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An Effective Alternate Method for Solving the Classic Three Reservoirs and Multi-Reservoirs Problem

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الملخص العربي

تتلخص الطريقة الجديدة لحل المسألة التقليدية للخزانات الثلاثة والخزانات المتعددة باختيار أحد الأنابيب المرتبطة بالخزانات كأنبوب محوري لبدء عملية الحسابات وافتراض قيمة أولية للتدفق في ذلك الأنبوب ثم حساب الفاقد فيه والمنسوب البيزومتري لنقطة التقاء الأنابيب (العقدة) ، و بناءا على ذلك يتم ايجاد التدفقات في بقية الانابيب والتأكد من مدى تحقق معادلة الاستمرارية عند العقدة من عدمه. يتم تعديل التدفق في الأنبوب المحوري بناء على نسبة المقاومة والمنسوب للأنبوب المحوري مع بقية الأنابيب ومن ثم ايجاد قيم مصححة لكل من المنسوب البيزومتري لنقطة التقاء الانابيب والتدفقات في بقية الأنابيب وتكرر هذه الخطوة حتى تتحقق معادلة الاستمرارية أو يكون فارق التدفقات ضمن حدود خطأ مسموح به. تم اختبار هذه الطريقة الجديدة باستخدام 26 مثال وسؤال أخذت من كتب دراسية جامعية مشهورة لمادتي ميكانيكا الموائع والهيدروليكا ومقارنة ذلك بالحل باستخدام طريقة العقدية Nodal Method وقد وأظهرت نتيجة المقارنة تطابق الحلين والوصول الى نفس النتيجة عند نفس الخطوة وفي أسوأ الحالات تسبق طريقة Nodal Method الطريقة الجديدة بخطوة واحدة أو خطوتين. عند مقارنة هاتين الطريقتين بطرق المحاولة والخطأ المتبعة في كتب الموائع والهيدروليكا وجد أن هاتين الطريقتين أفضل وأسهل كما انهما توفران الوقت في الحساب اليدوي لأن كل من المنسوب البيزومتري عند نقطة التقاء الأنابيب وتدفقات الأنابيب المصححة في خطوة الحساب التالية تستنتج بطريقة آلية من خطوة الحساب السابقة ولذا ينصح تضمين هاتين الطريقتين كطرق اضافية او بديلة في كل من كتب ميكانيكا الموائع والهيدروليكا التي تدرس في الجامعات لطلاب الهندسة المدنية والميكانيكية.

ABSTRACT

Branched pipes problems, traditionally known as three classical and multi-reservoir problems, were presented in many fluid mechanic and hydraulic text books. The complexity of solving such problems arises when water levels for reservoirs via the properties of each pipe were known and it is required to predict the discharge value and direction in each pipe. Manual trial and error methods are advised in most of text books and used for more complex branched pipe problems than mathematical solution. In which an assumption is made for the value of one of the variables and the other quantities are then calculated in turn from that assumption. Adjustments of the initial trial value are made as necessary. These methods were revised and found that they are time consuming especially for manual calculation using only scientific calculator, then a new method was developed. The new method is summarized by selecting a pivot pipe, assuming some initial value for the discharge in that pipe and calculating the associated head loss and subsequently the head at the junction and the other discharges then check the continuity equation .The discharge is then adjusted based on the relative head-resistance of the pivot pipe to the other pipes and finding corrected values for head at the junction and discharges until the net discharge out of the Junction is zero or within tolerable error. This method was tested using 26 examples and problems from well known fluid mechanics and hydraulic text books and compared with nodal method. The result showed convergence was achieved in both methods at the same calculation step and, in worst cases, the new method converges one or two steps behind the nodal method. These two methods saves time in manual calculation as a new corrected head at the Junction and discharges in pipes were automatically predicted from previous

calculation step. It is then advised to include the two methods as alternative methods in fluid mechanics and hydraulic text books.

Keywords: Classical reservoir problem, Multi-reservoirs, Nodal method, Trial and error methods

1. Background

Branched pipe's problems, traditionally known as three classical and multi-reservoir problems, were introduced in many fluid mechanic, hydraulic text books and problems' work books (Victor L. Streeter, 1962; Bruce et al, 2002; Frank M. White, 2001; E Johon Fennimore and Joseph B. Franzini,year; Merle C. Potter, and David C. Wiggert,2008; Massey and Ward-Smith,2006, Jack B. Evett and Cheng Liu,1989). For simplicity, consider the three-reservoir problem shown in Figure. 1. The three reservoirs 1, 2 and 3 are connected together with three pipes at a single junction *J*. Typically, the piezometric heads of the reservoirs with the pipes properties, lengths, diameters, and roughness are considered to be known. The problem is to determine the discharges Q_1, Q_2, *and* Q_3 into or out of the reservoirs.

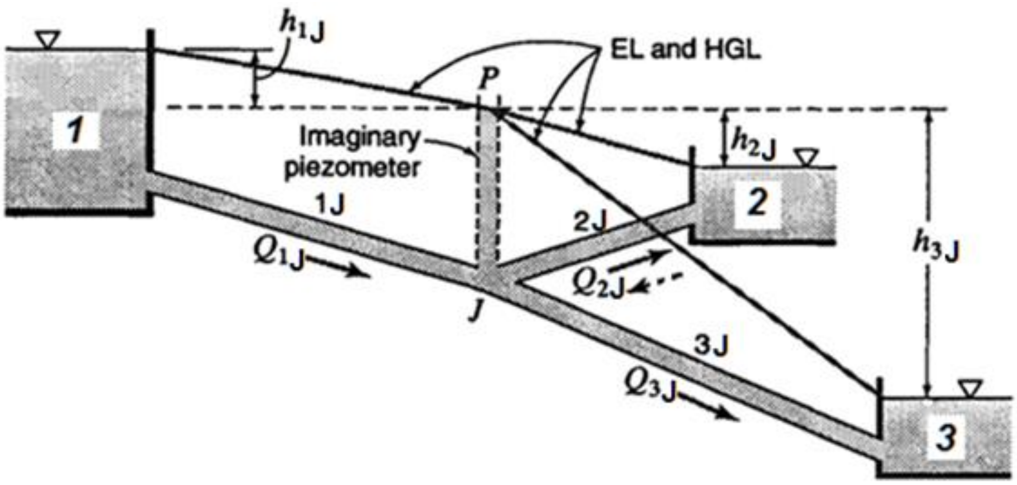


Figure 1: Three reservoir problem

It is clear that fluid flows from *Res.1* because the other two reservoir levels are lower but the flow direction in pipe *2J* is not generally obvious. Whether the fluid flows into or out of *Res. 2* depends on the piezometric heads of *Reservoirs 2 and 3* and the properties length, diameter and roughness of the three pipes (Bruce *et al.*2002).

General principles applied for solving the three reservoir problem and other problems with branching pipes are:

1-The discharge in each pipe must obey the headless formula with additional minor losses in fittings. Using Darcy-Weisbach formulae the head loss *h* in pipe *nJ* can be written in terms of discharge *Q* as,

$$h_{nJ} = R_{nJ}\, Q_{nJ}^2 \qquad (1)$$

Where R_{nJ} is the pipe resistance due to pipe and fluid properties in additional to minor losses which defined as,

$$R_{nJ} = \frac{1}{2gA^2}\left(f\frac{L}{D} + \sum K\right) \qquad (2a)$$

Or, alternatively,

$$R_{nJ} = \frac{8}{\pi g D^4}\left(f\frac{L}{D} + \sum K\right) \qquad (2b)$$

Where,

D is the pipe diameter, *L* is the length of pipe, g is the acceleration due to gravity, $\sum K$ is the sum of minor losses coefficients along the pipe and *f* is the Darcy friction factor which can be defined by another form of Haaland Equation (Haaland, S.E.,1983),

$$f_{nJ} = \left[-2\, log_{10}\left(\frac{e}{3.7D} + \left(\frac{6.9\, v}{V\, D}\right)^{0.9}\right)\right]^{-2} \qquad (3)$$

Where,

V is the average velocity in the pipe, *e* the pipe roughness and *v* is the kinematic viscosity of the liquid.

2-The uniqueness of the total head; this means that the value of the total head at a junction point is the same for all pipes connected to that junction and thus Equation *(1)* can be rewritten as,

$$h_{nJ} = H_n - H_J = R_{nJ}\, Q_{nJ}^2 \qquad (4a)$$

Or, alternatively,

$$H_J = H_n - R_{nJ}\, Q_{nJ}^2 \qquad (4b)$$

Or, when a pump of head *Hp* is added,

$$H_J = H_n - R_{nJ}\, Q_{nJ}^2 - Hp \qquad (4c)$$

Where,

H is the piezometric head, *n,* the reservoir number or name connected to the junction *J* through pipe *nJ.*

3- The continuity equation at a junction point must be satisfied. That is the discharge into the junction is equal to discharge out of the junction.

$$\sum Q_{nJ} = 0 \qquad (5)$$

Where,

Q_{nJ} is the discharge from *n* to *J* with a proper sign convention.

1.1. Mathematically (simultaneous equations):

Considering Figure 1, this problem can be solved mathematically by assuming a direction of flow in the pipe *2J* then accordingly formulating and solving a number of simultaneous equations equal to the number of unknowns. The unknowns are the discharges in pipes and the head at the junction *J*. Hence, there must be a number of equations equal to the number of pipes written in the form of Equation *(4b)* in addition to the continuity Equation *(5)* at the Junction *J*. However, if the assumed direction of flow was not correct there will be no physical solution and therefore it is needed to reverse the direction of flow in that pipe and reformulate the equations and solve them again based on the new direction. This makes the mathematical solution tedious and time consuming especially for the manual calculation and thus it will be no further discussed here.

1.2. Manual Trial and Error Method (Absolute Head Difference)

Trial and error methods are used for more complex branched pipe problems where an assumption is made for the value of one of the variables and the other quantities are then calculated in turn from that assumption. Adjustments of the initial trial value are made as necessary.

Therefore, a trial and error method is more favorable than mathematical solution and advised by many fluid mechanics, hydraulic text books and problems work books.

The method is summarized by assuming some initial value for H_J , calculating the discharges and then checking the continuity equation then adjust H_J until the net discharge out of the Junction is zero or within tolerable error.

In more detail, analysis proceeds by writing a head loss equation for each pipe as,

$$\left.\begin{array}{lcl} |H_1 - H_J| = R_{1J}Q_{1J}^2 & & Q_{1J} = sign\sqrt{\frac{|H_1-H_J|}{R_{1J}}} \\ |H_2 - H_J| = R_{2J}Q_{2J}^2 & or & Q_{2J} = sign\sqrt{\frac{|H_2-H_J|}{R_{2J}}} \\ |H_3 - H_J| = R_{3J}Q_{3J}^2 & & Q_{3J} = sign\sqrt{\frac{|H_3-H_J|}{R_{3J}}} \end{array}\right\} \qquad (6)$$

and the continuity equation at Junction, *J,*

$$Q_{1J} + Q_{2J} + Q_{3J} = 0 \qquad (7)$$

It should be noted here that Q_{2J} is the flow from *2 to J*; it will be negative if the flow from *J* to *2* and vice versa. In other words, the flow entering the Junction is positive and leaving the junction negative.

Solution procedure:

1-Establish the head-loss vs discharge equations for each pipe as in Equation group *(6)*;

2- Assume initial value for the head at the Junction H_J.

3-Calculate discharges in all pipes (from the head differences);

4-Calculate net discharge out of *J* using Equation *(7)*, which is generally not equal to zero.

5- Adjust H_J to reduce any discharge imbalance ΔQ and repeat from 3 until ΔQ is zero or within a predefined error limit.

If the direction of flow in a pipe, say *2J*, is not obvious then a good initial guess is to set $H_J = H_2$ so that there is initially no flow in this pipe. The first discharge calculation will then establish whether H_J should be lowered or raised and subsequently the direction of flow in this pipe can be determined.

To reduce number of iterations and hence calculation time some writers (Victor L. Streeter, 1962; Frank M. White ,2001; Jack B. Evett and Cheng Liu,1989) advice interpolating between two iteration steps to find a closer value of H_J to the real for the new iteration of calculation.

Some other writers (E Johon Fennimore and Joseph B. Franzini, 1998 and Massey and Ward-Smith, 2006) recommend using at least three trial points and plot a graph of $\sum Q_{nJ}$ as a function of H_J. The solution and hence the required H_J will be an intersection of this graph with the horizontal axis i.e. at $\sum Q_{nJ} = 0$.

Trial and error method with or without interpolation is subjected to the experience of a student. Familiar student with such problems will reach a solution in shorter time than the inexperienced student who may reach the solution in longer time or runs in an empty circle.

Recently, trial and error using Nodal Method (Cornish, 1939) has introduced in undergraduate hydraulic text books, such as Nalluri and Featherstone (1998). It is an effective method with such kind of problems and well presented in Nalluri and Featherstone (1998) with variety of examples and only used here for comparison with the new method introduced here after.

2. New Method Development

It is known that the discharge in a single pipe is directly proportional with the headloss across that pipe and inversely proportional with the pipe resistance (Equation *(1)*) and when a reservoir is connected to one end of this pipe, then the discharge is directly proportional with the reservoir head and inversely proportional with the pipe resistance.
It is also noticed that when branched pipes with known properties are connected at one end by a single junction and the other end by reservoirs of known fixed heads the discharge in each pipe and direction depend on the pipe resistance relative to the other pipes' resistances and also on the reservoir head, connected to that pipe, relative to the other reservoirs' heads. This implies that in each pipe of which the discharge is directly proportional with the ratio of the associated reservoir head to the sum of other reservoirs' heads and inversely proportional with the ratio of the pipe resistance to the sum of resistances of other pipes. This is also applicable to the discharge correction

portion (share) out of the discharges imbalance at the junction. Based on that, the new method is termed as to Relative Head-Resistance Method (RHRM).

The method is summarized by selecting a pivot pipe, preferably one of the lower pipes, assuming some initial value for Q in that pipe and calculating the associated head loss and subsequently H_J and the discharges in the other pipes and checking the continuity equation, then adjust Q in the pivot pipe and find corrected values for H_J and the other pipes' discharges until the net discharge out of the Junction is zero or within tolerable error.

In more detail, the corrected discharge in the pivot pipe *nJ* can be written as,

$$(Q_{nJ})_{i+1} = (Q_{nJ})_i \pm \alpha_{nJ}\Delta Q_i \qquad (8)$$

Where,

$(Q_{nJ})_{i+1}$ = the corrected discharge in a pivot pipe *nJ* at calculation step *i+1*

$(Q_{nJ})_i$ = the assumed or calculated discharge in the same pipe at previous calculation step *i*

ΔQ_i = the discharge imbalance at the Junction *J* calculated in a previous step *i*

α_{nJ} = discharge correction factor constant for the pipe *nJ* based on the initial pipes' resistance values and can be defined by the equation,

$$\alpha_{nJ} = \frac{\sqrt{\frac{H_n}{R_{nJ}}}}{\sqrt{\frac{H_{(n+1)}}{R_{(n+1)J}}} + \sqrt{\frac{H_{(n+2)}}{R_{(n+2)J}}} + \sqrt{\frac{H_{(n+3)}}{R_{(n+3)J}}} + \sqrt{\frac{H_{(n+4)}}{R_{(n+4)J}}} + \cdots + \cdots} \qquad (9)$$

The solution procedure explained here after can be applied for any number of reservoirs, however for simplicity consider the three reservoirs shown in Figure 1.
The continuity equation at Junction *J* and for calculation step *i* can be written as,

$$(Q_{1J})_i + (Q_{2J})_i + (Q_{3J})_i = \Delta Q_i \qquad (10a)$$

By considering pipe *2J* as a pivot pipe Equation *(10a)* can rewritten as,

$$\left(sign\sqrt{\frac{|H_1 - H_J|}{R_{1J}}}\right)_i + \left(Q_{2J}\right)_i + \left(sign\sqrt{\frac{|H_3 - H_J|}{R_{3J}}}\right)_i = \Delta Q_i \qquad (10b)$$

Where,

$$(Q_{1J})_i = \left(sign\sqrt{\frac{|H_1 - H_J|}{R_{1J}}}\right)_i, \quad (Q_{3J})_i = \left(sign\sqrt{\frac{|H_3 - H_J|}{R_{3J}}}\right)_i \qquad (11)$$

Utilizing principle 2 and Equation *(4)* the common head at the junction *J* is,

$$(H_J)_i = H_2 - (R_{2J}\,Q_{2J}^2)_i \qquad (12)$$

And the corrected discharge in next calculation step *i+1* is,

$$(Q_{2J})_{i+1} = (Q_{2J})_i - \alpha_{2J}\,\Delta Q_i \qquad (13)$$

Where the discharge correction factor for the pivot pipe *2J* is,

$$\alpha_{2J} = \frac{\sqrt{\frac{H_2}{R_{2J}}}}{\sqrt{\frac{H_1}{R_{1J}}}+\sqrt{\frac{H_3}{R_{3J}}}} \qquad (14)$$

Solution Procedure

1. Assume an initial discharge, $(Q_{2J})_i$ for the pivot pipe *2J* equals to or close to zero.
2. Calculate the head loss in the pivot pipe then piezometric head at Junction *J* using this discharge and Equation *(12)*.
3. Substitute the discharge value and the calculated piezometric head in step 2 into Equation *(10b)* and check for continuity balance taking into consideration the sign convention (i.e. positive for discharge entering the junction and negative for leaving the junction); generally the discharge imbalance ΔQ at the junction will not equal to zero.
4. Find the discharges in the other pipes, Q_{1J} and Q_{3J} using Equation *(11)*.
5. Find a corrected value for the discharge $(Q_{2J})_{i+1}$ using Equation *(13)*, which will be the new discharge for next calculation step ($i+1$).
6. Repeat steps 2 up to 5 until ΔQ is zero or within desired error limit.

It is worth noting to avoid selecting a pivot pipe connected to a reservoir with head equals to zero. In this case the discharge correction factor will be zero unless all the heads of the reservoirs are neglected from Equations *(9 & 14)* and hence the correction factor only depends on the pipes' relative resistances. If a discharge in the pivot pipe is assumed other than zero, the velocity in the friction factor equation (Equation *(3)*) must be assumed in the first calculation step for the whole pipes (say 1m/s). On the other hand, if the initial discharge in the pivot pipe is set as zero this velocity assumption (1m/s) must be maintained in the pivot pipe for the first two calculation steps. This is because the friction factor f is corrected iteratively based on the velocity of previous calculation step and thus to avoid repeating this velocity assumption it is recommended to assume the initial discharge in the pivot pipe close to zero or any other value based on the person judgment.

2.1. Method Verification and Discussion

The RHR Method is verified using 26 examples and problems taken from the sources mentioned above and compared with the Nodal method. The result showed that convergence is achieved in both methods mostly at the same calculation step and in worse cases the RHR method converges one or two steps behind the Nodal Method. Generally, it was noticed that convergence in both methods is achieved when the absolute ratio of the discharge of any pipe to the sum of the discharges of other pipes is equal to unity (1.0). Based on that the discharge correction factor α in the RHR Method must be taken more than zero and less or equal to unity ($0 < \alpha \leq 1.0$). In other words, α in Equations *(9)* and *(13)* is just this ratio of discharges with neglecting of the variable H_J and by taking only the pipes' resistance values calculated at the first calculation step (i.e. constant R). If, instead, the ratio of discharges was taken as a correction factor (variable correction factor) convergence achieved but too late.

To demonstrate application of this method, two examples were selected, Example 5.6 which is taken from Nalluri and Featherstone (1998) and Problem 10.15 taken from Merle Potter and David Wiggert (2008). These examples were solved using the RHR method and compared with the Nodal method.

To show that in the RHR method any pipe can be selected as a pivot pipe, Example 5.6 is solved twice, once by selecting pipe 2 as a pivot pipe (Table 1a) then by selecting pipe 4 as a pivot pipe (Table 2a). When compared, similar result was obtained which illustrates that no need to select the pipe connected to the middle reservoir as a pivot pipe, as the Manual trial and methods recommends, but any pipe connected to the junction. This example is also solved twice by the Nodal Method (Tables, 1b & 2b) using the same initial heads as RHR Method. When compared with the RHR Method (Tables 2a & 2b) in a respected manner it was noticed that similar results were obtained and convergence achieved at the same calculation step (i.e. the 7th step).

In some other cases, it was noticed that the RHR method works better than the Nodal Method especially when a pump with known characteristic equation is installed in one of the pipes, but it is compulsory here to select that pipe as a pivot pipe. The Nodal Method is very sensitive to such case and care must be taken with the initial guess of head otherwise it will not converge. An example of such case is Problem 10.15 which is solved using both methods in Tables (3a &3b; 4a &4b).

Using the RHR method, it can be seen in Table 3a that with the calculated discharge correction factor $\alpha = 1.39$ convergence is approached late i.e. starting from the 14th calculation step, but when the correction factor α is adjusted to the maximum allowed value, $\alpha = 1.0$ (Table 4a) convergence was achieved earlier, at the 7th calculation step (Compare Table 3a with Table 4a).

To show how sensitive is the Nodal Method to the initial guess of H_J compare Table 3b with Table 4b. In Table 3b, although an initial H_J value was assumed equals to that in the RHR method (Table 3a) i.e.140, no convergence was achieved. Convergence only approached at the 15th calculation step when the initial H_J is reduced down to 135 and then achieved at the 9th calculation step when H_J is reduced further down to 101 (see Table 4b). Therefore, with Nodal Method care must be taken with the initial guess of H_J otherwise no convergence will be achieved.

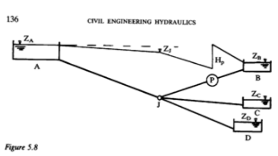
Generally, it can conclude that both the RHR method and Nodal method are more efficient in solving multi reservoirs problems than Manual Trial and Error methods. This is because a corrected head H_J at the Junction is automatically predicted from a previous calculation step and thus reducing the calculation time especially for a student sitting for the exam and using only scientific calculator. It is thus recommended to include these two methods as alternative or additional methods for solving multi reservoirs problems in hydraulic and fluid mechanics texts books to those traditionally introduced i.e. Manual Trial and Error methods.

3. Conclusions and Recommendations

Manual Trial and Error methods for solving branching pipes problem known as three and multi-reservoir problems were studied. An alternative method was then developed depends on estimating of a discharge in a pipe then adjust it iteratively by a correction factor based on the relative head- resistance of that pipe to the others until convergence is achieved. The method is termed as to Relative-Head Resistance method (RHRM). The RHR method is compared with the Nodal Method using various multi reservoir examples and problems from well known text books and found that:

1. Manual Trial and Error Methods with or without interpolation are time consuming in manual calculation and depend largely on the experience of a student and familiarity with such problems.
2. The RHR method can be applied to any number of reservoirs connected to a single junction with or without a pump.
3. With such problems, the RHRM is as efficient as the Nodal Method and they mostly converge at the same calculation step.
4. Using only scientific calculator the RHRM and the Nodal Methods consume less calculation time than the Manual trial and error methods.
5. It is advised to include the RHRM and the Nodal Methods as alternative methods for solving multi reservoir problems in fluid mechanic and hydraulic text books.

Table 1a: Solution of Example 5.6 using Relative Head Resistance Method and taking Pipe 2 as a pivot pipe



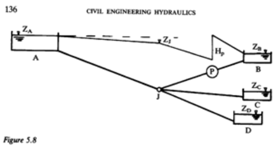
RESERVOIR 1			RESERVOIR 2			RESERVOIR 3			RESERVOIR 4		
H_1 =	200	m	H_2 =	120	m	H_3 =	100	m	H_4 =	75	m
Hp_1 =		m	Hp_2 =	10	m	Hp_3 =		m	Hp_4 =		m
L_1 =	10000	m	L_2 =	2000	m	L_3 =	3000	m	L_4 =	3000	m
D_1 =	450	mm	D_2 =	350	mm	D_3 =	300	mm	D_4 =	250	mm
$\sum K_m$ =	0		$\sum K_m$ =	0		$\sum K_m$ =	0		$\sum K_m$ =	0	
e	0.0600	mm	e	0.0600	mm	e	0.0600	mm	e	0.0600	mm
T	15	C^0	T	15	C^0	T	15	C^0	T	15	C^0
ν (m^2/s)	1.15E-06		ν (m^2/s)	1.15E-06		ν (m^2/s)	1.15E-06		ν (m^2/s)	1.15E-06	
			α =	0.5325							

Trial no	$HJ_{i+1}=H_2-hL2_{i+1}-Hp$	f_1	R_1	$H_1-H_J+H_p$	$Q_1(m^3/s)$	f_2	R_2	hL_2-Hp (Assume initial Q_2)	$Q_2(m^3/s)$ (Assume initial Q_2)	f_3	R_3	$H_3-H_J-H_p$	$Q_3(m^3/s)$	f_4	R_4	$H_4-H_J-H_p$	$Q_4(m^3/s)$	$\sum Q=\Delta Q\ (m^3/s)$	Error %
1	130.000	0.0150	672.602	70.00	0.32260	0.0158	497.313	-10.000	0.00000	0.0163	1664.615	-30.00	-0.13425	0.0170	4304.377	-55.00	-0.11304	0.07532	23.35
2	110.800	0.0140	628.095	89.20	0.37685	0.0158	497.313	9.200	-0.04011	0.0153	1560.758	-10.80	-0.08319	0.0156	3972.342	-35.80	-0.09493	0.15862	42.09
3	118.726	0.0139	620.795	81.27	0.36183	0.0179	562.188	1.274	-0.12458	0.0160	1634.158	-18.73	-0.10705	0.0159	4027.287	-43.73	-0.10420	0.02600	7.19
4	119.263	0.0139	622.629	80.74	0.36010	0.0154	483.393	0.737	-0.13843	0.0156	1592.634	-19.26	-0.10998	0.0157	3997.118	-44.26	-0.10523	0.00646	1.79
5	119.627	0.0139	622.848	80.37	0.35922	0.0152	478.322	0.373	-0.14187	0.0156	1588.572	-19.63	-0.11115	0.0157	3994.038	-44.63	-0.10570	0.00049	0.14
6	119.640	0.0139	622.960	80.36	0.35916	0.0152	477.189	0.360	-0.14213	0.0156	1586.992	-19.64	-0.11125	0.0157	3992.646	-44.64	-0.10574	0.00004	0.01
7	119.642	0.0139	622.968	80.36	0.35916	0.0152	477.104	0.358	-0.14216	0.0156	1586.870	-19.64	-0.11125	0.0157	3992.548	-44.64	-0.10574	0.00000	0.00
8	119.642	0.0139	622.969	80.36	0.35916	0.0152	477.096	0.358	-0.14216	0.0156	1586.859	-19.64	-0.11126	0.0157	3992.538	-44.64	-0.10574	0.00000	0.00
9	119.642	0.0139	622.969	80.36	0.35916	0.0152	477.095	0.358	-0.14216	0.0156	1586.858	-19.64	-0.11126	0.0157	3992.538	-44.64	-0.10574	0.00000	0.00
10	119.642	0.0139	622.969	80.36	0.35916	0.0152	477.095	0.358	-0.14216	0.0156	1586.858	-19.64	-0.11126	0.0157	3992.538	-44.64	-0.10574	0.00000	0.00

Table 1b: Solution of Example 5.6 using Nodal Method

Trial no	$HJ_{i+1}=HJ_i+\Delta z_i$ (Asume initial H_J)	f_1	R_1	$H_1-H_J+H_p$	$Q_1(m^3/s)$	f_2	R_2	$H_2-H_J-H_p$	$Q_2(m^3/s)$	f_3	R_3	$H_3-H_J-H_p$	$Q_3(m^3/s)$	f_4	R_4	$H_4-H_J-H_p$	$Q_4(m^3/s)$	$\sum Q=\Delta Q\ (m^3/s)$	$\sum(Q/h_L)$	Δz	Error %
1	130.000	0.0150	672.602	70.00	0.32260	0.0158	497.313	-20.000	-0.20054	0.0163	1664.615	-30.000	-0.13425	0.0170	4304.377	-55.000	-0.11304	-0.12522	0.0212	-11.8324	38.82
2	118.168	0.0140	628.095	81.83	0.36095	0.0147	463.029	-8.168	-0.13281	0.0153	1560.758	-18.168	-0.10789	0.0156	3972.342	-43.168	-0.10425	0.01600	0.0290	1.1027	4.43
3	119.270	0.0139	622.740	80.73	0.36005	0.0153	480.274	-9.270	-0.13893	0.0156	1591.447	-19.270	-0.11004	0.0157	3996.979	-44.270	-0.10524	0.00584	0.0275	0.4239	1.62
4	119.694	0.0139	622.854	80.31	0.35907	0.0152	478.153	-9.694	-0.14239	0.0156	1588.489	-19.694	-0.11135	0.0157	3994.007	-44.694	-0.10578	-0.00045	0.0272	-0.0330	0.12
5	119.661	0.0139	622.980	80.34	0.35911	0.0152	477.022	-9.661	-0.14231	0.0156	1586.736	-19.661	-0.11131	0.0157	3992.412	-44.661	-0.10577	-0.00029	0.0272	-0.0211	0.08
6	119.640	0.0139	622.975	80.36	0.35916	0.0152	477.045	-9.640	-0.14216	0.0156	1586.779	-19.640	-0.11125	0.0157	3992.465	-44.640	-0.10574	0.00001	0.0272	0.0005	0.00
7	119.641	0.0139	622.968	80.36	0.35916	0.0152	477.096	-9.641	-0.14215	0.0156	1586.860	-19.641	-0.11125	0.0157	3992.540	-44.641	-0.10574	0.00001	0.0272	0.0010	0.00
8	119.642	0.0139	622.968	80.36	0.35916	0.0152	477.098	-9.642	-0.14216	0.0156	1586.862	-19.642	-0.11126	0.0157	3992.541	-44.642	-0.10574	0.00000	0.0272	0.0000	0.00
9	119.642	0.0139	622.969	80.36	0.35916	0.0152	477.095	-9.642	-0.14216	0.0156	1586.858	-19.642	-0.11126	0.0157	3992.538	-44.642	-0.10574	0.00000	0.0272	0.0000	0.00
10	119.642	0.0139	622.969	80.36	0.35916	0.0152	477.095	-9.642	-0.14216	0.0156	1586.858	-19.642	-0.11126	0.0157	3992.538	-44.642	-0.10574	0.00000	0.0272	0.0000	0.00

Table 2a: Solution of Example 5.6 using Relative Head Resistance Method and taking Pipe 4 as a pivot pipe



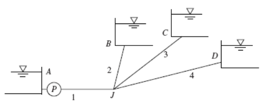
RESERVOIR 1			RESERVOIR 4			RESERVOIR 3			RESERVOIR 2		
H_1 =	200	m	H_2 =	75	m	H_3 =	100	m	H_4 =	120	m
Hp_1 =		m	Hp_2 =		m	Hp_3 =		m	Hp_4 =	10	m
L_1 =	10000	m	L_2 =	3000	m	L_3 =	3000	m	L_4 =	2000	m
D_1 =	450	mm	D_2 =	250	mm	D_3 =	300	mm	D_4 =	350	mm
$\sum K_m$ =	0		$\sum K_m$ =	0		$\sum K_m$ =	0		$\sum K_m$ =	0	
e	0.0600	mm	e	0.0600	mm	e	0.0600	mm	e	0.0600	mm
T	15	C^0	T	15	C^0	T	15	C^0	T	15	C^0
ν (m^2/s)	1.15E-06		ν (m^2/s)	1.15E-06		ν (m^2/s)	1.15E-06		ν (m^2/s)	1.15E-06	
			α =	0.1030							

Trial no	$HJ_{i+1}=H_4-hL4_{i+1}-Hp$	f_1	R_1	$H_1-H_J+H_p$	$Q_1(m^3/s)$	f_4	R_4	hL_4-Hp	$Q_4(m^3/s)$ (Assume initial Q_4)	f_3	R_3	$H_3-H_J-H_p$	$Q_3(m^3/s)$	f_2	R_2	$H_2-H_J-H_p$	$Q_2(m^3/s)$	$\sum Q=\Delta Q$ (m^3/s)	Error %
1	75.000	0.0150	672.602	125.00	0.43110	0.0170	4304.377	0.000	0.00000	0.0163	1664.615	25.00	0.12255	0.0158	497.313	35.00	0.26529	0.81894	308.70
2	105.623	0.0137	615.103	94.38	0.39171	0.0170	4304.377	-30.623	-0.08435	0.0154	1573.009	-5.62	-0.05979	0.0144	453.840	4.38	0.09821	0.34578	352.09
3	133.548	0.0138	619.101	66.45	0.32762	0.0160	4068.542	-58.548	-0.11996	0.0167	1699.122	-33.55	-0.14051	0.0158	496.125	-23.55	-0.21786	-0.15072	46.00
4	118.140	0.0140	627.334	81.86	0.36123	0.0156	3955.165	-43.140	-0.10444	0.0152	1554.909	-18.14	-0.10801	0.0146	460.111	-8.14	-0.13300	0.01578	4.37
5	119.957	0.0139	622.704	80.04	0.35853	0.0157	3996.403	-44.957	-0.10606	0.0156	1591.280	-19.96	-0.11199	0.0153	480.206	-9.96	-0.14399	-0.00352	0.98
6	119.596	0.0139	623.049	80.40	0.35923	0.0157	3991.599	-44.596	-0.10570	0.0155	1585.889	-19.60	-0.11116	0.0151	476.511	-9.60	-0.14191	0.00046	0.13
7	119.648	0.0139	622.959	80.35	0.35914	0.0157	3992.658	-44.648	-0.10575	0.0156	1586.983	-19.65	-0.11127	0.0152	477.175	-9.65	-0.14220	-0.00007	0.02
8	119.641	0.0139	622.970	80.36	0.35916	0.0157	3992.519	-44.641	-0.10574	0.0156	1586.839	-19.64	-0.11125	0.0152	477.083	-9.64	-0.14215	0.00001	0.00
9	119.642	0.0139	622.968	80.36	0.35916	0.0157	3992.540	-44.642	-0.10574	0.0156	1586.861	-19.64	-0.11126	0.0152	477.097	-9.64	-0.14216	0.00000	0.00
10	119.642	0.0139	622.969	80.36	0.35916	0.0157	3992.537	-44.642	-0.10574	0.0156	1586.857	-19.64	-0.11126	0.0152	477.095	-9.64	-0.14216	0.00000	0.00

Table 2b: Solution of Example 5.6 using Nodal Method

Trial no	$HJ_{i+1}=HJ_i+\Delta z_i$ (Asume initial H_J)	f_1	R_1	$H_1-H_J+H_p$	$Q_1(m^3/s)$	f_2	R_2	$H_2-H_J-H_p$	$Q_2(m^3/s)$	f_3	R_3	$H_3-H_J-H_p$	$Q_3(m^3/s)$	f_4	R_4	$H_4-H_J-H_p$	$Q_4(m^3/s)$	$\sum Q=\Delta Q$ (m^3/s)	$\sum(Q/h_L)$	Δz	Error %
1	76.000	0.0150	672.602	124.00	0.42937	0.0158	497.313	34.000	0.26147	0.0163	1664.615	24.000	0.12007	0.0170	4304.377	-1.000	-0.01524	0.79567	0.0314	50.6828	5220.24
2	126.683	0.0137	615.265	73.32	0.34520	0.0144	454.271	-16.683	-0.19164	0.0154	1575.858	-26.683	-0.13012	0.0202	5138.895	-51.683	-0.10029	-0.07684	0.0230	-6.6785	22.26
3	120.004	0.0140	624.816	80.00	0.35781	0.0148	464.702	-10.004	-0.14673	0.0153	1564.864	-20.004	-0.11306	0.0158	4009.278	-45.004	-0.10595	-0.00792	0.0271	-0.5838	2.21
4	119.420	0.0139	623.141	80.58	0.35960	0.0151	475.664	-9.420	-0.14073	0.0155	1584.486	-19.420	-0.11071	0.0157	3991.933	-44.420	-0.10549	0.00267	0.0275	0.1945	0.74
5	119.615	0.0139	622.912	80.38	0.35923	0.0152	477.559	-9.615	-0.14189	0.0156	1587.586	-19.615	-0.11115	0.0157	3993.285	-44.615	-0.10570	0.00048	0.0273	0.0355	0.13
6	119.651	0.0139	622.959	80.35	0.35914	0.0152	477.181	-9.651	-0.14221	0.0156	1586.992	-19.651	-0.11128	0.0157	3992.659	-44.651	-0.10575	-0.00010	0.0272	-0.0072	0.03
7	119.643	0.0139	622.971	80.36	0.35915	0.0152	477.078	-9.643	-0.14217	0.0156	1586.831	-19.643	-0.11126	0.0157	3992.512	-44.643	-0.10574	-0.00003	0.0272	-0.0019	0.01
8	119.641	0.0139	622.969	80.36	0.35916	0.0152	477.091	-9.641	-0.14216	0.0156	1586.851	-19.641	-0.11125	0.0157	3992.531	-44.641	-0.10574	0.00000	0.0272	0.0002	0.00
9	119.642	0.0139	622.969	80.36	0.35916	0.0152	477.096	-9.642	-0.14216	0.0156	1586.859	-19.642	-0.11125	0.0157	3992.539	-44.642	-0.10574	0.00000	0.0272	0.0001	0.00
10	119.642	0.0139	622.969	80.36	0.35916	0.0152	477.096	-9.642	-0.14216	0.0156	1586.858	-19.642	-0.11126	0.0157	3992.538	-44.642	-0.10574	0.00000	0.0272	0.0000	0.00

Table 3a: Solution of Problem 10.15 using Relative Head Resistance Method, Pipe 1 as a pivot pipe and with the calculated α (α = 1.3909)



Hp = 120 - 0.5 Q^2

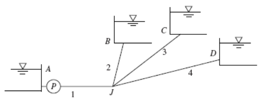
RESERVOIR 1			RESERVOIR 2			RESERVOIR 3			RESERVOIR 4		
H_1 =	20	m	H_2 =	50	m	H_3 =	100	m	H_4 =	40	m
Hp_1 =		m	Hp_2 =		m	Hp_3 =		m	Hp_4 =		m
L_1 =	250	m	L_2 =	700	m	L_3 =	2000	m	L_4 =	1500	m
D_1 =	500	mm	D_2 =	300	mm	D_3 =	300	mm	D_4 =	350	mm
$\sum K_m$ =	0		$\sum K_m$ =	0		$\sum K_m$ =	0		$\sum K_m$ =	0	
e	0.6	mm	e	0.35	mm	e	0.35	mm	e	0.4	mm
T	20	C^0	T	20	C^0	T	20	C^0	T	20	C^0
ν (m^2/s)	1.01E-06		ν (m^2/s)	1.01E-06		ν (m^2/s)	1.01E-06		ν (m^2/s)	1.01E-06	
α =	1.3909										

Trial no	$HJ_{i+1}=H_1-hL1_{i+1}+Hp$	f_1	R_1	$Q_1(m^3/s)$ (Assume initial Q_1)	Hp	hL_1	f_2	R_2	$H_2-H_J-H_p$	$Q_2(m^3/s)$	f_3	R_3	$H_3-H_J-H_p$	$Q_3(m^3/s)$	f_4	R_4	$H_4-H_J-H_p$	$Q_4(m^3/s)$	$\sum Q=\Delta Q$ (m^3/s)	Error %
1	140.000	0.0210	13.909	0.00000	120.00	0.00	0.0212	504.782	-90.000	-0.42225	0.0212	1442.235	-40.00	-0.16654	0.0210	495.731	-100.00	-0.44914	-1.03792	#DIV/0!
2	109.968	0.0210	13.909	1.44366	118.96	28.99	0.0206	489.604	-59.968	-0.34998	0.0208	1412.675	-9.97	-0.08400	0.0205	483.307	-69.97	-0.38049	0.62919	43.58
3	135.428	0.0206	13.645	0.56850	119.84	4.41	0.0206	490.262	-85.428	-0.41743	0.0211	1434.269	-35.43	-0.15717	0.0205	483.940	-95.43	-0.44406	-0.45016	79.18
4	119.719	0.0207	13.711	1.19463	119.29	19.57	0.0206	489.640	-69.719	-0.37734	0.0208	1414.014	-19.72	-0.11809	0.0205	483.347	-79.72	-0.40612	0.29308	24.53
5	131.234	0.0207	13.654	0.78698	119.69	8.46	0.0206	489.983	-81.234	-0.40717	0.0209	1421.808	-31.23	-0.14821	0.0205	483.679	-91.23	-0.43431	-0.20271	25.76
6	123.797	0.0207	13.681	1.06894	119.43	15.63	0.0206	489.722	-73.797	-0.38819	0.0208	1415.446	-23.80	-0.12966	0.0205	483.427	-83.80	-0.41634	0.13475	12.61
7	128.996	0.0207	13.660	0.88152	119.61	10.61	0.0206	489.884	-78.996	-0.40157	0.0209	1419.025	-29.00	-0.14295	0.0205	483.583	-89.00	-0.42899	-0.09199	10.44
8	125.558	0.0207	13.673	1.00947	119.49	13.93	0.0206	489.768	-75.558	-0.39278	0.0208	1416.370	-25.56	-0.13433	0.0205	483.472	-85.56	-0.42067	0.06169	6.11
9	127.916	0.0207	13.664	0.92366	119.57	11.66	0.0206	489.843	-77.916	-0.39883	0.0209	1418.034	-27.92	-0.14031	0.0205	483.544	-87.92	-0.42640	-0.04187	4.53
10	126.339	0.0207	13.669	0.98190	119.52	13.18	0.0206	489.791	-76.339	-0.39479	0.0208	1416.858	-26.34	-0.13634	0.0205	483.494	-86.34	-0.42258	0.02819	2.87
11	127.412	0.0207	13.665	0.94269	119.56	12.14	0.0206	489.826	-77.412	-0.39754	0.0208	1417.627	-27.41	-0.13905	0.0205	483.527	-87.41	-0.42518	-0.01908	2.02
12	126.690	0.0207	13.668	0.96924	119.53	12.84	0.0206	489.802	-76.690	-0.39569	0.0208	1417.097	-26.69	-0.13724	0.0205	483.505	-86.69	-0.42343	0.01287	1.33
13	127.179	0.0207	13.666	0.95133	119.55	12.37	0.0206	489.818	-77.179	-0.39695	0.0208	1417.450	-27.18	-0.13847	0.0205	483.520	-87.18	-0.42462	-0.00870	0.91
14	126.850	0.0207	13.667	0.96344	119.54	12.69	0.0206	489.807	-76.850	-0.39610	0.0208	1417.209	-26.85	-0.13764	0.0205	483.510	-86.85	-0.42382	0.00587	0.61
15	127.072	0.0207	13.667	0.95527	119.54	12.47	0.0206	489.814	-77.072	-0.39667	0.0208	1417.370	-27.07	-0.13820	0.0205	483.517	-87.07	-0.42436	-0.00397	0.42

Table 3b: Solution of Example 10.15 using Nodal Method and using initial H_J value as in Table 3a

Trial no	Asume initial H_J $HJ_{i+1}=HJ_i+\Delta z_i$	f_1	R_1	Hp	H_1-H_J+Hp	$Q_1(m^3/s)$	f_2	R_2	H_2-H_J-H_p	$Q_2(m^3/s)$	f_3	R_3	H_3-H_J-H_p	$Q_3(m^3/s)$	f_4	R_4	H_4-H_J-H_p	$Q_4(m^3/s)$	$\Sigma Q=\Delta Q\ (m^3/s)$	$\Sigma(Q/h_L)$	Δz	Error %
1	140.000	0.0210	13.909		-120.00	-2.93721	0.0212	504.782	-90.000	-0.42225	0.0212	1442.235	-40.000	-0.16654	0.0210	495.731	-100.000	-0.44914	-3.97513	0.0378	-210.1954	135.34
2	-70.195	0.0210	13.909	115.69	205.88	3.84728	0.0206	489.604	120.195	0.49547	0.0208	1412.675	170.195	0.34710	0.0205	483.307	110.195	0.47750	5.16735	0.0292	354.1504	1082.17
3	283.955	0.0206	13.619	112.60	-151.36	-3.33376	0.0206	489.131	-233.955	-0.69160	0.0206	1400.839	-183.955	-0.36238	0.0205	483.097	-243.955	-0.71062	-5.09836	0.0299	-341.4273	152.93
4	-57.472	0.0206	13.621	114.44	191.92	3.75362	0.0205	488.355	107.472	0.46912	0.0206	1400.373	157.472	0.33534	0.0204	482.007	97.472	0.44969	5.00777	0.0307	326.5926	1113.60
5	269.120	0.0206	13.619	112.96	-136.17	-3.16200	0.0206	489.284	-219.120	-0.66921	0.0206	1401.227	-169.120	-0.34741	0.0205	483.302	-229.120	-0.68853	-4.86715	0.0313	-310.6504	153.93
6	-41.530	0.0206	13.622	115.00	176.53	3.59990	0.0205	488.421	91.530	0.43290	0.0206	1400.829	141.530	0.31786	0.0204	482.079	81.530	0.41124	4.76190	0.0324	293.8361	1157.92
7	252.306	0.0206	13.620	113.52	-118.79	-2.95324	0.0206	489.525	-202.306	-0.64286	0.0206	1401.855	-152.306	-0.32961	0.0205	483.630	-212.306	-0.66256	-4.58828	0.0333	-275.3689	155.36
8	-23.063	0.0206	13.623	115.64	158.70	3.41310	0.0205	488.504	73.063	0.38674	0.0206	1401.425	123.063	0.29633	0.0204	482.169	63.063	0.36165	4.45782	0.0349	255.1538	1232.64
9	232.091	0.0206	13.621	114.18	-97.92	-2.68119	0.0206	489.897	-182.091	-0.60967	0.0206	1402.729	-132.091	-0.30687	0.0205	484.155	-192.091	-0.62988	-4.22761	0.0363	-232.7137	157.68
10	-0.623	0.0206	13.626	116.41	137.03	3.17124	0.0205	488.619	50.623	0.32188	0.0206	1402.286	100.623	0.26787	0.0204	482.294	40.623	0.29022	4.05121	0.0393	206.1283	1395.90
11	205.505	0.0206	13.622	114.97	-70.53	-2.27551	0.0206	490.597	-155.505	-0.56300	0.0206	1404.097	-105.505	-0.27412	0.0206	485.221	-165.505	-0.58403	-3.69666	0.0420	-175.9952	162.45
12	29.510	0.0206	13.630	117.41	107.90	2.81365	0.0205	488.803	20.490	0.20474	0.0206	1403.773	70.490	0.22409	0.0204	482.492	10.490	0.14745	3.38992	0.0533	127.1925	2299.07
13	156.703	0.0206	13.624	116.04	-20.66	-1.23145	0.0207	492.962	-106.703	-0.46524	0.0207	1406.869	-56.703	-0.20076	0.0208	490.351	-116.703	-0.48785	-2.38530	0.0717	-66.5509	193.70
14	90.152	0.0207	13.653	119.24	49.09	1.89621	0.0206	489.308	-40.152	-0.28646	0.0207	1408.829	9.848	0.08361	0.0205	483.027	-50.152	-0.32222	1.37114	0.0607	45.1952	425.52
15	135.347	0.0206	13.635	118.20	2.86	0.45760	0.0206	491.111	-85.347	-0.41687	0.0211	1434.470	-35.347	-0.15697	0.0205	484.686	-95.347	-0.44353	-0.55978	0.1742	-6.4251	122.33

Table 4a: Solution of Problem10.15 sing Relative Head Resistance Method, Pipe 1 as a pivot pipe and with maximum allowed $\boldsymbol{\alpha}$ $(\boldsymbol{\alpha}=\mathbf{1})$



Hp = 120 - 0.5 Q^2

RESERVOIR 1			RESERVOIR 2			RESERVOIR 3			RESERVOIR 4		
H_1 =	20	m	H_2 =	50	m	H_3 =	100	m	H_4 =	40	m
Hp_1 =		m	Hp_2 =		m	Hp_3 =		m	Hp_4 =		m
L_1 =	250	m	L_2 =	700	m	L_3 =	2000	m	L_4 =	1500	m
D_1 =	500	mm	D_2 =	300	mm	D_3 =	300	mm	D_4 =	350	mm
ΣK_m =	0		ΣK_m =	0		ΣK_m =	0		ΣK_m =	0	
e	0.6	mm	e	0.35	mm	e	0.35	mm	e	0.4	mm
T	20	C^0	T	20	C^0	T	20	C^0	T	20	C^0
$\nu\ (m^2/s)$	1.01E-06		$\nu\ (m^2/s)$	1.01E-06		$\nu\ (m^2/s)$	1.01E-06		$\nu\ (m^2/s)$	1.01E-06	
α =	1.0000										

Trial no	$HJ_{i+1}=H_1-hL1_{i+1}+Hp$	f_1	R_1	Assume initial Q_1 $Q_1(m^3/s)$	Hp	hL_1	f_2	R_2	H_2-H_J-H_p	$Q_2(m^3/s)$	f_3	R_3	H_3-H_J-H_p	$Q_3(m^3/s)$	f_4	R_4	H_4-H_J-H_p	$Q_4(m^3/s)$	$\Sigma Q=\Delta Q\ (m^3/s)$	Error %
1	140.000	0.0210	13.909	0.00000	120.00	0.00	0.0212	504.782	-90.000	-0.42225	0.0212	1442.235	-40.00	-0.16654	0.0210	495.731	-100.00	-0.44914	-1.03792	#DIV/0!
2	124.477	0.0210	13.909	1.03792	119.46	14.98	0.0206	489.604	-74.477	-0.39002	0.0208	1412.675	-24.48	-0.13163	0.0205	483.307	-84.48	-0.41808	0.09819	9.46
3	127.494	0.0207	13.662	0.93973	119.56	12.06	0.0206	489.867	-77.494	-0.39773	0.0209	1418.599	-27.49	-0.13922	0.0205	483.568	-87.49	-0.42536	-0.02258	2.40
4	126.880	0.0207	13.668	0.96231	119.54	12.66	0.0206	489.800	-76.880	-0.39618	0.0208	1417.066	-26.88	-0.13773	0.0205	483.503	-86.88	-0.42390	0.00451	0.47
5	127.004	0.0207	13.667	0.95781	119.54	12.54	0.0206	489.814	-77.004	-0.39650	0.0208	1417.354	-27.00	-0.13803	0.0205	483.516	-87.00	-0.42419	-0.00091	0.10
6	126.979	0.0207	13.667	0.95872	119.54	12.56	0.0206	489.811	-76.979	-0.39643	0.0208	1417.295	-26.98	-0.13797	0.0205	483.513	-86.98	-0.42413	0.00018	0.02
7	126.984	0.0207	13.667	0.95853	119.54	12.56	0.0206	489.811	-76.984	-0.39645	0.0208	1417.307	-26.98	-0.13798	0.0205	483.514	-86.98	-0.42414	-0.00004	0.00
8	126.983	0.0207	13.667	0.95857	119.54	12.56	0.0206	489.811	-76.983	-0.39644	0.0208	1417.304	-26.98	-0.13798	0.0205	483.514	-86.98	-0.42414	0.00001	0.00
9	126.983	0.0207	13.667	0.95856	119.54	12.56	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.98	-0.13798	0.0205	483.514	-86.98	-0.42414	0.00000	0.00
10	126.983	0.0207	13.667	0.95857	119.54	12.56	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.98	-0.13798	0.0205	483.514	-86.98	-0.42414	0.00000	0.00
11	126.983	0.0207	13.667	0.95857	119.54	12.56	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.98	-0.13798	0.0205	483.514	-86.98	-0.42414	0.00000	0.00
12	126.983	0.0207	13.667	0.95857	119.54	12.56	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.98	-0.13798	0.0205	483.514	-86.98	-0.42414	0.00000	0.00
13	126.983	0.0207	13.667	0.95857	119.54	12.56	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.98	-0.13798	0.0205	483.514	-86.98	-0.42414	0.00000	0.00
14	126.983	0.0207	13.667	0.95857	119.54	12.56	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.98	-0.13798	0.0205	483.514	-86.98	-0.42414	0.00000	0.00
15	126.983	0.0207	13.667	0.95857	119.54	12.56	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.98	-0.13798	0.0205	483.514	-86.98	-0.42414	0.00000	0.00

Table 4b: Solution of Example 10.15 using Nodal Method and using suitable initial H_J value

Trial no	Asume initial H_J $HJ_{i+1}=HJ_i+\Delta z_i$	f_1	R_1	Hp	H_1-H_J+Hp	$Q_1(m^3/s)$	f_2	R_2	H_2-H_J-H_p	$Q_2(m^3/s)$	f_3	R_3	H_3-H_J-H_p	$Q_3(m^3/s)$	f_4	R_4	H_4-H_J-H_p	$Q_4(m^3/s)$	$\Sigma Q=\Delta Q\ (m^3/s)$	$\Sigma(Q/h_L)$	Δz	Error %
1	101.000	0.0210	13.909		-81.00	-2.41316	0.0212	504.782	-51.000	-0.31786	0.0212	1442.235	-1.000	-0.02633	0.0210	495.731	-61.000	-0.35079	-3.10814	0.0681	-91.2721	128.80
2	9.728	0.0210	13.909	117.09	127.36	3.02595	0.0206	490.649	40.272	0.28649	0.0224	1520.015	90.272	0.24370	0.0205	484.290	30.272	0.25002	3.80616	0.0418	181.9757	1522.36
3	191.704	0.0206	13.623	115.42	-56.28	-2.03259	0.0206	491.110	-141.704	-0.53716	0.0207	1405.506	-91.704	-0.25543	0.0206	486.084	-151.704	-0.55865	-3.38383	0.0464	-145.9393	166.48
4	45.764	0.0206	13.633	117.93	92.17	2.60016	0.0205	488.919	4.236	0.09308	0.0207	1404.789	54.236	0.19649	0.0205	482.616	-5.764	-0.10929	2.78044	0.0728	76.4200	2544.14
5	122.184	0.0206	13.626	116.62	14.44	1.02926	0.0210	500.534	-72.184	-0.37976	0.0207	1409.237	-22.184	-0.12547	0.0209	493.898	-82.184	-0.40792	0.11611	0.0872	2.6637	11.28
6	124.848	0.0207	13.663	119.47	14.62	1.03453	0.0206	489.961	-74.848	-0.39085	0.0209	1419.976	-24.848	-0.13228	0.0205	483.662	-84.848	-0.41884	0.09255	0.0862	2.1466	8.95
7	126.995	0.0207	13.662	119.46	12.47	0.95538	0.0206	489.860	-76.995	-0.39646	0.0209	1418.460	-26.995	-0.13795	0.0205	483.561	-86.995	-0.42415	-0.00318	0.0917	-0.0693	0.33
8	126.925	0.0207	13.667	119.54	12.62	0.96086	0.0206	489.811	-76.925	-0.39630	0.0208	1417.310	-26.925	-0.13783	0.0205	483.514	-86.925	-0.42400	0.00273	0.0913	0.0598	0.28
9	126.985	0.0207	13.667	119.54	12.55	0.95839	0.0206	489.813	-76.985	-0.39645	0.0208	1417.333	-26.985	-0.13798	0.0205	483.515	-86.985	-0.42415	-0.00019	0.0915	-0.0041	0.02
10	126.981	0.0207	13.667	119.54	12.56	0.95864	0.0206	489.811	-76.981	-0.39644	0.0208	1417.304	-26.981	-0.13797	0.0205	483.514	-86.981	-0.42414	0.00008	0.0915	0.0018	0.01
11	126.983	0.0207	13.667	119.54	12.56	0.95856	0.0206	489.811	-76.983	-0.39644	0.0208	1417.306	-26.983	-0.13798	0.0205	483.514	-86.983	-0.42414	-0.00001	0.0915	-0.0002	0.00
12	126.983	0.0207	13.667	119.54	12.56	0.95857	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.983	-0.13798	0.0205	483.514	-86.983	-0.42414	0.00000	0.0915	0.0001	0.00
13	126.983	0.0207	13.667	119.54	12.56	0.95857	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.983	-0.13798	0.0205	483.514	-86.983	-0.42414	0.00000	0.0915	0.0000	0.00
14	126.983	0.0207	13.667	119.54	12.56	0.95857	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.983	-0.13798	0.0205	483.514	-86.983	-0.42414	0.00000	0.0915	0.0000	0.00
15	126.983	0.0207	13.667	119.54	12.56	0.95857	0.0206	489.811	-76.983	-0.39644	0.0208	1417.305	-26.983	-0.13798	0.0205	483.514	-86.983	-0.42414	0.00000	0.0915	0.0000	0.00

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