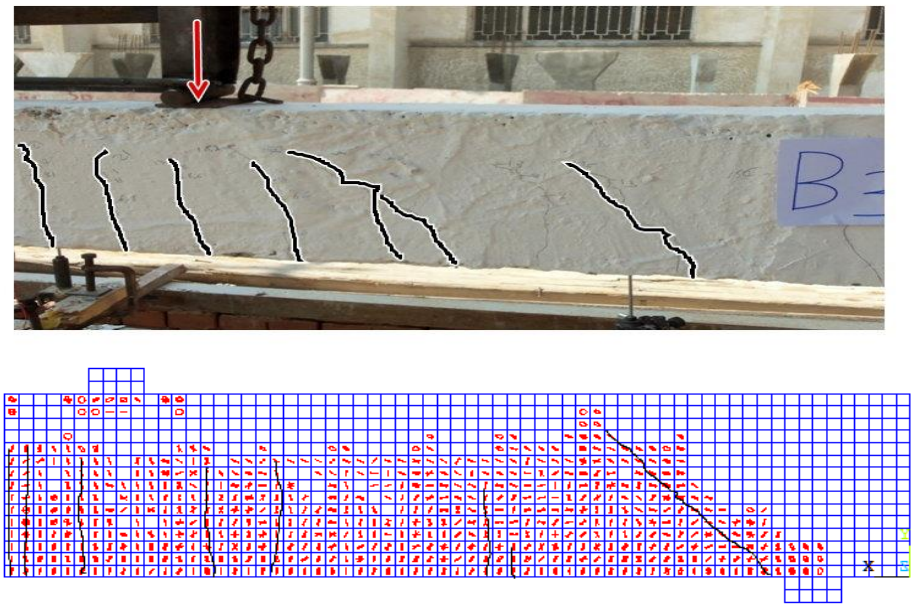

Fig. 9- Crack Pattern of Beam at Failure EXP. vs. FEM

AXIAL-TRANSVERSE FORCE INTERACTION

In order to find the effect of the type and magnitude of the axial force on the behavior of a reinforced concrete beam, a non-linear finite element model of a single span beam was analyzed under simultaneous vertical and axial loads. Axial loads ranging from 0.1 up to 10 times of the vertical load were analyzed in both tension and compression. For all cases, the failure loads, deflection and strains were compared to study the non-linear effect of the different axial loads on the beam. The change in the axial stiffness as the load increased was also studied.

Failure Load

The failure load obtained from the analysis for various axial load ratios are presented in Fig. 10 In the figure negative ratios indicate tension while positive ratios indicate compression. The figure shows that the maximum failure load was obtained when the axial load was compression with a value of 5 times the vertical load, V. The failure load in that case was 164 KN. Increasing the ratio of the axial compression caused the value of the failure load to decrease reaching 123 KN when the compression force was doubled (10 V). On the other hand, when the axial load was decreased below 5 V, the failure load started to decrease. The failure load decreased further as the axial load was changed to tension reaching 14 KN as the beam was subjected to an axial tension of 10 V.

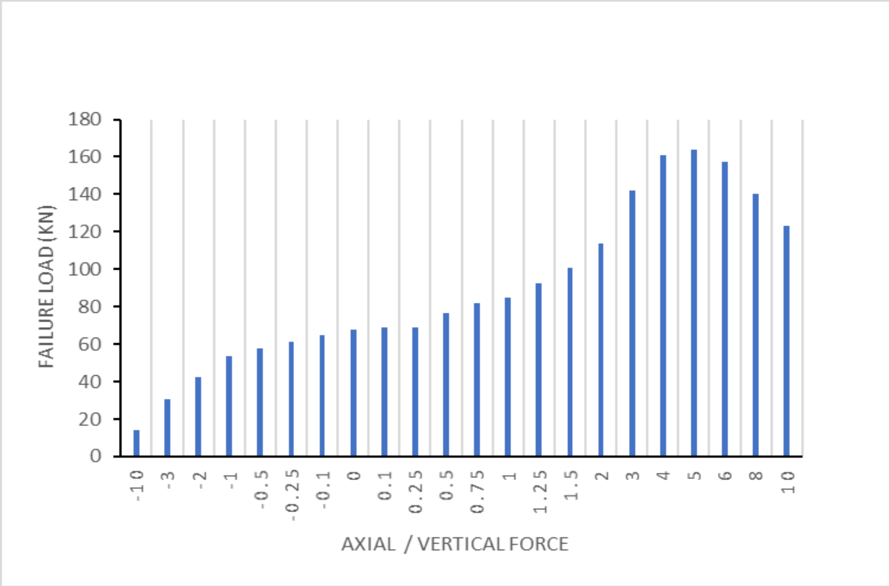

Fig. 10 – Failure Load for Varying Axial Ratios

The failure moment (calculated from the failure vertical load V) was calculated and plotted against the failure axial load for the various cases analyzed. The results are shown in Fig. 11. The figure is typical of the theoretical interaction diagrams for sections subjected to Moment and Normal force. The figure shows that the maximum capacity of the beam was reached at a axial force of 820 KN (5 times the vertical load of 164 KN).

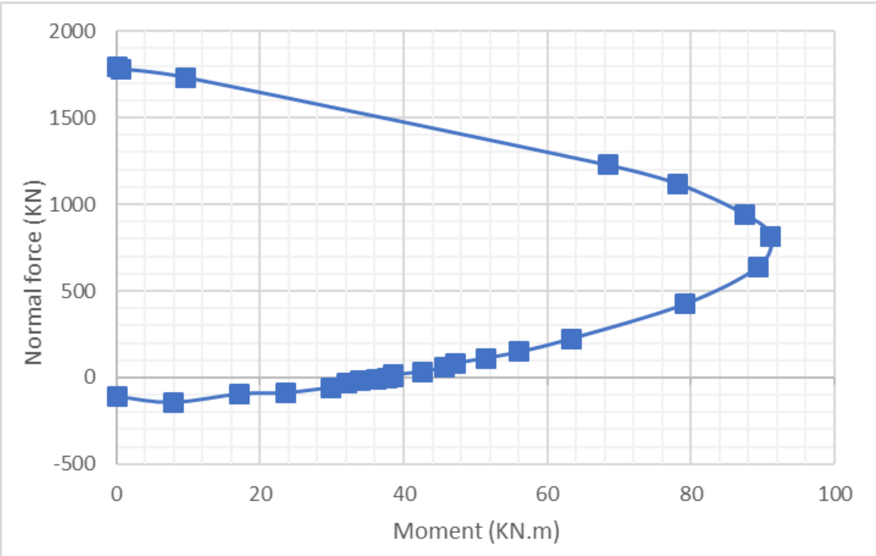

Fig.11 – M-N Interaction Diagram

Mid Span-Deflection and Reinforcement Strains

Fig. 12 shows the mid-span deflection for beams with various axial load ratios ranging from Compression = 10 V up to a tension of 10 V, where V is the magnitude of the vertical load. For beams with tension axial load or no axial load, the relation between the load and deflection is initially linear followed by a nearly horizontal branch probably due to extensive cracking, then it is followed by a stiffening branch with a considerable increase in load but with a lower stiffness than the initial linear part. This is then followed by a horizontal curve when the lower reinforcement starts yielding as indicated by the reinforcement strain relationship shown in Fig. 13. For axial compression loads, the load-deflection relation and the bottom reinforcement strain relation showed a smooth transition between the initial linear part and the final horizontal relation due to yielding. In general, the load deflection relation showed a decrease in vertical stiffness as the compression axial load decreased and the stiffness decreased further as the axial load became zero and changed into tension. The top reinforcement strain is shown in Fig. 14. The figure shows that in the cases of high compression values (N= 5V and 10V), the top compression yielded in compression during failure. For small values of the axial load, the strain in the top reinforcement was in the linear stage. For high values of axial tension, the top reinforcement yielded in tension.

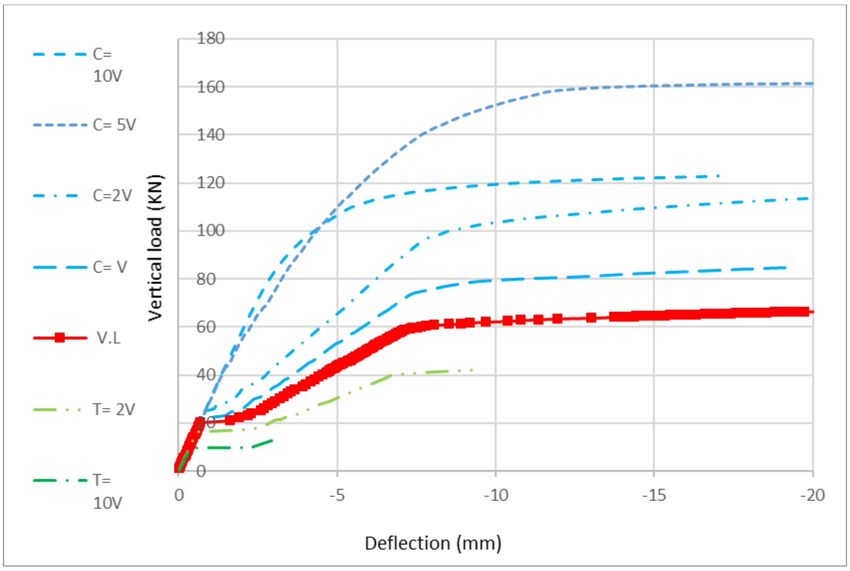

Fig. 12 – Mid-Span Deflection for Different Axial Load Ratios

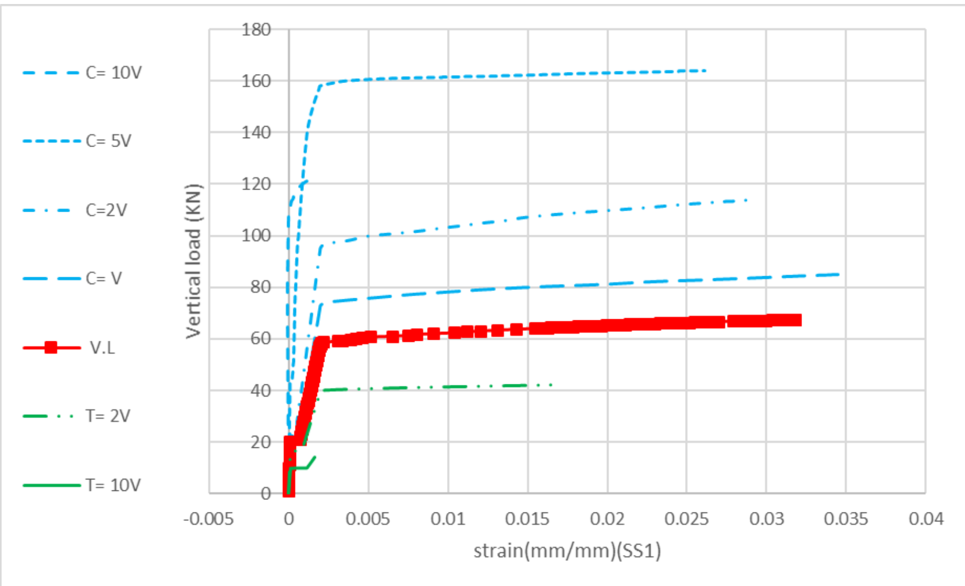

Fig. 13 – Bottom Reinforcement Strain for Different Axial Load Ratios

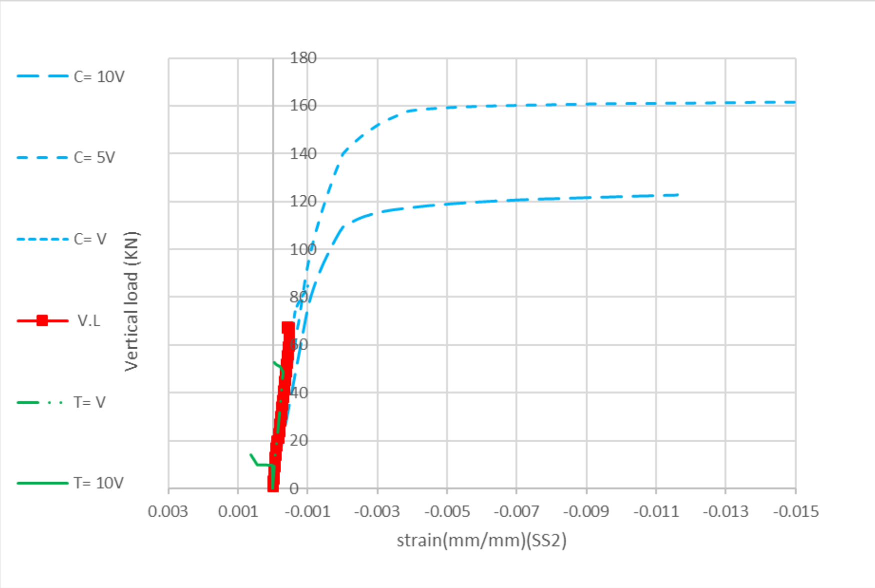

Fig. 14 – Top Reinforcement Strain for Different Axial Load Ratios

Axial Stiffness

Fig. 15 shows the axial stiffness for beams with various axial load ratios ranging from Compression = 10 V up to a tension of 10 V. As the vertical load increased, the stiffness started to decrease for cases of both compression and tension axial loads. The decrease in stiffness, however, was larger in the case of axial tension. The axial stiffness for an axial tension load = V reached about 10% of its original value as the beam approached failure. It appears from the figure the axial stiffness is not affected by the magnitude of the tension force. For axial compression, however, the stiffness was similarly decreased as the vertical load increased, but the value of the reduced stiffness depended on the magnitude of the axial compression applied. For axial compression of 10 V the axial stiffness was decreased to 50% of its original value, while for an axial compression of C=V, the stiffness was decreased to 20% of its original value as the beam approached failure.

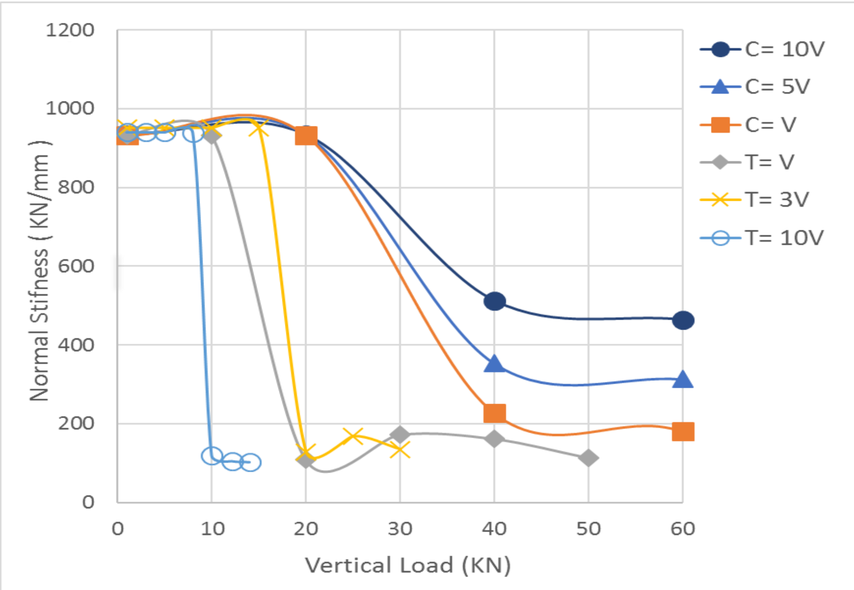

Fig. 15 – Axial Stiffness for Different Axial Load Ratios

AXIAL EFFECTS IN INDERTERMINATE STRUCTURES

To evaluate the interaction of transverse and axial loads in indeterminate structures, the two-span beam shown in Fig. 16 was analyzed. The beam was restrained in the X-direction at the two outer supports. The intermediate support, however, was a roller support. The beam was subjected to a vertical load V at the middle of each span and a horizontal load, H, at the intermediate roller support. Two cases were analyzed with values of H= V and H= 2V.

The horizontal force, H, caused tension in the left span (Span 1) and compression in the right span (span 2), and therefore the behavior of the beam with respect to the axial force was not symmetric.

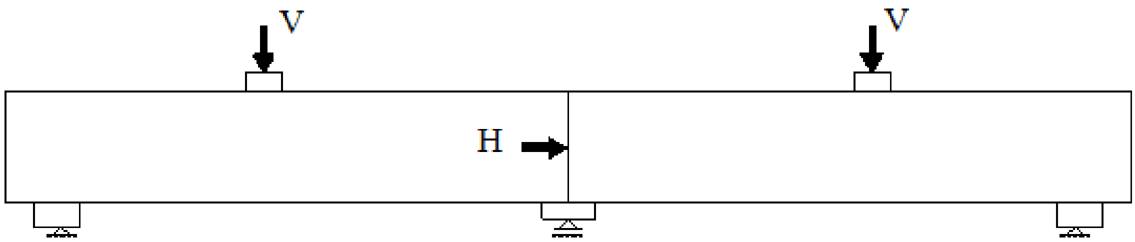

Fig. 16 – Two-Span Indeterminate Beam

Distribution of Horizontal Force between Spans

The distribution of the axial force, H, between the left span (Span1) and the right span (Span2) is shown in Fig. 17 and Fig. 18 for the case of H=V and H=2V, respectively. In both cases the left span which is subjected to tension received a considerably lower portion of the horizontal load, H, than the left span. This is in contrast to the linear solution in which both spans will take 50% of the horizontal load H.

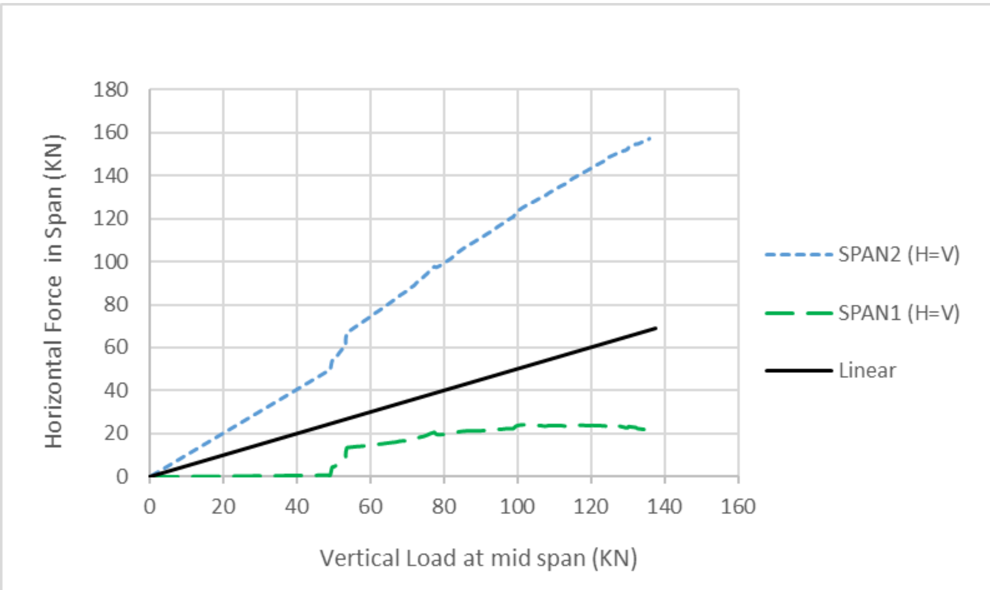

Fig. 17- Distribution of Horizontal Force Case 1 (H=V)

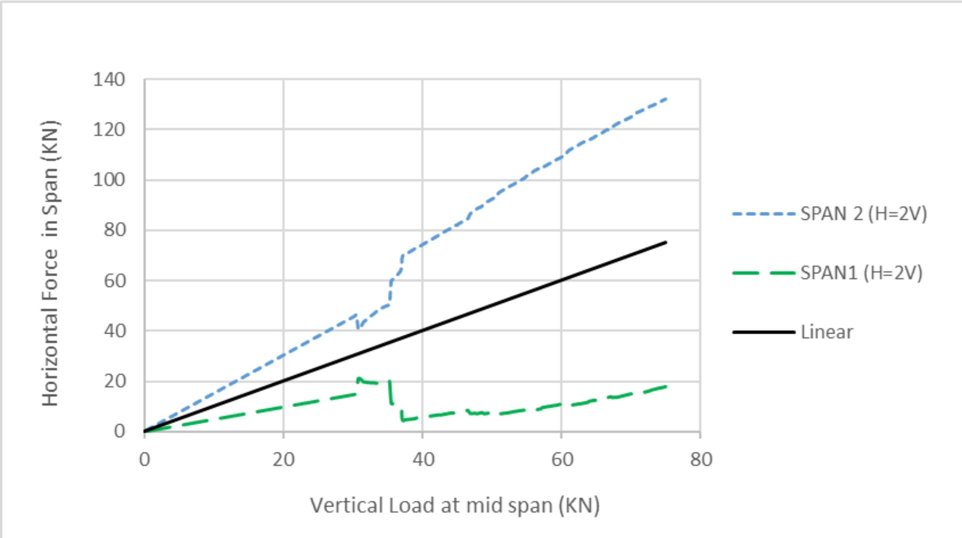

Fig. 18- Distribution of Horizontal Force Case 2 (H=2V)

Distribution of Axial Stiffness

Fig. 19 and Fig. 20 show the distribution of the horizontal stiffness between span 1 and span 2 for the case of H=V, and H= 2V respectively. The figure shows that the stiffness in the non-linear analysis was considerably lower than the linear stiffness considering full concrete section, both in the tension as well as the compression span. Span1, however, which was subjected to tension had a considerably lower stiffness than span2.
For case1 (H=V), the difference between the tension and compression stiffnesses was larger than case2 (H=2V) at the start of the analysis at low values of H. As the load increased, both case 1 and case 2 started to converge to the same stiffness values.
It is also noticed in both cases, that as the load increased (both Hz. and Vertical), the axial stiffness started to decrease differing substantially than the linear value calculated using the full concrete section.

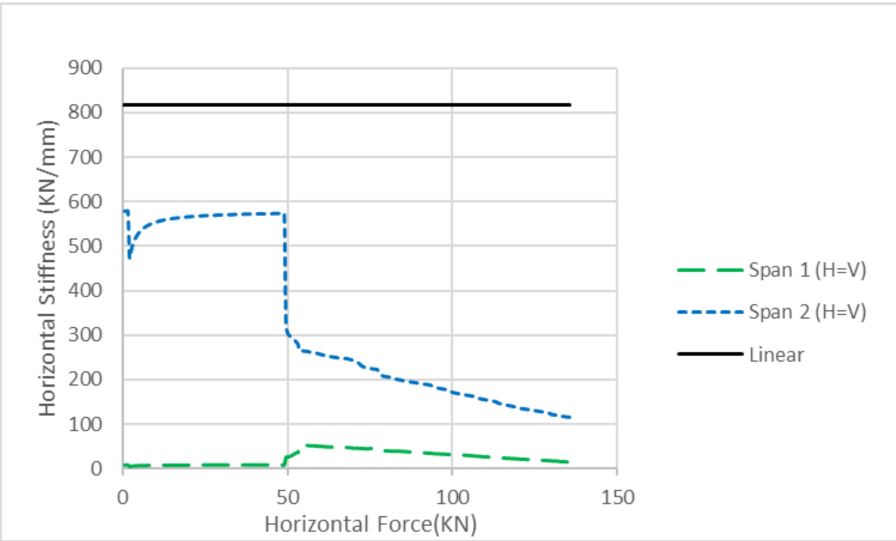

Fig. 19- Distribution of Horizontal Stiffness Case 1 (H=V)

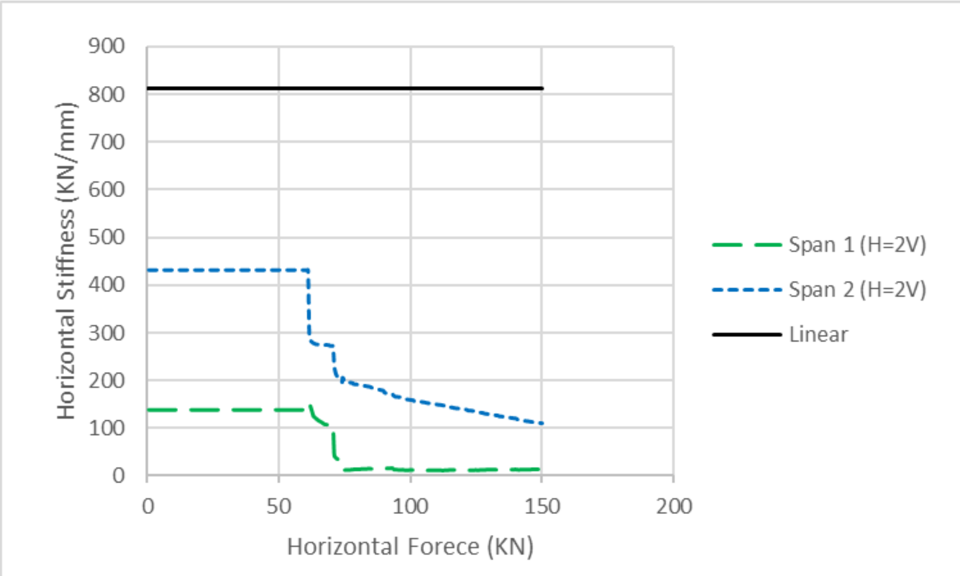

Fig. 20- Distribution of Horizontal Stiffness Case 2 (H=2V)

SUMMARY AND CONCLUSIONS

In this research, the behavior of concrete elements subjected to axial and transverse loads in varying ratios was studied using a non-linear 3D finite element model. The non-linear finite element model was constructed in ANSYS[9] using solid elements capable of simulating cracking behavior using the smeared crack approach. The model was verified by comparing its results to previous laboratory experiment. The model was analyzed under vertical and axial loads of different proportions to study the structural behavior of the beam and the effect of cracking on axial stiffness in each case. The analysis showed that the capacity of the beam in the non-linear analysis was affected by the ratio of the axial force, producing an interaction diagram similar to the typical theoretical Moment-Axial force diagrams. The effect of the axial force ratio on the deflection and reinforcement strain was also shown. The axial stiffness of the beam was also studied and was found to deviate considerably from the linear stiffness values. The axial stiffness was generally less for beams subjected to tension than that subjected to compression. But in both cases the axial stiffness decreased as the vertical load increased leading to increased cracking.

Additionally, a model of a 2-span beam was analyzed to study the distribution of an axial load applied on the middle support on the two different spans of the beam, and the dependence of this distribution on the extent of cracking of the two spans. The analysis showed that the distribution of the horizontal load was not equal as suggested in a linear analysis. The span subjected to compression carried a considerably larger portion of the load than the other span subjected to tension. A comparison of the axial stiffness in both spans showed that the axial stiffness of the span subjected to tension was much lower than the compression span. The axial stiffnesses of both spans were considerably lower than the linear stiffness calculated based on the full concrete section. Overall, the study showed the importance of stiffness, which depends on the extent of cracking, on the structural behavior of different concrete elements.

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