Al-Azhar University Civil Engineering Research Magazine (CERM)
Vol. (40) No. (1) January, 2018

Experimental study of new reinforcement details for reinforced concrete deep beams with shear opening

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ملخص البحث

يقدم هذا البحث، استنادا إلى الدراسات التجريبية والنظرية، طريقتين جديدتين لتسليح الكمرات العميقة ذات فتحات القص مختلفة الأحجام؛ الطريقة الأولى عبارة عن ضواغط مسلحة مدفونة في مسار الحمل يرافقها زيادة قطر صلب التسليح الملاصق للفتحات، والطريقة الثانية عبارة عن تكثيف حديد الكانات أسفل وأعلي الفتحات في منطقة القص يرافقها زيادة قطر صلب التسليح المتاخم للفتحات.

أثبتت نتائج الدراسة التجريبية كفاءة الأساليب المقترحة من خلال الزيادة في قدرة تحمل الكمرات المختبرة بنسبة تتراوح من 23-50٪ عن الكمرات المرجعية في كل المجموعات. وشمل هذا البرنامج التجريبي دراسة مجموعتان مختلفتان من اجهاد الخرسانة، وتسليحين مختلفين للشداد السفلي الرئيسي في الكمرات، وثلاثة أحجام مختلفة من مقاس فتحة القص لمقارنة النتائج مع نموذج الضاغط والشداد بالكود الأمريكي 318-14 [1] في حالة الكمرات العميقة التي ليس بها فتحات ومع معادلة Kong and Sharp [2] في حالة الكمرات العميقة التي بها فتحات.

وكشفت المقارنة مع النماذج النظرية عن دقة نموذج الضاغط والشداد بالكود الأمريكي 318-14 [1] في التنبؤ بحمل الكسر للكمرات العميقة المصنوعة من خرسانة عادية المقاومة، في حين أنه متحفظ قليلا في التنبؤ بحمل الانهيار للكمرات العميقة المصنوعة من خرسانة عالية المقاومة.

كما وجد أن معادلة Kong and Sharp [2] متحفظة جدا في التنبؤ بحمل الانهيار للكمرات العميقة التي بها فتحات.

Abstract

This paper present, based on experimental and theoretical investigations, two new methods to reinforce deep beams with different sizes of shear opening; first method was embedded struts in the load path accompanied with increasing steel bar diameter adjacent to the opening, second method was intensifying web reinforcement below and above the opening in the shear zone accompanied with increasing steel bar diameter adjacent to the opening.

Results of experimental study proved efficiency of the proposed methods by increasing the load capacity of the tested deep beams by 23-50% from the reference cases in all groups. Two different concrete strengths, two different main tie reinforcement, and three sizes of shear web opening were involved in the experimental program to compare the results with strut-and-tie model in the ACI 318-14 [1] in case of solid deep beams and with equation of Kong and Sharp [2] in case of deep beams with opening

Comparison revealed that the ACI 318-14 [1] give precise predictions of load capacity of solid deep beam with normal strength concrete, while it a little bite conservative in prediction of HSC solid deep beams; more conservatism of the equation of Kong and Sharp [2] in predicting load capacity of deep beams with opening was observed.

Keywords: *solid deep beams, deep beams with shear openings, shear reinforcement, strut-and-tie model.*

1. Introduction

Nowadays it is important to study the effect of critical web openings on the load capacity of reinforced concrete deep beams, because of using it widely in high-rise building and bridge transfer girders to achieve the ability of passing civil services.

The ACI 318-14 [1] defines the deep beam with two conditions as follow: beam with shear span-to-depth ratio less than or equal to 2 or beam with clear span less than or equal to four times its height.

ACI 318-14 [1] illustrated the method of strut-and-tie for designing and detailing of solid deep beams which based on balancing between forces in chosen truss model, it will be used to predict the shear strength of solid deep beams in this study.

Previous researches concentrated upon studying the effect of opening existence on the load capacity of the deep beam such as; Kong and Sharp 1977 [2] which conducted experimental program to obtain the effect of web openings on the shear strength of deep beams, This study revealed that the effect of opening in shear zone depending on the length of opening interfering the load path as presented in figure 1, they proposed formula to predict the capacity of deep beam with and without opening as mentioned in equations 2, 1

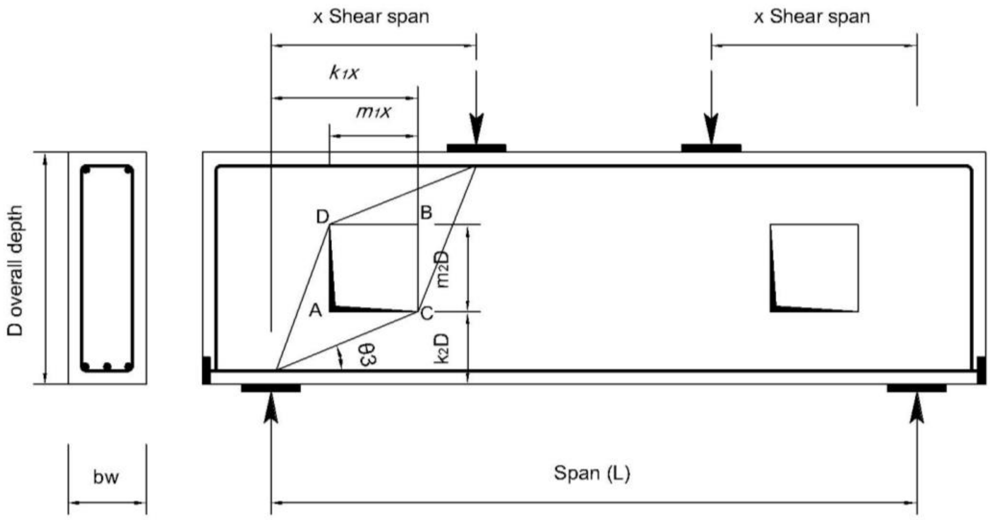


Fig.1: Factors defining size and location of openings in shear zone of deep beams according to Kong and Sharp [2] equations

$$Q_u = C_1\left[1 - 0.35\frac{x}{D}\right] f_t bD + C_2 \sum_{all\ bars}\left(A\frac{y}{D}sin^2\theta\right) \quad \text{(eq. 1)}$$

$$Q_u = C_1\left[1 - 0.35\frac{k_1 x}{k_2 D}\right] f_t b k_2 D + C_2 \sum_{all\ bars}\left(\lambda A\frac{y}{D}sin^2\theta_3\right) \quad \text{(eq. 2)}$$

Where:

A is area of individual longitudinal or web bar

b is breadth of deep beam

C_1 is empirical coefficient (equals 1.4 for normal-weight concrete and 1.35 for lightweight concrete)

C_2 is empirical coefficient (equal 130 N/mm^2 for plain reinforcement and 300 N/mm^2 for deformed bars)

D is the overall depth of deep beam

f_t is the cylinder splitting tensile strength

k_1, k_2 are coefficient defining the position of openings in the shear zone of deep beams

x is clear shear span

y is depth at which a typical bar intersects the potential diagonal crack, it is forms approximately along the line jointing the load and the support points
θ is the angle of intersection between a typical bar and the diagonal crack
λ is empirical coefficient (equal 1 in case of longitudinal main steel bars and 1.5 in case of web bars)
m_1, m_2 are factors defining size of openings.
Previous equations are applicable in critical shear zone of deep beams, which extend for area of (0.6 D multiply X), and also for limited opening size as follow

$$m_1 \leq x/2$$
$$m_2 \leq 0.3D$$

Yang et al (2006) [4] executed an experimental program of 32 specimens of normal and high strength concrete deep beams to attain the effect of concrete strength, shear span-to-depth ratio, and size of the openings on the shear capacity of deep beams, results from experimental program were compared with Kong and Sharp [2] equations, Comparison revealed that efficiency of this equation has lower bound limit of the inclination angle of the load path of $\theta_3 \geq 30°$
G. Campione and G. Minafo (2012) [5] tested twenty deep beams with and without openings in flexure under four-point loading to investigate the effect of Circular openings. They found that the effect of hole in deep beams depends on its position, they also suggested equation to determine the transverse tension of reinforced/unreinforced concrete struts.
M. E. El- Zoughiby et al. (2014) [6] Proposed new values for struts effectiveness factor βs and compared the predicted load capacities with previous experimental data.
El-Demerdash W. E. et al (2016) [7] made verifications on previous experimental results by finite element analysis program.
This research contributes in reinforcing of the deep beams with shear openings based on new applicable methods including study different parameters such as opening size, concrete compressive strength, reduction in shear strength of deep beams with openings relative to the solid ones, percentage of gained shear strength resulted in using proposed reinforcement detailing methods.

2. Experimental Study

2.1. Specimen details

Experimental program consisted of 14 deep beams divided to two solid deep beams and four groups which was designed to Ph.D. program [8], each group contain three deep beams with openings: one beam represents the reference case with Φ6 mm steel bars adjacent to the openings (the same web bars) and the other two beams represent the two proposed reinforcing methods; first method involve using embedded struts accompanied with replace steel bars adjacent to the openings to Φ10 and Φ12 mm in the vertical and horizontal directions respectively, second method consisted of intensifying 8 mm ties to confine shear zone regions in addition to the same replacement of steel bars adjacent to openings in the first method.
Table 1 illustrates the properties and designations of all specimens in the experimental program.
Reinforcement detailing methods and section configurations of all groups of the experimental program shown in figures 2 to 6. Test setup for the experimental program will be explained in figure 7.
All solid and opened deep beams were designed according to the ACI 318-14 [1] requirements including main steel ratio, web reinforcement ratio, spacing of web reinforcement, tie anchorage length, and within boundary of concrete strength.

Large opening size of the deep beams represent the boundary limits of opening size in equation of Kong and Sharp [2] which is 0.3D and 0.5X.

Table 1: Properties and details of deep beams in the experimental study

Group Descriptions	Specimen name	Dimension of openings (x*h) mm	Main tensile rft.	Comp. rft.	Web rft. (except shear zone)	Reinforcement of the openings (in Shear zone)			
						Closed short ties below & above openings	Add. Horizontal web rft. below& above openings	Add. Vertical web rft. adjacent to openings	Embedded struts
Solid NSC deep beam	NSD		2Φ16	2Φ12	Φ6@110 mm				
Group A (NSC deep beams with large openings)	NLR	200x180				2Φ6	2Φ6	2Φ6	
	NLS					2Φ6	2Φ12	2Φ10	4Φ12
	NLT					15Φ8	2Φ12	2Φ10	
Solid HSC deep beam	HSD		3Φ16	2Φ12					
Group B (HSC deep beams with large openings)	HLR	200x180				2Φ6	2Φ6	2Φ6	
	HLS					2Φ6	2Φ12	2Φ10	4Φ12
	HLT					15Φ8	2Φ12	2Φ10	
Group C (HSC deep beams with Medium openings)	HMR	150x150				1Φ6	2Φ6	2Φ6	
	HMS					1Φ6	2Φ12	2Φ10	4Φ12
	HMT					15Φ8	2Φ12	2Φ10	
Group D (HSC deep beams with Small openings)	HSR	100x120				1Φ6	2Φ6	2Φ6	
	HSS					1Φ6	2Φ12	2Φ10	4Φ12
	HST					15Φ8	2Φ12	2Φ10	

* Section dimensions are 150x600 mm for all HSC and 120x600 mm for all NSC deep beams.
* Large openings represent max. Limit of Kong and Sharp [2] equation.
*All embedded struts have ties of 6mm @ 50 mm.

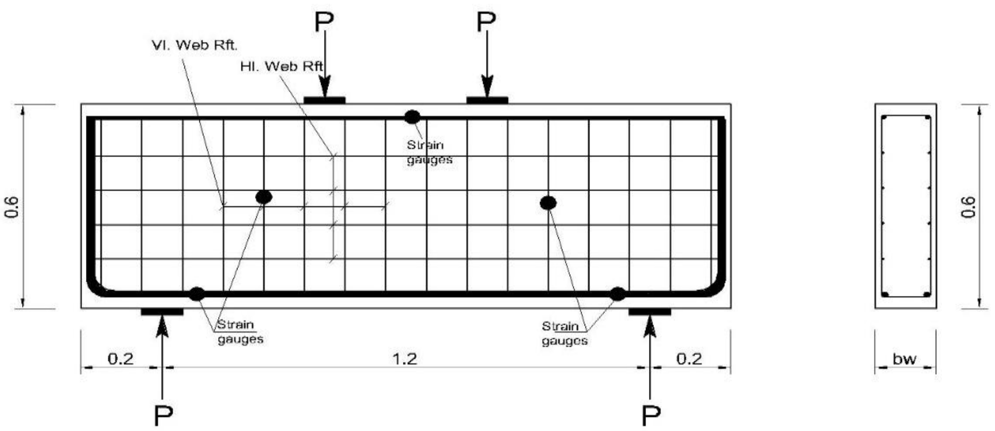


Fig. 2: Location of installed strain gauges and web reinforcement details for solid deep beams NSD and HSD.

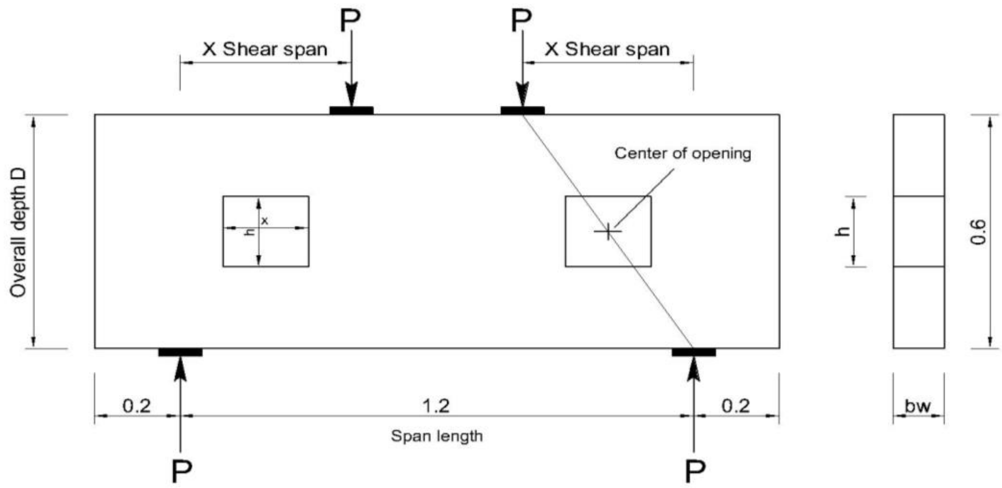


Fig. 3: General layout and section details for all deep beams with openings.

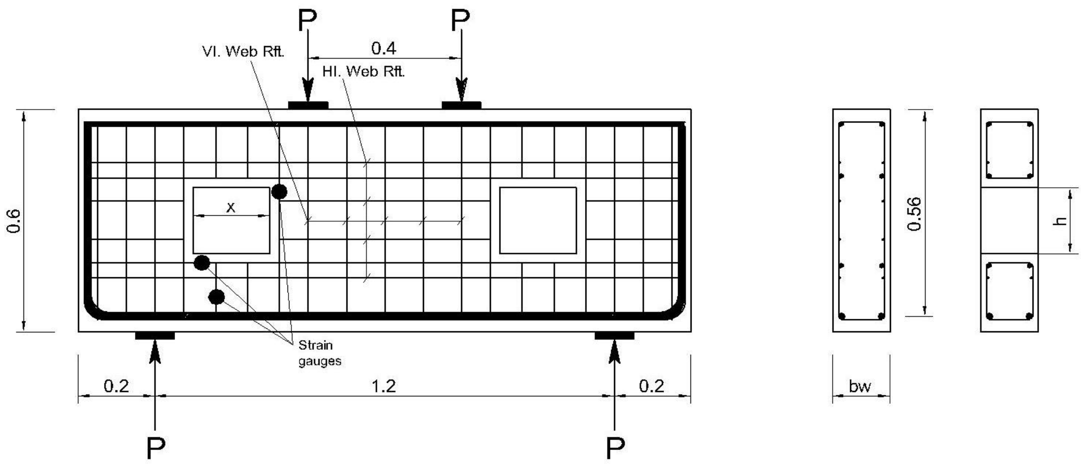


Fig. 4: Location of installed strain gauges and reinforcement details for all reference deep beams with openings.

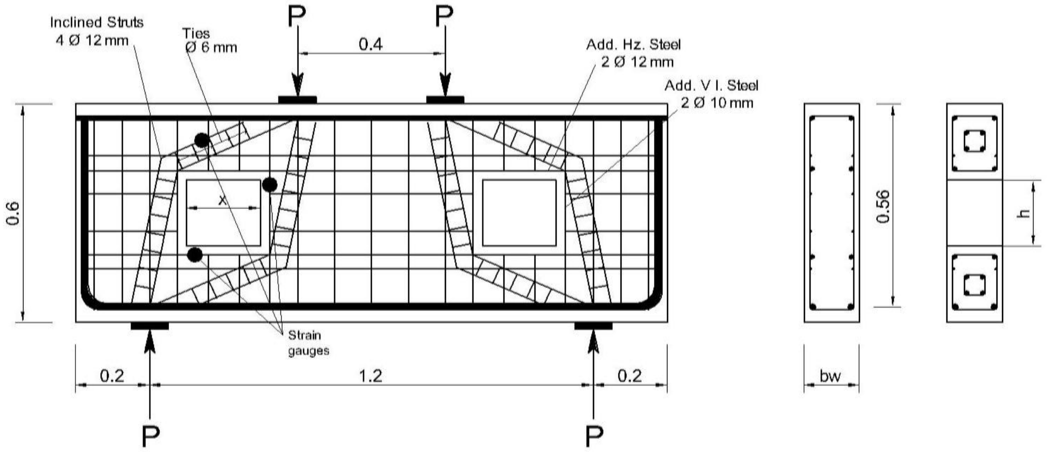


Fig. 5: Location of installed strain gauges and reinforcement details for all deep beams with openings reinforced with embedded struts (method one).

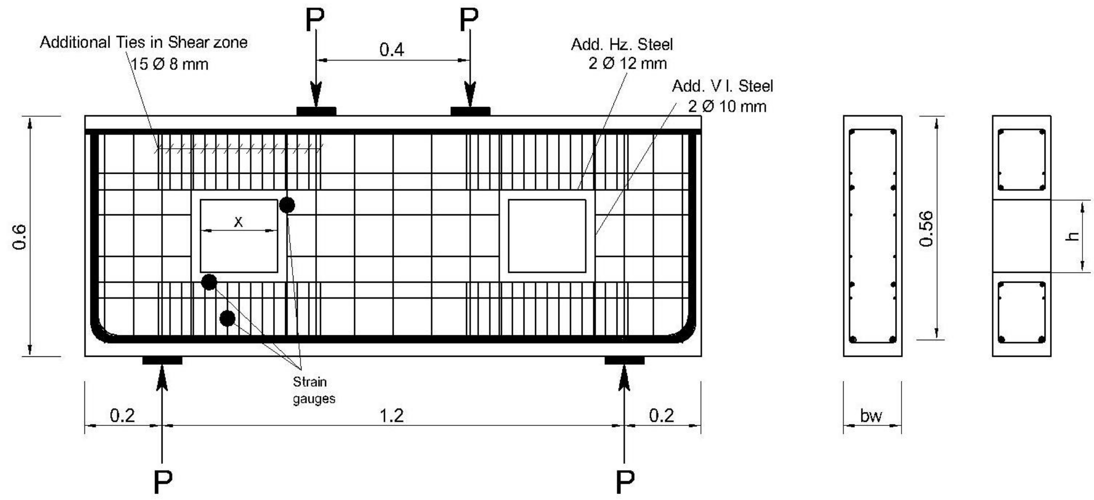


Fig. 6: Location of installed strain gauges and reinforcement details for all deep beams with openings reinforced with intensify ties in shear zone (method two)

2.2. Properties of used material

Material properties of steel bars used in the experimental program are illustrated in table 2, the cylinder compressive strengths of used concrete in the experimental study were 40, 53 MPa based on compressive test of three cylinders as well be mentioned in table 3.

Concrete strengths represent the upper limit of normal strength concrete and the lower limit of high compressive strength concrete covered in the ACI 318-14 [1]

Table 2: Mechanical properties of used steel bars in the tested deep beams

Bar diameter (mm)	Steel Grade	Actual area section (mm^2)	Yield strength (MPa)	Ultimate strength (MPa)	Elongation at failure %
Φ6	24/35	28.3	371	577	37.6
Φ8	24/35	50.29	324	464	35
Φ10	40/60	78.5	536	638	24.3
Φ12	40/60	113	560	692	18.9
Φ16	40/60	201	512	640	22.4

* Results are the average of three different batches of steel bars.

3. Experimental results

All deep beams were loaded till failure in the HBRC concrete structures laboratory under loading frame of 1000 kN capacity and Universal testing machine of 5000 kN capacity for specimens with higher predicted failure load.

All deep beams have the same full span length, depth, and shear span length, the only two differences between NSC and HSC specimens are width of the beam and main tie steel. Figure 7 illustrate the general setup and instrument configuration of the specimen just before the loading.

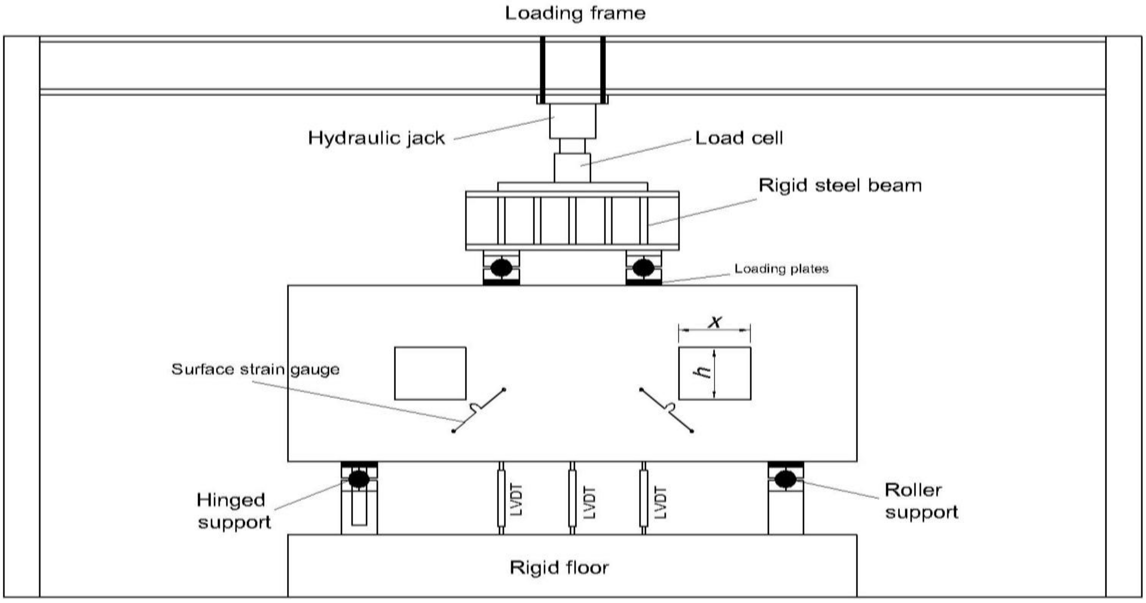


Fig. 7: Setup and specimen configuration details for all deep beams with openings including location of concrete surface strain gauges

All results of the deep beams were gathered in table 3 such as specimen first crack load, ultimate load, deflection at mid-span of the specimen at peak load, state of the main tie steel in case of solid deep beams, and failure type of the specimen.

Table 3: Experimental results for all deep beams in the study

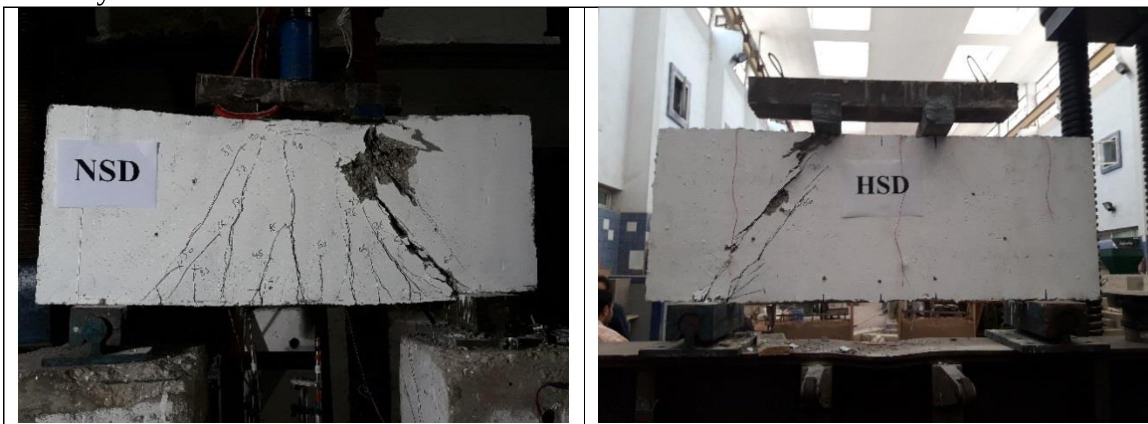
Group Description	Specimen name	f'_c (MPa)	First cracking load P_{cr}(kN)	Peak load P_u (kN)	$\frac{P_{cr}}{P_u}$	Deflection at ultimate load Δ (mm)	Percentage of load increase %	Main steel state	Failure Type
Solid NSC deep beam	NSD	40	180	455	0.396	6.00		yield	*Flexural shear*
Group A (NSC deep beams with large openings)	NLR		55	270	0.204	5.02			*shear splitting at openings*
	NLS		165	357	0.462	3.64	32.22		*shear splitting at openings*
	NLT		175	381	0.459	5.8	41.11		*shear splitting at openings*
Solid HSC deep beam	HSD	53	130	552	0.236	3.58		yield	*Diagonal shear splitting*
Group B (HSC deep beams with large openings)	HLR		100	270	0.370	5.13			*shear splitting at openings*
	HLS		130	359	0.362	4.41	32.96		*shear splitting at openings*
	HLT		125	407	0.307	8.1	50.74		*shear splitting at openings*
Group C (HSC deep beams with Medium openings)	HMR		100	325	0.308	5.1			*shear splitting at openings*
	HMS		175	400	0.438	3.53	23.08		*shear splitting at openings*

	HMT		105	422	0.249	5.9	29.85		*shear splitting at openings*
Group D (HSC deep beams with Small openings)	HSR		120	440	0.273	5.09			*shear splitting at openings*
	HSS		125	623	0.201	5	41.59		*shear splitting at openings*
	HST		150	554	0.271	6.2	25.91		*shear splitting at openings*

* The first letter in the specimen designation refer to the two concrete strengths of 40 and 53 Mpa

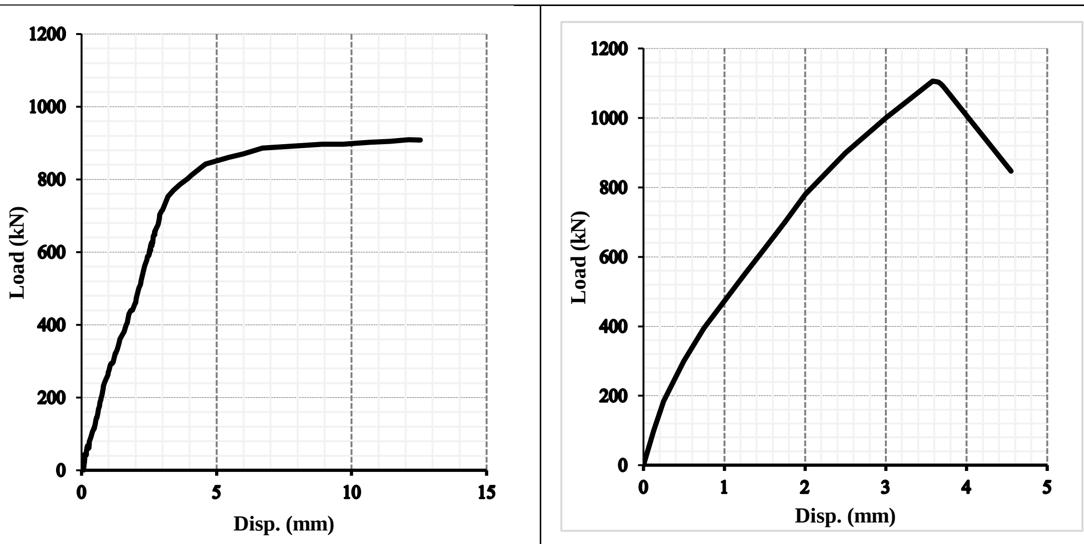
3.1. Behavior of the solid deep beams NSD and HSD

In case of beam NSD, it failed by flexural shear failure showing more flexural cracks in the middle third of the beam during the test. But in case of beam HSD, the failure type was diagonal shear splitting showing less flexural cracks in the middle third during test. Figures 8 to 11 show the crack pattern of those specimens, load-displacement, main steel strain, and concrete compressive strain for only beam HSD because of beam NSD reached strain equal to 10 times concrete maximum strain before crushing then it was destroyed.



NSD	HSD

Fig. 8: Crack pattern of the solid deep beams in the study NSD and HSD



NSD	HSD

Fig. 9: Load-displacement for the solid deep beams in the study NSD and HSD

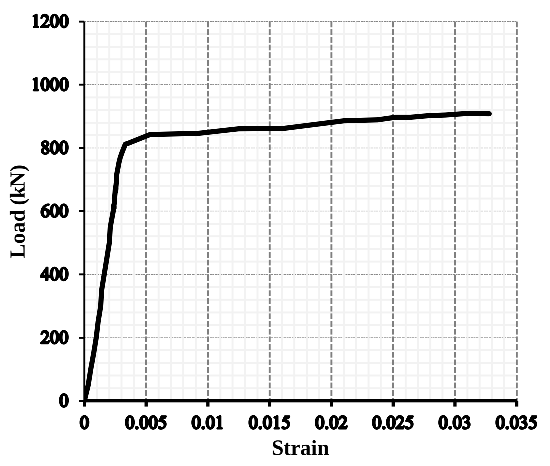
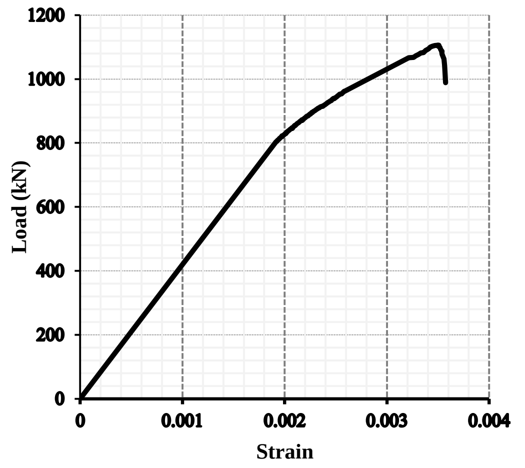
	
NSD	HSD

Fig. 10: Load-strain of main steel for beams NSD and HSD

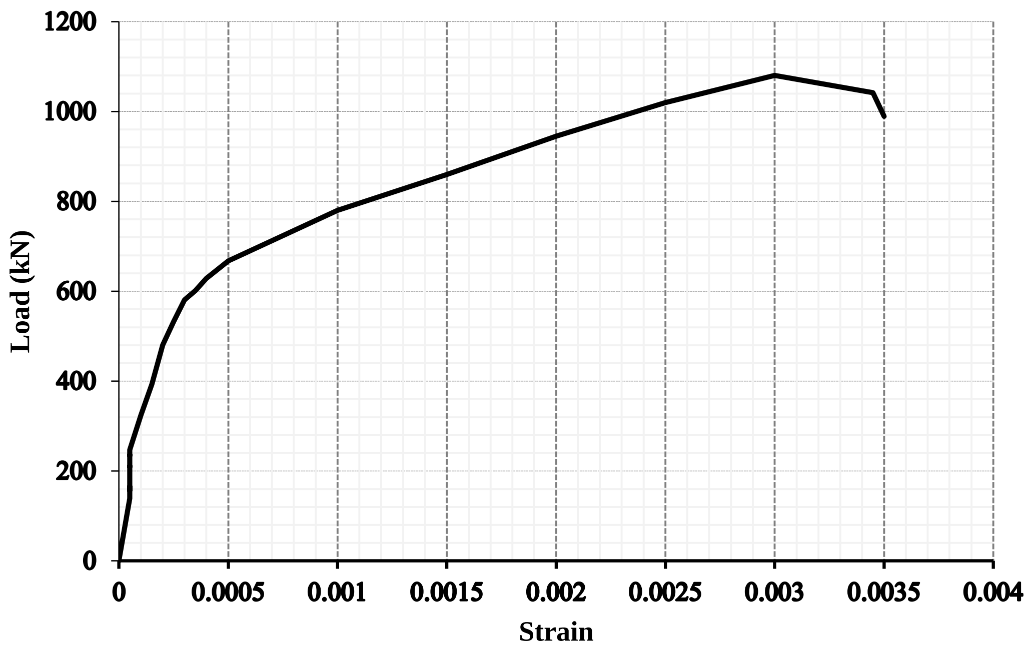


Fig. 11: Load-compressive strain of inclined strut between load and support of beam HSD

3.2. Behavior of beams in the group A- NSC deep beams with large openings

All beams failed by shear splitting at line connecting between both of load and support points and openings diagonal corners as shown in figure 12, only beam NLS exhibited less destructive failure than the other beams.

Figure 13 shows the load-displacement response, tensile strain of additional steel bar adjacent to the openings, concrete compressive strain at opening lower corner, and compressive strain of Φ12 mm embedded strut steel bars for beams in the group A

It was observed that in both of NLR and NLT failure cause opening of ties lock in regions below and above the shear openings.

	
NLR	NLS
	
NLT	

Fig. 12: Crack patterns for beams in group A

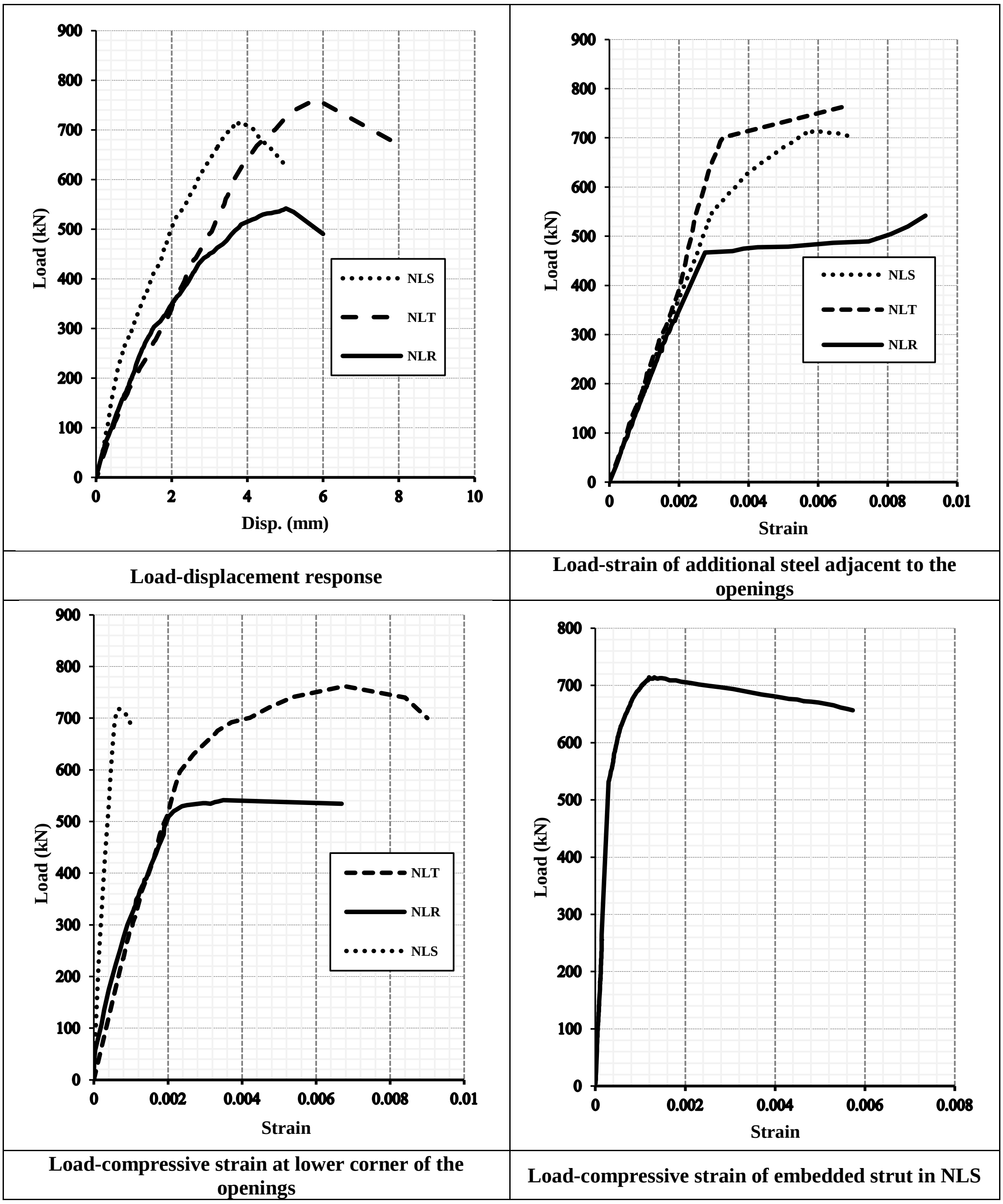


Fig. 13: The different responses for beams in group A

3.3. Behavior of beams in the group B-HSC deep beams with large openings

All beams failed by shear splitting at line connecting between both of load and support points and openings diagonal corners as shown in figure 14, only beam HLS exhibited less destructive failure than other beams.

Figure 15 shows the load-displacement response, tensile strain of additional steel bar adjacent to the openings, concrete compressive strain at opening lower corner, and compressive strain of Φ12 mm embedded strut steel bars for all beams in group B.

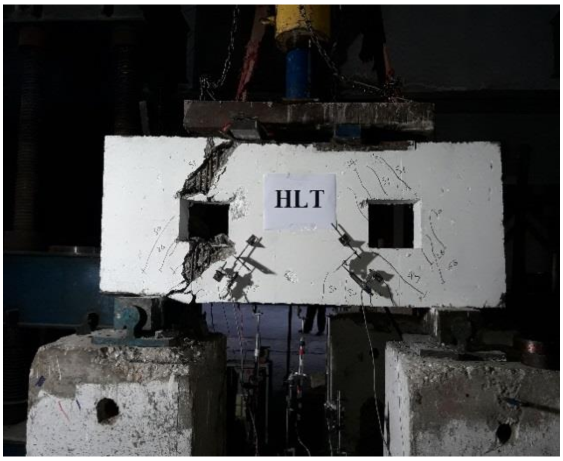
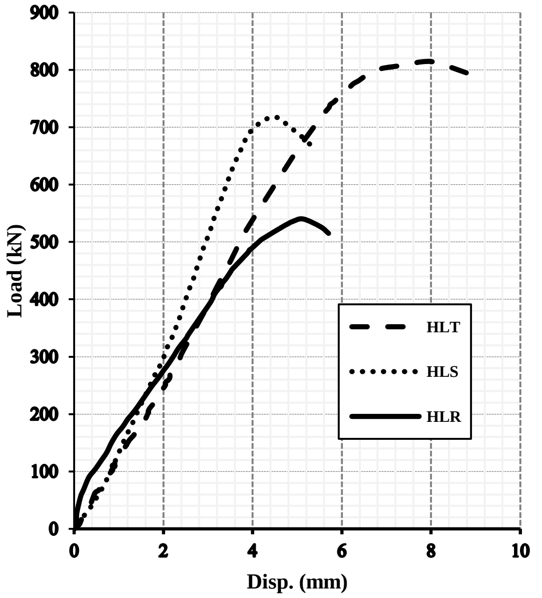
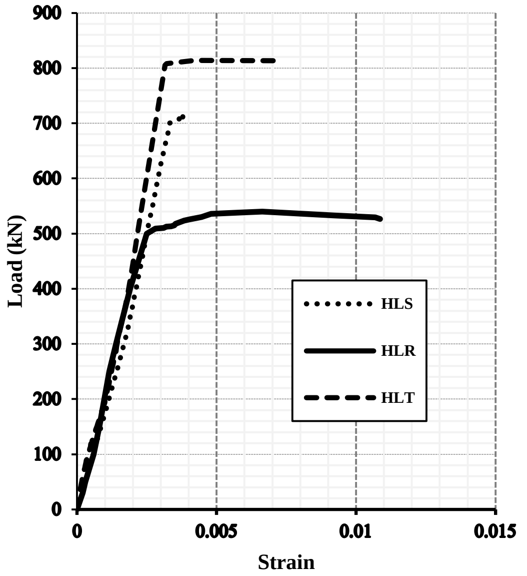
	
HLR	HLS
	
HLT	

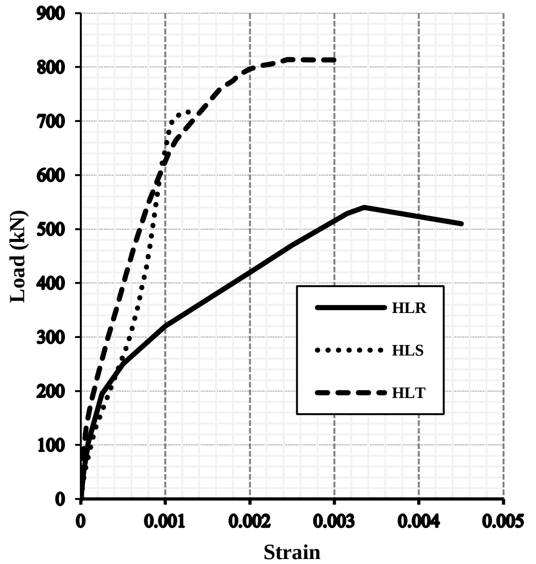
Fig. 14: Crack patterns for beams in group B



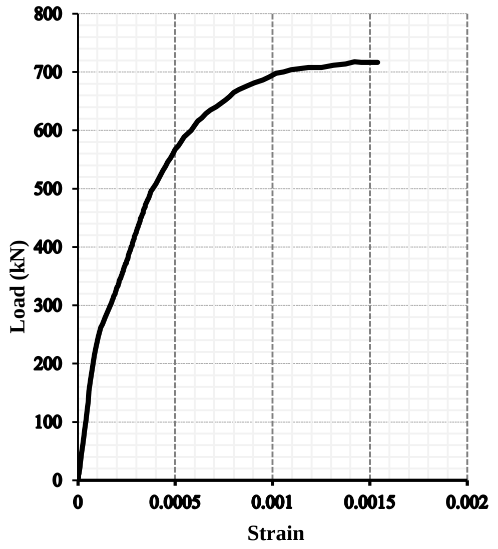
Load-displacement response



Load-strain of additional steel adjacent to the openings



Load-compressive strain at lower corner of the openings



Load-compressive strain of embedded strut in HLS

Fig. 15: The different responses for beams in group B

3.4. Behavior of beams in the group C-HSC deep beams with medium openings

All beams failed by shear splitting at line connecting between both of load and support points and openings diagonal corners as shown in figure 16, only beam HMS exhibited less destructive failure than other beams.

Figure 17 shows the load-displacement response, tensile strain of additional steel bar adjacent to the openings, concrete compressive strain at opening lower corner, and compressive strain of Φ12 mm embedded strut steel bars for all beams in group C.

HLR

HLS

HLT

Fig. 16: Crack patterns for beams in group C

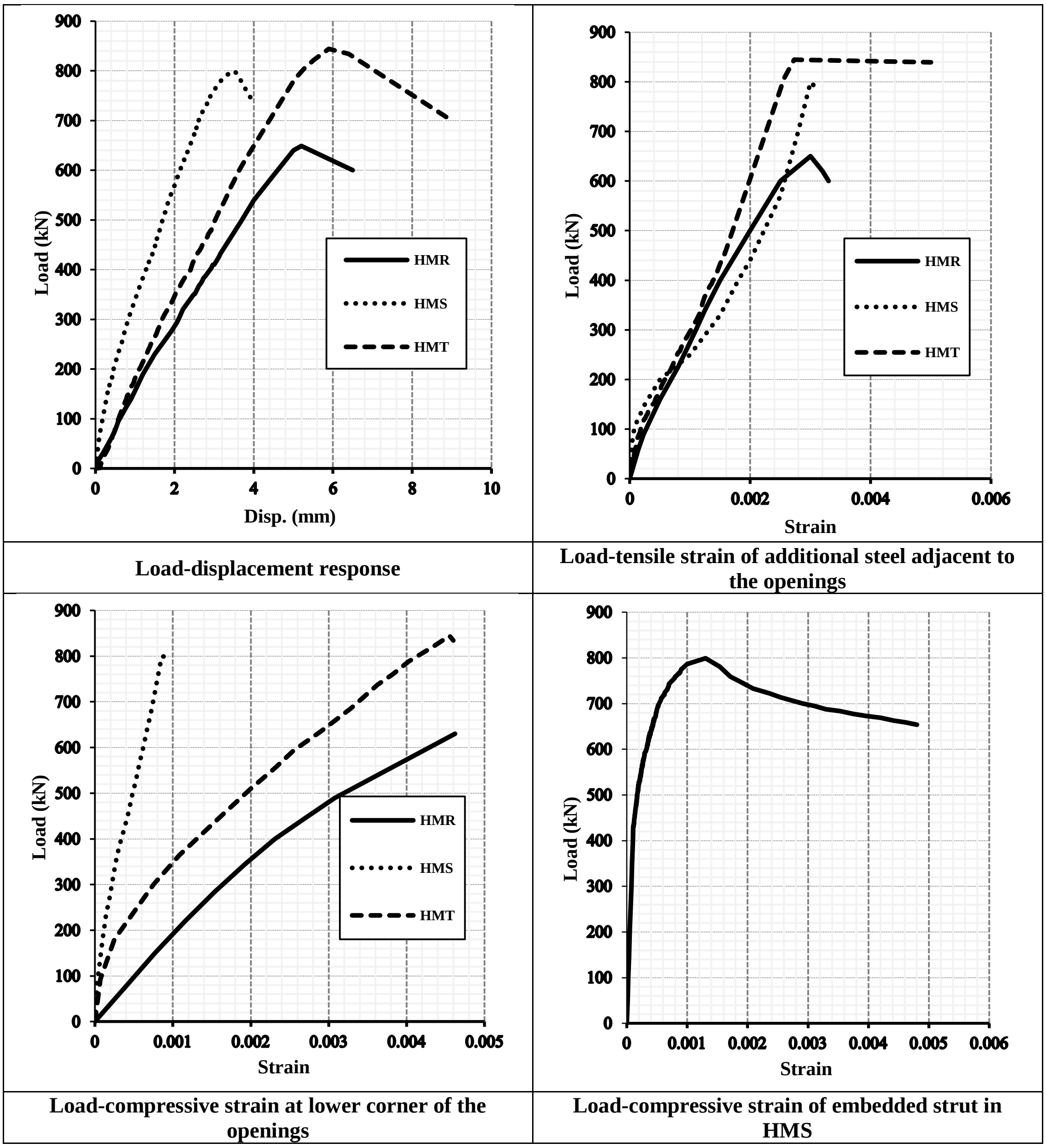


Fig. 17: The different responses for beams in group C

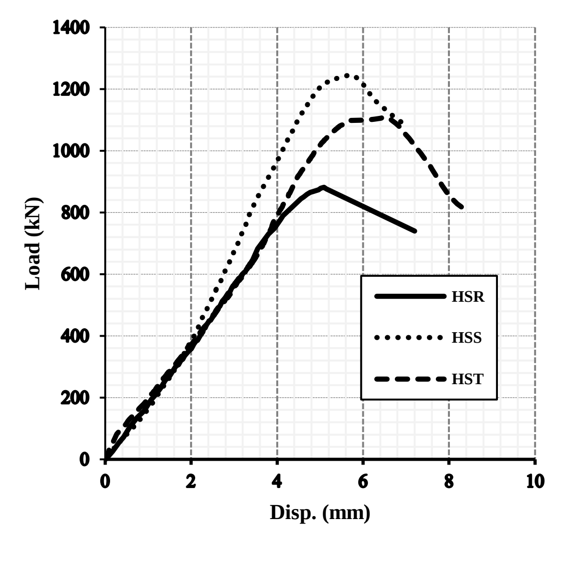
3.5. Behavior of beams in the group D-HSC deep beams with small openings

All beams failed by shear splitting at line connecting between both of load and support points and openings diagonal corners as shown in figure 18, only beam HSS exhibited less destructive failure than other beams.

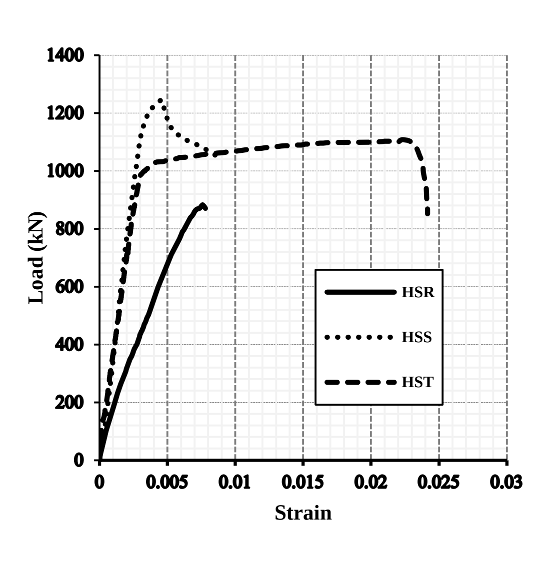
Figure 19 shows the load-displacement response, tensile strain of additional steel bar adjacent to the openings, concrete compressive strain at opening lower corner, and compressive strain of Φ12 mm embedded strut steel bars for all beams in group D.

 	
HSR	HSS
 	
HST	

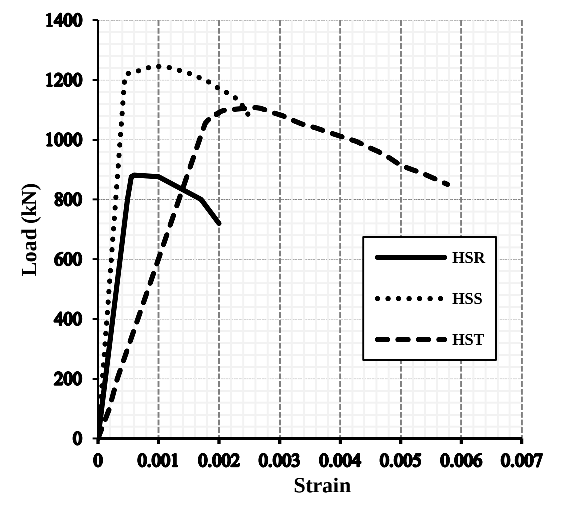
Fig. 18: Crack patterns for beams in group D



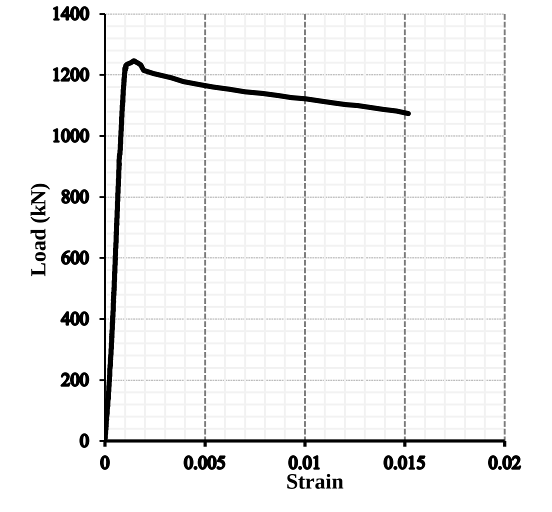
Load-displacement response



Load-tensile strain of additional steel adjacent to the openings



Load-compressive strain at lower corner of the openings



Load-compressive strain of embedded strut in HSS

Fig. 19: The different responses for beams in group D

4. Analysis and discussion

4.1. Comparison between results of experimental study

Figure 20 give comparison between peak loads for tested deep beams in the experimental study, it can be observed that existence of the openings in shear zone of both normal and high strength concrete deep beams reduce load capacity in proportion of opening size, it will be compared with Kong and Sharp [2] equation.

From figures 20 and 21, it can be illustrated that the using first method is more efficient way to enhance load capacity of deep beam with medium and large openings than the second method of using embedded strut, while the opposite is true in case of deep beams with small openings.

The increase in the load capacity due to the method of using embedded struts in shear zone is 32% in case of normal strength concrete deep beam with large openings, while it equal to 33%, 23%, and 41.6% in case of high strength concrete deep beams with large, medium, and small openings.

The increase in the load capacity due to the method of intensifying ties below and above the web openings in shear zone is 41% in case of normal strength concrete deep beam with large openings, while it equal to 50%, 30%, and 26% in case of high strength concrete with large, medium, and small openings.

The efficiency of the proposed reinforcement methods is more significant in case of deep beams with large openings.

The additional steel around the openings is the most efficient component in resisting stresses affecting the openings, because of yielding of it in all deep beams with shear openings at ultimate load.

Peak load and type of failure of solid deep beams depends on the balancing between forces in the strut-and-tie model (failure by flexural shear in case of 2Φ16mm tie, while pure diagonal shear splitting in case of 3Φ16mm tie)

Failure of the deep beams reinforced with additional short 8 mm ties in the shear zone is more destructive than failure of the deep beams reinforced with embedded struts because of its contribution in resisting compressive stresses.

It was found that efficiency of the second method of embedded strut depends on lower bound inclination of the strut in the lower path, it explain the superiority of this method in reinforcing of the deep beams with small openings where inclination of the lower embedded strut is 31^{o} with the horizontal direction, while it equal 24^{o} and 26^{o} in deep beams with large and medium openings respectively, it agrees with the findings of Yang. et al 2006 [4] in case of concrete struts.

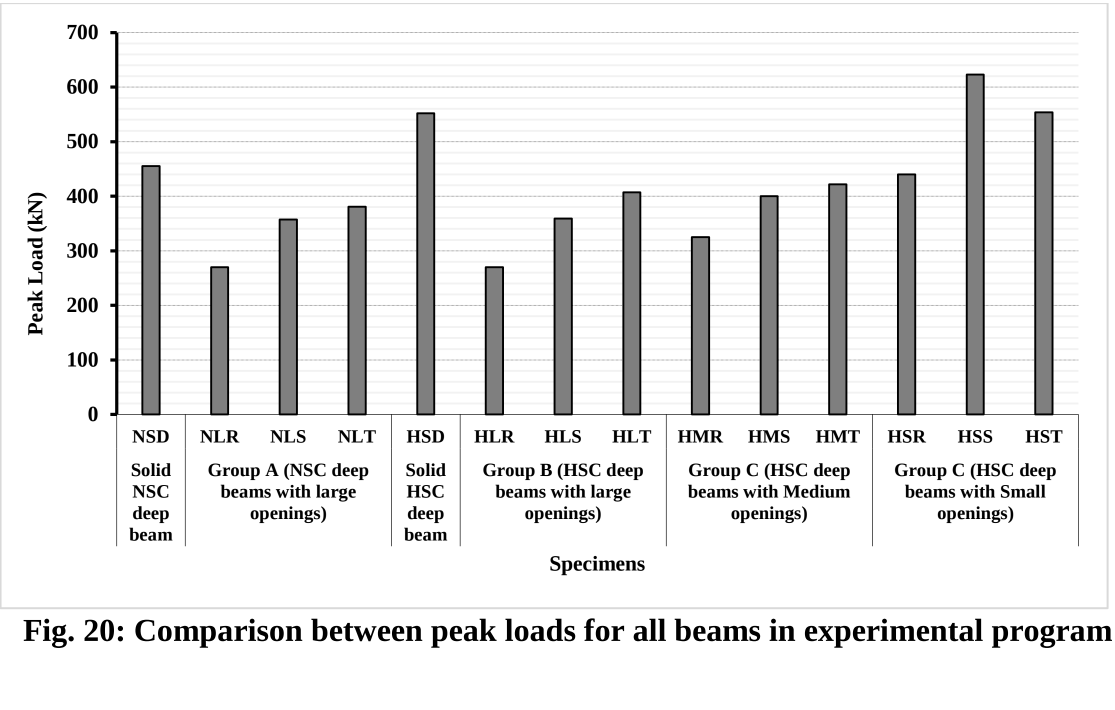


Fig. 20: Comparison between peak loads for all beams in experimental program

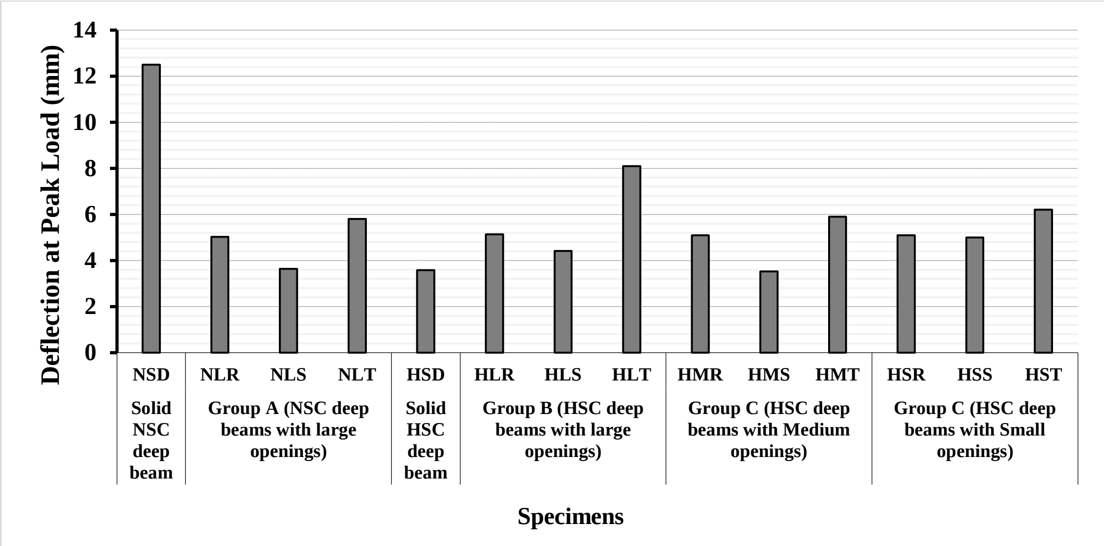


Fig. 21: Comparison between deflections at ultimate load for all beams in the experimental program

4.2. Comparison between experimental results and Kong & Sharp [2] equation for deep beams with openings

Excel spread sheet was assigned to implement the equation of Kong and Sharp [2] on all deep beams with openings in the experimental program. Table 4 indicates the predicted load capacity according to the previously mentioned Kong and Sharp [2] equation in introduction clause, in addition to comparison between those predicted load and experimental loads.

Table 4: Comparison between experimental load capacities of reference deep beams with shear openings and its predicted load capacities by Kong and Sharp [2]

			Kong & Sharp. Equation	Exp.	
Group Description	**Specimen name**	**Dim. of openings *x*h* (mm)**	**Load capacity, $L_{Th.}$ (kN)**	**Load capacity, L_{exp} (kN)**	$L_{exp.}/L_{th.}$
Group A (NSC deep beam with large openings)	**NLR**	**200x180**	**101.7**	**270**	**265%**
Group B (HSC deep beam with large openings)	**HLR**	**200x180**	**109.7**	**270**	**246%**
Group C (HSC deep beam with medium openings)	**HMR**	**150x150**	**137**	**325**	**237%**
Group D (HSC deep beam with small openings)	**HSR**	**100x120**	**166.5**	**440**	**246%**

The comparison between experimental and predicted load capacities by Kong and Sharp equation [2] in table 4 resulted in conservatism of this equation with average ratio of 248.5%

If we take in consideration strength reduction factor of 0.75 which represent the ratio between experimental and ultimate strengths as required in the ACI 318-14 [1] for all elements of strut-and-tie model and according to Sharp [3]

So, after dividing all predicted shear strengths by 0.75, the real average ratio between experimental and theoretical load capacities of deep beams with shear openings will be 1.86 which is a quite conservative but more safely in the design of such cases.

From table 4 it can be observed that predicted shear capacity of deep beams with shear openings has almost constant rate with decreasing of opening size.

This comparison verify findings of Yang.et al [4] and assure on a lower bound limit of inclination of the lower strut in the deep beam of 30º

4.3. Comparison between experimental results and ACI 318-14 [1] Strut-and-Tie model for solid deep beams

According to the ACI 318-14 [1] appendix A, a flowchart used to predict the load capacity of solid deep beams NSD and HSD.

Table 5 indicates the comparison between predicted load capacity according to the ACI 318-14 [1] and experimental ones.

It can be observed that ACI 318-14 [1] strut-and-tie model give reasonable conservative prediction of the load capacity of solid deep beams.

Table 5: Comparison between experimental load capacities of solid deep beams and predicted load capacities by ACI 318-14 [1] strut-and-tie model

		Strut-and-tie model in ACI 318	Experimental	
Description	**Specimen name**	**Load capacity $P_{ACI.}$(kN)**	**Load capacity $P_{Exp.}$ (kN)**	**$P_{Exp.}/P_{ACI.}$%**
Solid NSC deep beam	**NSD**	**440**	**455**	**103%**
Solid HSC deep beam	**HSD**	**405**	**552**	**136%**

5. Conclusions

From the experimental and theoretical study it can be conclude that:

- Efficiency of the new proposed methods in reinforcing deep beams with shear openings was proved by increasing load capacity within 23-50% relative to reference cases.
- The method of embedded struts is more effective in case of angle of inclination not less than 30º.
- The most important component of the two proposed methods is the steel bars adjacent to the shear openings.
- Efficiency of the proposed methods is significant in case of deep beams with large openings.
- Equation of Kong and Sharp [2] is quite conservative in predicting shear strength of deep beams with openings within its limit, however it can be recommended in design codes.
- ACI 318-14 [1] strut-and tie model gives precise prediction of load capacity of solid deep beams in case of NSC deep beams.

- ACI 318-14 [1] strut-and tie model gives conservative prediction in case of HSC solid deep beams due to overestimated values of effectiveness factor which agree with previous findings in El-Zoughiby et al [6].

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