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COMPUTATIONAL MODEL OF ULTIMATE CAPACITY AND BEHAVIOR OF REINFORCED CONCRETE BEAMS WITH MULTIPLE OPENINGS

Khaled Farouk El-Kashif[1]
1- Assistant Professor of Concrete Structures, Structural Engineering Dept., Cairo University, Giza, Egypt

ملخص البحث:
المباني الحديثة تتطلب العديد من المواسير والقنوات لتمرير انابيب المياه والغاز وغيرها من الخدمات. وقد تعلق هذه الخدمات تحت كمرات الاسقف ولكنها بذلك تشغل حيزا في الطابق الذي تمر فيه مما يؤثر على الارتفاع الخاص للدور. ويتم تمرير هذه المواسير او الانابيب من خلال فتحات في الكمرات. ووجود هذه الفتحات قد يؤثر على الأمان الإنشائي للكمرات. وبالتالي، فان هذا البحث يدرس تأثير المتغيرات الاتية على سلوك الكمرات الخرسانية المسلحة ذات الفتحات المتعددة وهذه المتغيرات تشمل:
مقاس الفتحة - شكل الفتحة (دائرية، مربعة) - المسافة بين الفتحات – قيمة حديد تسليح مقاومة القص.
تم تكوين نماذج تحليلية للكمرات باستخدام طريقة العناصر المحددة عن طريق برنامج ANSYS14 وتم عرض النتائج عن طريق الجداول والرسوم البيانية وتم استنتاج ان الحمل الأقصى الذي تتحمله الكمرة يزداد مع تقليل حجم الفتحة وزيادة المسافة بين الفتحات وزيادة حديد تسليح مقاومة القص واستخدام الفتحات الدائرية بدلا من المربعة.
ومنه تم استنتاج نموذج حسابي لحساب الحمل الأقصى للكمرات الخرسانية المسلحة ذات فتحات متعددة لجعل عملية التصميم أسهل وأسرع.

Abstract

Pipes and ducts associated with the mechanical, electrical, and plumping systems in a building are usually located underneath the floor beams, resulting in a considerable loss in the usable floor height. Passage of these pipes and ducts through web openings in floor beams gives an effective way to utilize the entire floor height, providing a more compact and economic design.

Steel beams with multiple web openings are commonly used for this reason. In this study, the effect of changing the following parameters on the ultimate load capacity and behavior of reinforced concrete beams is being studied analytically:

- Size of opening.
- Shape of opening (Circular, Square).
- Post width.
- Shear RFT value.

A computation analytical finite element model FEM has been done using ANSYS14, the results of the FEM are illustrated in tables, figures explaining the effect of each parameter on the behavior of the beam, and it was concluded that the ultimate load capacity of beams with circular openings are larger than beams with square openings, and the smaller the opening size, the larger the load capacity. It was also found that the increase in the post width or shear RFT value results in larger load capacity as well.

Furthermore, the analyzed data was used to formulate a mathematical model to predict the ultimate load capacity of the beam with multiple transversal openings to make the design process easier and faster.

Keywords: RC Beams, Opening, Finite Element Model FEM, Simplified method, Analytical.

1. Introduction

Nevertheless, the presence of opening(s) in a reinforced concrete (RC) beam reduces its load-carrying capacity and increases its service-load deflections. The studies on reinforced concrete beams with transverse openings focused on providing these beams with strengths and rigidities comparable to solid beams by proper reinforcement detailing. In this way, the negative effects of the stress concentrations around the openings could be decreased, the load-carrying capacities increased, and the deflections decreased.

In a comprehensive experimental study on continuous reinforced concrete beams with a large rectangular opening, Mansur et al. (1991) [1] concluded that the failure of these beams is generally related to Vierendeel-truss action. The deformations in a beam with an opening were shown to increase and the collapse load to decrease as the opening is moved to a more highly stressed part of span. As the opening length and depth increase, Mansur et al. (1991) [1] found that the Vierendeel action becomes more noticeable, and the decrease in the collapse load increases.

Mansur et al. (1992) [2] suggested that the deflections of an RC beam with a large rectangular opening can be approximately estimated by reducing flexural and shear rigidities to the parts containing the opening. Tan and Mansur (1996) [3] proposed design guidelines for the design of reinforced concrete beams with large openings.

Mansur (1998) [4] found different shear failure modes of reinforced concrete beams with web openings and formulated design equations. The tests carried out by Tan et al. (2001) [5] on reinforced concrete beams with circular openings indicated that the use of diagonal reinforcement enhances crack control.

Mansur and tan (1999) [6] formulated design equations for reinforced concrete beams subject to torsion in addition to bending and shear. The equations correspond to the beam failure as a whole, termed as beam-type, and failure of each chord (top and bottom) separately, termed as frame-type.

Mansur et al. (2006) [7] found that the flexural capacities of reinforced concrete beams with large circular openings can be predicted using strut and tie models.

Yang et al. (2006) [8] investigated the behavior and strength of reinforced concrete deep beams with openings, and showed that the failure of a deep reinforced concrete beam is caused by the diagonal cracks projecting from the corners of the opening.

Dundar 2008 [9]; Egriboz 2008 [10]; and Aykac and Yilmaz 2011 [11] studied the influence of multiple openings in the span and assumed to provide more efficient design by helping the stress concentrations around openings to be distributed to the entire beam span. Furthermore, the presence of openings in the central zone in addition to shear spans of the beam was assumed to shift the failure mode of the beam from brittle shear failure to ductile flexural failure. Trials were made to prevent shear failure (beam-type and frame-type), and the ductility of the beams was increased by proper detailing: short stirrups in the chords, and posts and full-depth stirrups next to openings. Furthermore, reinforced concrete beams with different opening geometries were tested within the scope of the program to establish the geometry which reduces the effect on the strength and ductility of reinforced concrete beam.

Different methods and different software packages can be used for beam analysis. The finite element method is a numerical method of structural analysis in which the solution of the problems of various boundary conditions and loadings is achieved by the analysis of an assemblage of finite elements which are interconnected at a finite number of nodal points and represent the solution domain of the problem.

Concrete is a brittle material and has different behavior in compression and tension. To model the concrete, ANSYS requires linear isotropic and multilinear isotropic as well as some additional concrete material properties to simulate the concrete behavior.
Steel reinforcement in the models was constructed with typical grade 52 steel reinforcing bars. The finite element models for the steel were assumed to be identical in tension and compression, an elastic-perfectly plastic material.

2. Parametric Study

The present work studies the effect of the presence of multiple openings in the span of the reinforced concrete beams. For this purpose, total of 39 simply supported reinforced concrete beams with rectangular cross section, 150 mm wide, 400 mm deep, and with a span 4000 mm subjected to 4 equally distributed concentrated loads as shown in Figure 1: Test setup and procedure.The description of these analyzed beams is presented and analyzed with the parameters which are representing the opening size, opening shape, post width, and shear RFT.

For the analyzed beams, the main reinforcement area of the beams A_s is 5T12 at the bottom and the compression reinforcement area $A_{s'}$ is 5T12 at the top of the beams. Stirrups are doubled at the external 800 mm of the span, additional chord reinforcement around opening (top and bottom) are used and equals to 5T12 at the external 800 mm of the span, and equals 2T12 at the rest of the span.

Control beams group contains three solid beams without any openings as shown in Figure 2 and the reinforcement detailing of a typical beam without openings is shown in Figure 3: Solid beam RFT details.

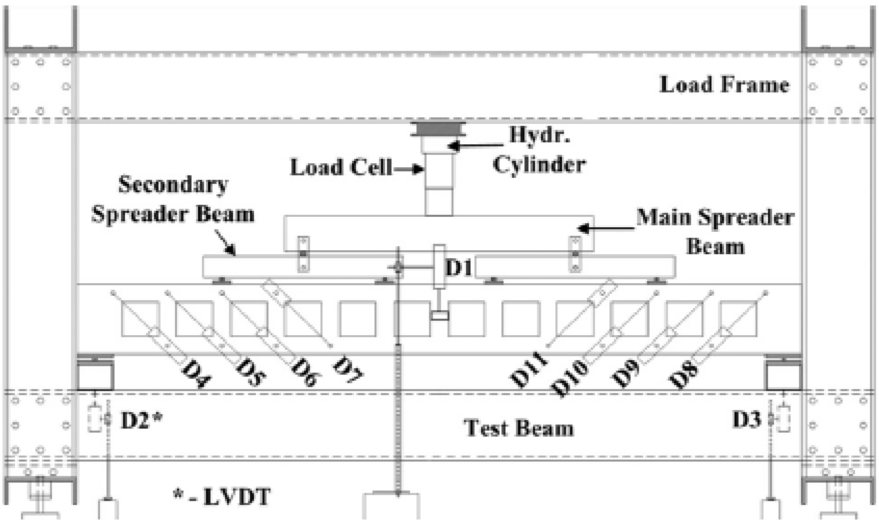

Figure 1: Test setup and procedure

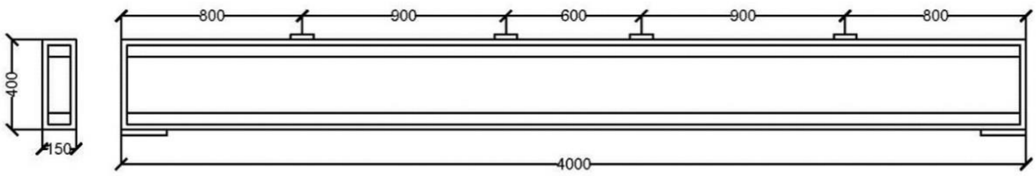

Figure 2: Control beams dimensions

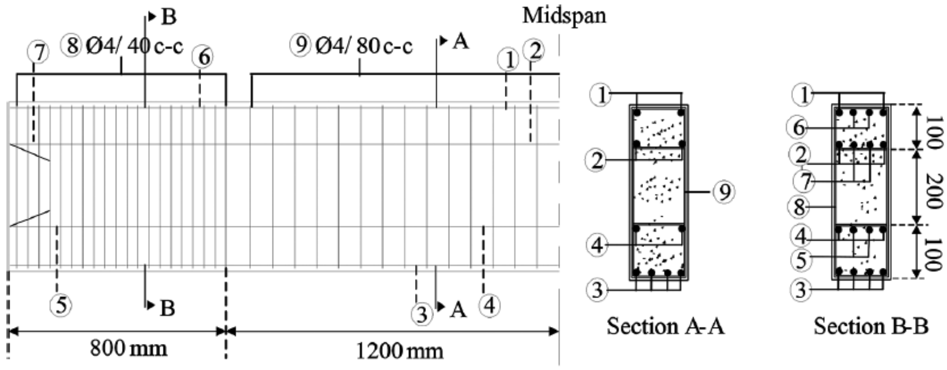

Figure 3: Solid beam RFT details

The concrete for the analyzed beams was assumed to have a characteristic strength; $f_{c'}$ = 30 MPa. Besides, the longitudinal reinforcement and stirrups were assumed to have elastic perfectly plastic materials with yield stresses equal to 420 MPa and 280 MPa, respectively.

The naming of the beams was chosen matching the variables as follows:

1. Size of opening (200 mm,150 mm, 100 mm) denoted by (1, 2, 3)
2. Width of post (100 mm, 200 mm) denoted by (V, W)
3. Shape of opening (Square, Circular) denoted by (R, C)
4. Shear RFT (ϕ4@80, ϕ6@80, ϕ8@80) denoted by (A, B, C)

Tables 1 and 2 show the analyzed beam models properties

Table 1: analyzed beam models properties

Group #	Beam #	Shear RFT	Opening Shape	# of Openings	Opening Size	Post Size
Control Beams	**NA**	□4 / 80	N/A			
	NB	□6 / 80				
	NC	□8 / 80				
A	**NRVA1**	□4 / 80	Square	12	200 mm	100 mm
	NRVB1	□6 / 80				
	NRVC1	□8 / 80				
B	**NRVA2**	□4 / 80	Square	14	150 mm	100 mm
	NRVB2	□6 / 80				
	NRVC2	□8 / 80				
C	**NRVA3**	□4 / 80	Square	18	100 mm	100 mm
	NRVB3	□6 / 80				
	NRVC3	□8 / 80				
D	**NRWA1**	□4 / 80	Square	9	200 mm	200 mm
	NRWB1	□6 / 80				
	NRWC1	□8 / 80				
E	**NRWA2**	□4 / 80	Square	10	150 mm	200 mm
	NRWB2	□6 / 80				
	NRWC2	□8 / 80				
F	**NRWA3**	□4 / 80	Square	12	100 mm	200 mm
	NRWB3	□6 / 80				
	NRWC3	□8 / 80				

Table 2: analyzed beams models properties

Group #	Beam #	Shear RFT	Opening Shape	# of Openings	Opening Size	Post Size
Control Beams	NA	□4 / 80	N/A			
	NB	□6 / 80				
	NC	□8 / 80				
G	NCVA1	□4 / 80	Circular	12	200 mm	100 mm
	NCVB1	□6 / 80				
	NCVC1	□8 / 80				
H	NCVA2	□4 / 80	Circular	14	150 mm	100 mm
	NCVB2	□6 / 80				
	NCVC2	□8 / 80				
I	NCVA3	□4 / 80	Circular	18	100 mm	100 mm
	NCVB3	□6 / 80				
	NCVC3	□8 / 80				
J	NCWA1	□4 / 80	Circular	9	200 mm	200 mm
	NCWB1	□6 / 80				
	NCWC1	□8 / 80				
K	NCWA2	□4 / 80	Circular	10	150 mm	200 mm
	NCWB2	□6 / 80				
	NCWC2	□8 / 80				
L	NCWA3	□4 / 80	Circular	12	100 mm	200 mm
	NCWB3	□6 / 80				
	NCWC3	□8 / 80				

These finite element models were divided into 12 groups containing 3 beam models each having different shear RFT ratios and 3 control beams. Groups (A, B, C) and groups (G, H, I) are beams with 100mm post width, while Groups (D, E, F) and groups (J, K, L) are beams with 200mm post width. Groups (A, B, C, D, E, F) are beams with square openings, while Groups (G, H, I, J, K, L) are beams with circular openings. Groups (A, D, G, J) are beams with 200mm opening size, While Groups (B, E, H, K) are beams with 150mm opening size, while Groups (C, F, I, L) are beams with 100mm opening size.

The following table shows the ultimate load capacity and ductility of analyzed FEM:

Table 3: Summary of ultimate load capacity and ductility of analyzed FEM

Group #	Beam #	$P_{ultimate}$ FEM (kN)	DuctilityFEM (kN.mm)	Group #	Beam #	$P_{ultimate}$ FEM (kN)	DuctilityFEM (kN.mm)
Control Beams	NA	238.53	4437.48	Control Beams	NA	238.53	4437.48
	NB	255.73	4548.50		NB	255.73	4548.50
	NC	293.67	5649.40		NC	293.67	5649.40
A	NRVA1	128.64	2804.64	G	NCVA1	184.55	3178.29
	NRVB1	155.72	3503.87		NCVB1	206.94	4304.01
	NRVC1	197.87	5265.79		NCVC1	231.02	5716.80
B	NRVA2	132.47	1212.96	H	NCVA2	189.43	3076.06
	NRVB2	169.68	2161.86		NCVB2	211.61	3657.32
	NRVC2	206.69	3105.22		NCVC2	249.00	5337.96
C	NRVA3	159.62	2932.26	I	NCVA3	193.07	4628.19
	NRVB3	179.99	4030.18		NCVB3	221.49	5212.54
	NRVC3	212.24	5096.40		NCVC3	267.06	6577.51
D	NRWA1	147.68	3487.67	J	NCWA1	210.74	2628.85
	NRWB1	186.85	4097.64		NCWB1	233.25	6142.36
	NRWC1	223.87	4770.61		NCWC1	255.57	8420.21
E	NRWA2	181.67	2600.99	K	NCWA2	218.65	5374.47
	NRWB2	198.55	3642.29		NCWB2	243.00	6072.24
	NRWC2	236.54	4437.09		NCWC2	270.00	6749.99
F	NRWA3	185.22	2363.82	L	NCWA3	225.68	3404.87
	NRWB3	208.56	4783.13		NCWB3	251.65	4245.69
	NRWC3	238.47	5160.32		NCWC3	282.35	5770.74

The following figures show comparison between load deflection relationships for different beam models in order to study graphically the effect of changing each parameter. Figure 4 shows load deflection curves for control beams.

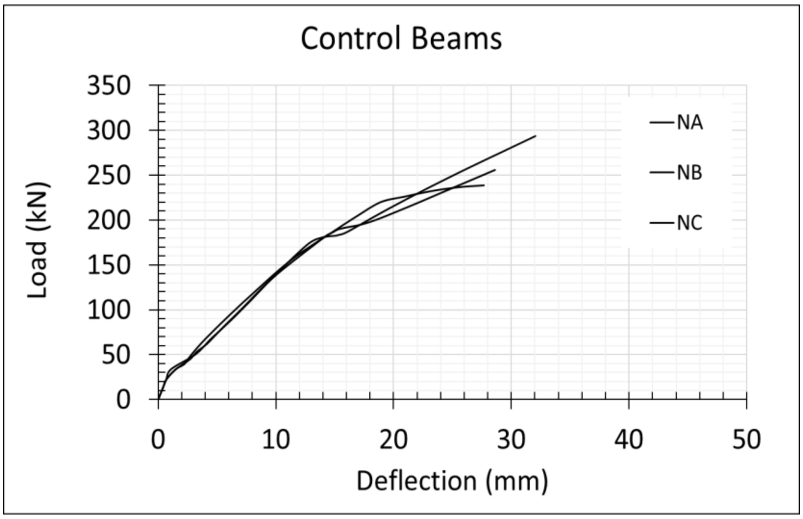

Figure 4: Load deflection curves for control beams

2.1 Effect of opening size and number of opening

To study the effect of opening size and number of openings on the behavior of reinforced concrete beams, thirty-six specimens was divided into four groups with the same shear reinforcement, post width and opening shape but different opening size (200, 150 and 100 mm) with different number of opening (9, 10, 12, 14, and 18) as shown in Tables 1 and 2. Figure 5 shows the load – deflection for different beam models group 1.

Figure 5 (1-1) shows a comparison between beam models having 100 mm post size, and square opening shape. Relative to NRVA1, increasing the no. of opening and decreasing opening size causes an increase in the load capacity and a decrease in ductility for specimen NRVA2 however for specimen NRVA3 both load capacity and ductility were increased by increasing the no. of openings and decreasing its size. Relative to NRVB1, increasing no. of opening and decreasing opening size causes an increase in the load capacity and a decrease in ductility for specimen NRVB2 but an increase in both load capacity and ductility for specimen NRVB3 was obtained. Relative to NRVC1, increasing no. of opening and decreasing opening size causes an increase in the load capacity and a decrease in ductility for specimens NRVC2 and NRVC3.

In figure 5 (1-2), a comparison between beam models having 200 mm post size, and square opening shape. Relative to NRWA1, increasing the no. of opening and decreasing its size causes an increase in the load capacity and a decrease in ductility for specimens NRWA2 and NRWA3. Relative to NRWB1, increasing no. of opening and decreasing opening size causes an increase in the capacity and decrease ductility for specimen NRWB2 and causes an increase in the capacity and ductility for specimen NRWB3. Relative to NRWC1, increasing no. of opening and decreasing opening size causes an increase in the load capacity and a decrease in ductility for specimen NRWC2 and increase in the load capacity and ductility for specimen NRWC3.

In figure 5 (1-3), a comparison between beam models having 100 mm post size, and circular opening shape, relative to NCVA1, increasing no. of opening and decreasing opening size causes an increase in the capacity and decrease ductility for specimen

NCVA2 and an increase in the capacity and ductility for specimen NCVA3.Relative to NCVB1, increasing no. of opening and decreasing opening size causes an increase in the capacity and decrease ductility for specimen NCVB2 and increase capacity and ductility for specimen NCVB3. Relative to NCVC1, increasing no. of opening and decreasing opening size causes an increase in the capacity and decrease ductility for specimen NCVC2 and an increase in the capacity and ductility for specimen NCVC3.

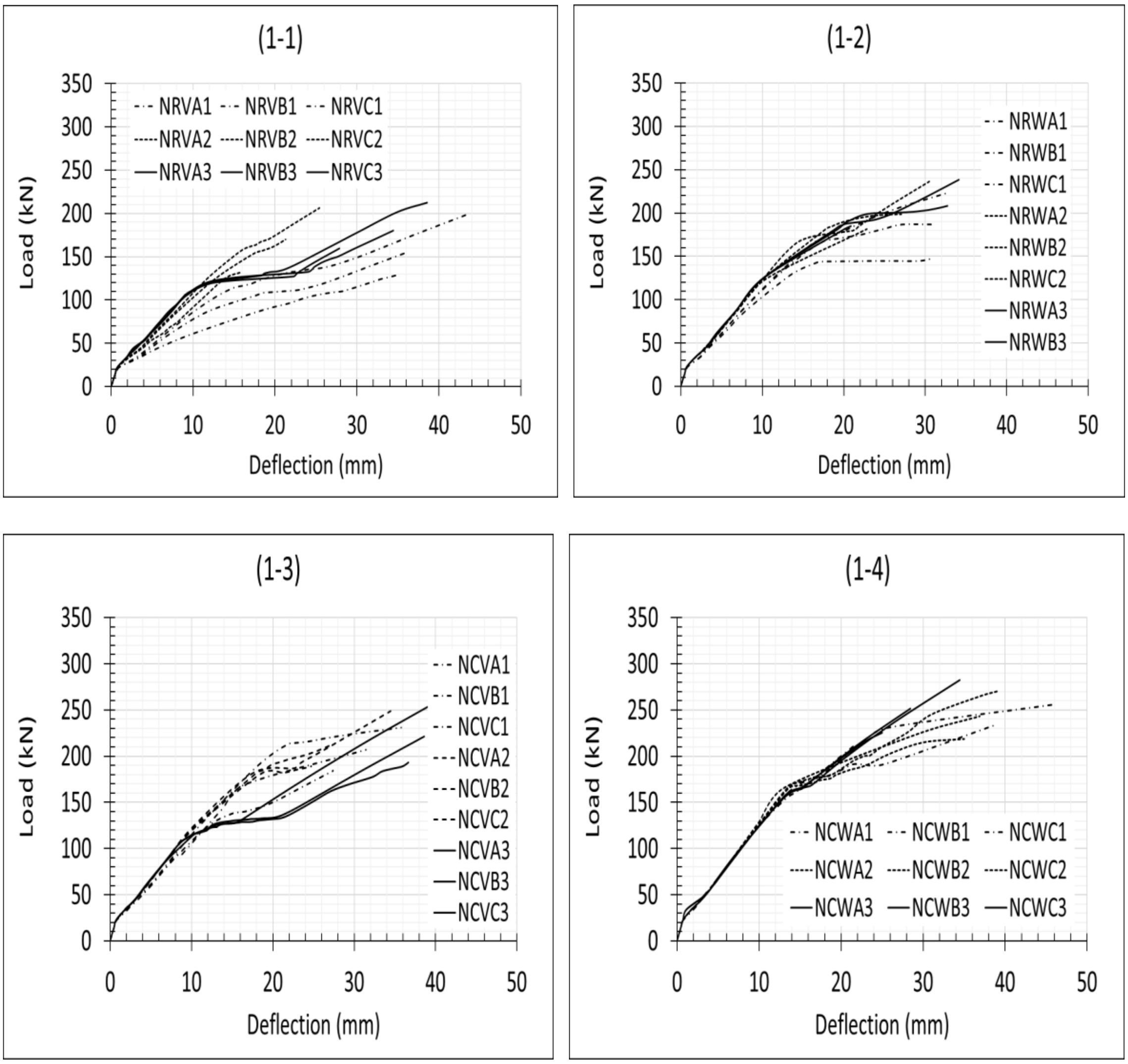

Figure 5: Load deflection curves for beam models group 1

In figure 5 (1-4), a comparison between beam models having 200 mm post size, and circular opening shape. Relative to NCWA1, increasing no. of opening and decreasing opening size causes an increase in the load capacity and ductility for specimens NCWA2 and NCWA3. Relative to NCWB1, increasing no. of opening and decreasing opening size causes an increase in the load capacity and decrease in ductility for specimens NCWB2 and NCWB3. Relative to NCWC1, increasing no. of opening and decreasing opening size causes an increase in the load capacity and decrease in ductility for specimen NCWC2 and NCWC3.

The ductility is calculated in terms of the strain energy, i.e., the area under the load displacement curve.

Nazar Oukaili and Abeer Shammari (2013) [13] studied the response of reinforced concrete T-beams with multiple web openings to static load and they found that increasing the no. of un-strengthened circular openings from four to six with diameter of 0.48 the web depth in the shear zone reduces the strength capacity by 30% and 41% respectively. The beam deflection and the no. of intensive shear cracks around the opening were increased by increasing the number of opening as well. Rezwana Hafiz et al. (2014) [14] concluded that beam of circular opening with diameter ≤ 44% the beam depth behave similar to the beams without opening, however increasing the openings diameter > 44% the beam depth reduces the load capacity by 34.3 %.

2.2 Effect of post size and number of opening

To study the effect of post size and number of opening on the behavior of reinforced concrete beams, thirty-six specimens was divided into six groups with the same shear reinforcement, opening width and opening shape but different post size (100 and 200 mm) and different number of opening (9, 10, 12, 14, and 18) as shown in Tables 1 and 2. Figure 6 shows the load – deflection for different beam models group 2.

Figure 6 (2-1) shows comparison between beam models having 200 mm opening size, and square opening shape; relative to NRVA1, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NRWA1.Relative to NRVB1, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NRWB1. Relative to NRVC1, decreasing no. of opening and increasing post size causes an increase in the load capacity and decrease in ductility for specimen NRWC2.

Figure 6 (2-2) shows comparison between beam models having 150 mm opening size, and square opening shape; relative to NRVA2, decreasing no. of opening and increasing post size causes an increase in the load capacity and ductility for specimen NRWA2. Relative to NRVB2, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NRWB2. Relative to NRVC2, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NRWC2.

Figure 6 (2-3) shows comparison between beam models having 100 mm opening size, square opening shape; relative to NRVA3, decreasing no. of opening and increasing post size causes an increase in the capacity and a decrease in ductility for specimen NRWA3.Relative to NRVB3, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NRWB3. Relative to NRVC3, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NRWC3.

Figure 6 (2-4) shows comparison between beam models having 200mm opening size with circular opening shape; relative to NCVA1, decreasing no. of opening and increasing post size causes an increase in the load capacity and a decrease in ductility for specimen NCWA1. Relative to NCVB1, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NCWB1. Relative to NCVC1, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NCWC1.

Figure 6 (2-5) shows comparison between beam models having 150mm opening size with circular opening shape; relative to NCVA2, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NCWA2. Relative to NCVB2, decreasing no. of opening and increasing post size causes an increase in the load capacity and ductility for specimen NCWB2. Relative to NCVC2, decreasing no. of opening and increasing post size causes an increase in the capacity and ductility for specimen NCWC2.

Figure 6 (2-6) shows comparison between beam models having 100mm opening size, and circular opening shape; relative to NCVA3, decreasing no. of opening and increasing post size causes an increase in the capacity and decrease in ductility for specimen NCWA3.Relative to NCVB3, decreasing no. of opening and increasing post size causes an increase in the capacity and decrease in ductility for specimen NCWB3. Relative to NCVC3, decreasing no. of opening and increasing post size causes an increase in the load capacity and a decrease in ductility for specimen NCWC3.

Mansur and Tan (1999) [6] illustrated the selection of the size and location of web openings for rectangular beams as following: the depth of openings should be limited to 50% of the overall beam depth and when the opening becomes bigger, it is better to use multiple openings with the same passage instead of single one. In such case, the post size should not be less than 0.5 the diameter to ensure that each opening has independent behavior.

2.3 Effect of opening shape

To study the effect of opening shape on the behavior of reinforced concrete beams, thirty-six specimens was divided into six groups with the same shear reinforcement, opening width and post width but different opening shape (square and circular) as shown in Tables 1 and 2. Figure 7 shows the load – deflection for different beam models group 3.

Comparison between beam models having 12 opening, 200mm opening size, and post size 100 mm is shown in Figure 7 (3-1); relative to NRVA1, using circular opening causes an increase in the load capacity and ductility for specimen NCVA1. Relative to NRVB1, using circular opening causes an increase in the capacity and ductility for specimen NCVB1. Relative to NRVC1, using circular opening causes an increase in the load capacity and ductility for specimen NCVC1.

Figure 7 (3-2) shows comparison between beam models having 14 opening, 150 mm opening size, and post size 100 mm; relative to NRVA2, using circular opening causes an increase in the capacity and ductility for specimen NCVA2. Relative to NRVB2, using circular opening causes an increase in the capacity and ductility for specimen NCVB2. Relative to NRVC2, using circular opening causes an increase in the capacity and ductility for specimen NCVC2.

Figure 7 (3-3) shows a comparison between beam models having 18 opening, 100mm opening size, and post size 100 mm; Relative to NRVA3, using circular opening causes an increase in the capacity and ductility for specimen NCVA3. Relative to NRVB3, using circular opening causes an increase in the load capacity and ductility for specimen NCVB3. Relative to NRVC3, using circular opening causes an increase in the load capacity and ductility for specimen NCVC3.

Figure 7 (3-4) shows a comparison between beam models having 9 opening, 200 mm opening size, and post size 200 mm; Relative to NRWA1, using circular opening causes an increase in the load capacity and a decrease in ductility for specimen NCWA1. Relative to NRWB1, using circular opening causes an increase in the load capacity and ductility for specimen NCWB1. Relative to NRWC1, using circular opening causes an increase in the capacity and ductility for specimen NCWC1.

Figure 7 (3-5) shows a comparison between beam models having 10 opening, 150 mm opening size, and post size 200 mm; relative to NRWA2, using circular opening causes an increase in the capacity and ductility for specimen NCWA2.Relative to NRWB2, using circular opening causes an increase in the load capacity and ductility for specimen NCWB2. Relative to NRWC2, using circular opening causes an increase in the load capacity and ductility for specimen NCWC2.

Figure 7 (3-6) shows a comparison between beam models having 12 opening, 100 mm opening size, and post size 100 mm, relative to NRWA3, using circular opening causes an increase in the load capacity and ductility for specimen NCWA3.Relative to NRWB3, using circular opening causes an increase in the load capacity and a decrease in ductility for specimen NCWB3. Relative to NRWC3, using circular opening causes an increase in the load capacity and ductility for specimen NCWC3.

Rezwana Hafiz et al. (2014) [14] studied RC beam with square opening (width = 133 mm) and compared to its equivalent circular opening of diameter 150 mm; they reported that the ultimate load capacity for square opening from ANSYS analysis was 42270 N, while the corresponding value for the circular opening was 46750 N. The difference in the ultimate load capacity between circular and square opening is about 9.58% which may be due to the stress concentration at the corners of the square.

2.4 Effect of shear reinforcement

To study the effect of shear reinforcement on the behavior of reinforced concrete beams, thirty-nine specimens was divided into seven groups with the same number of opening, opening width, post width and opening shape but different in shear reinforcement (ϕ4/80, ϕ6/80 and ϕ8/80) as shown in Tables 1 and 2. Figure 1 and figure 7 show the load – deflection for different beam models.

From figures 1,7 and table 3; It was found that by increasing shear reinforcement causes an increase in load capacity and ductility of specimens.

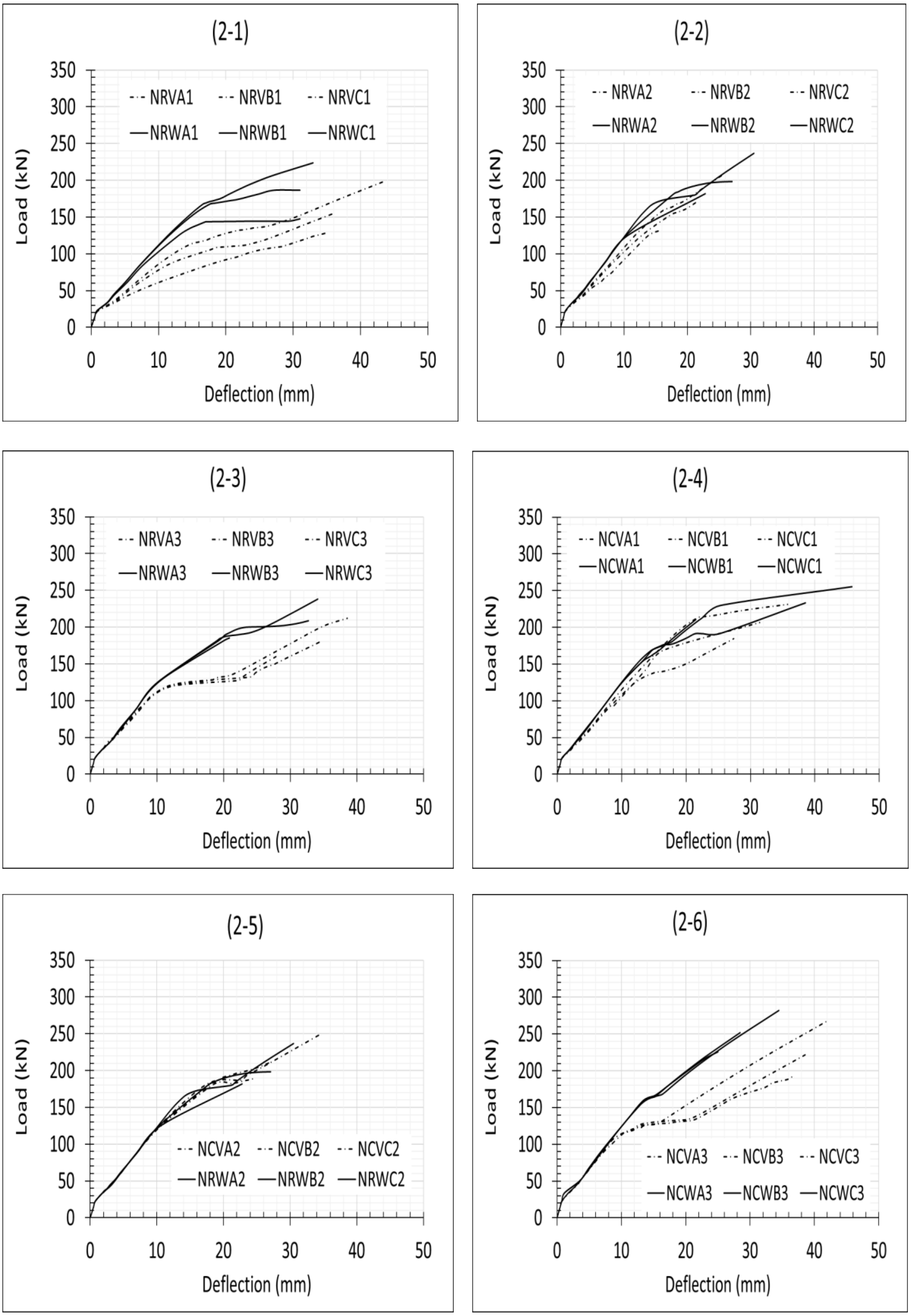

Figure 6: Load deflection curves for beam models group 2

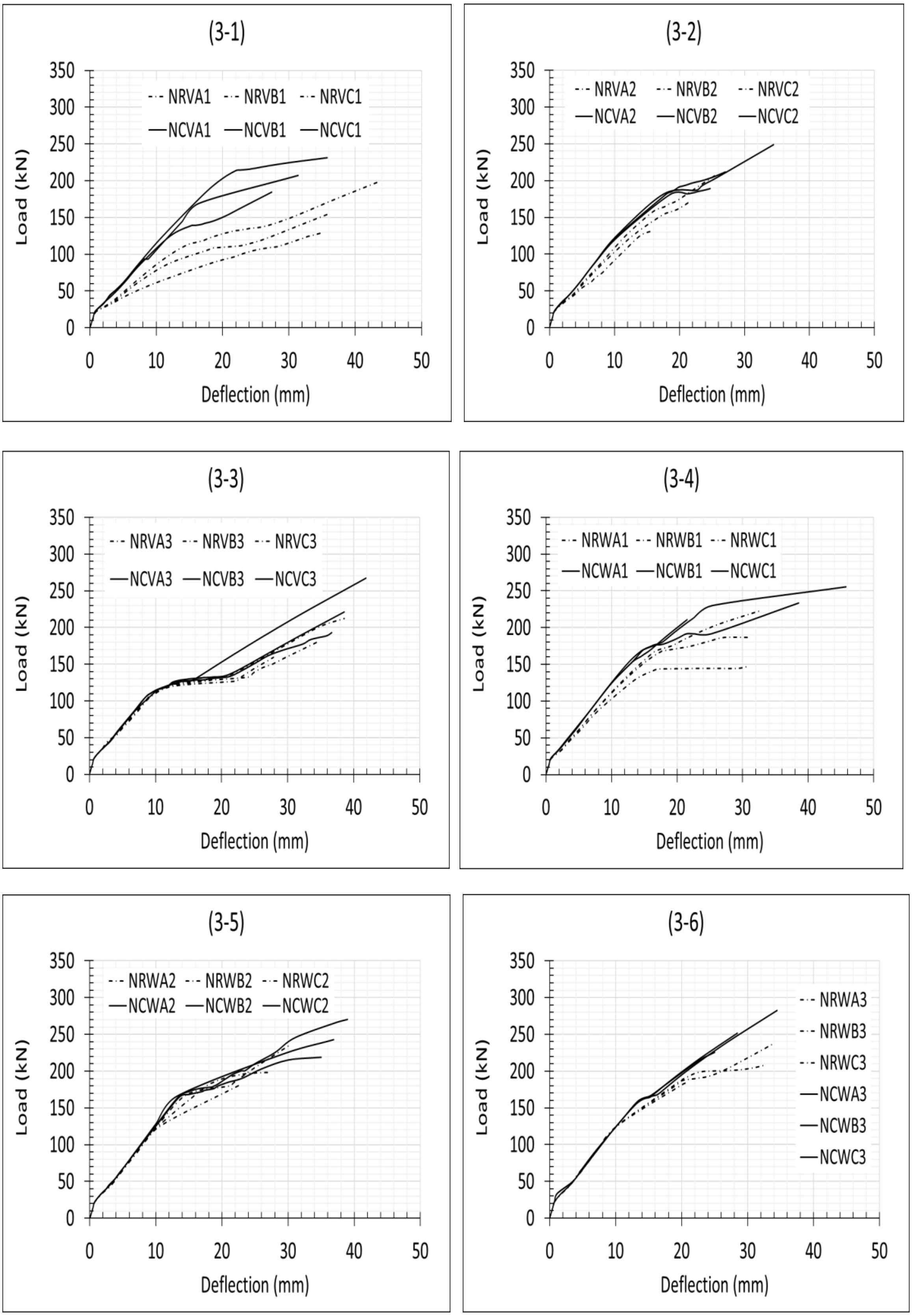

Figure 7: Load deflection curves for beam models group 3

3. Computational model

A computational model is developed using analyzed data to predict the ultimate load capacity of a beam having multiple transversal openings.

It was found that ultimate load capacity of beams having multiple transversal openings is controlled by shear capacity, not flexure capacity due to the reduction of concrete depth at opening location. Therefore, the following factors affect the ultimate shear capacity of the beam:

- Opening size.
- Shear RFT.
- Post width.
- Opening shape.

The effect of each factor will be studied separately and an equation for each reduction factor will be formulated. And then all these factors will be multiplied by the ultimate shear capacity of Solid beam as per ACI 318-11. [18].

3.1 Effect of opening size (R_{Size}):

From the analysis of the Finite element models formed, it was found that with increasing the opening size, the ultimate load capacity decreases.

Therefore, charts have been created between the ratio of opening depth to total depth on X-axis, and percentage of ultimate load capacity on Y-axis where solid beam was taken as reference for ultimate load capacity.

For each chart, linear trend line has been formed between these points and its equation has been calculated.

Average has been calculated for all these trend lines to get the reduction factor $\mathbf{R_{size}}$

$$\boldsymbol{R}_{Size} = \mathbf{1} - \mathbf{0.22} \times \frac{\boldsymbol{d_o}}{\boldsymbol{t}} \qquad \textit{(Equation 1)}$$

Where:

d_o= Depth of opening.

t = Total depth of beam.

3.2 Effect of shear RFT ($R\mu v$):

From the analysis of the Finite element models formed, it was found that with increasing the shear RFT (Stirrups), the ultimate load capacity increases.

Therefore, charts have been created between the ratio of shear rft ratio to 6.7 * min shear rft ratio on X-axis, and percentage of ultimate load capacity on Y-axis.

For each chart, linear trend line has been formed between these points and its equation has been calculated.

Average has been calculated for all these trend lines to get the reduction factor $\mathbf{R\mu_v}$

$$\boldsymbol{R\mu}_V = \mathbf{0.67} + \mathbf{0.33} \times \frac{\boldsymbol{\mu}_V}{\mathbf{6.7} \times \boldsymbol{\mu}_{v\,min}} \leq \mathbf{1.0} \qquad \textit{(Equation 2)}$$

Where:

$$\mu_v = \frac{n \times Av}{b \times s}$$

$$\mu_{v\ min} = \text{greater of} \begin{cases} \dfrac{0.062 \times \sqrt{fc'}}{fyt} \\ \dfrac{0.35}{fyt} \end{cases}$$

n = No. of stirrup branches.
Av = Stirrups cross sectional area, mm^2.
b = Beam width, mm.
s = Stirrups spacing, mm.
f_c' = Specified compressive strength of concrete, N/mm^2.
f_{yt}= Specified yield strength of transverse reinforcement, N/mm^2.

3.3 Effect of post width (Rbp):

From the analysis of the Finite element models formed, it was found that with increasing the post width, the ultimate load capacity increases.

Therefore, charts have been created between (post width/200) on X-axis, and percentage of ultimate load capacity on Y-axis.

For each chart, linear trend line has been formed between these points and its equation has been calculated.

Average has been calculated for all these trend lines to get the reduction factor $\mathbf{R_{bp}}$.

$$\boldsymbol{R}_{bp} = \mathbf{0.74 + 0.26} \times \frac{\boldsymbol{b}_p}{\mathbf{200}} \quad \leq \mathbf{1.0} \qquad \text{(Equation 3)}$$

Where:

b_p= width of post between openings, mm.

3.4 Effect of opening shape (R shape):

From the analysis of the Finite element models formed, it was found that beams with circular openings have higher ultimate load capacity than beams with square openings due to the lack of stresses concentration at square opening edges.

Therefore, percentage of ultimate load capacity has been calculated for each two beams.

Average has been calculated for all these beams to get the reduction factor $\mathbf{R_{shape}}$

$$\boldsymbol{R}_{shape} = \begin{cases} \mathbf{1}, & \boldsymbol{for\ circular\ openings} \\ \mathbf{0.8}, & \boldsymbol{for\ square\ openings} \end{cases} \qquad \text{(Equation 4)}$$

3.5 Verification

Calculated results Vs. Analyzed results from FEM are being compared in table 4 and table 5:

Table 4: Verification of mathematical model results

Group #	Beam #	$P_{ultimate}^{FEM}$ (kN)	R_{Size}	$R_{\mu V}$	R_{Shape}	R_{bp}	$P_{ultimate}^{Calculated}$ (kN)	Calc. Error
Control Beams	NA	238.53	1.0	0.753	1.0	1.0	221.14	17%
	NB	255.73	1.0	0.856	1.0	1.0	251.45	23%
	NC	293.67	1.0	1.001	1.0	1.0	293.89	33%
A	NRVA1	128.64	0.891	0.753	0.814	0.870	139.55	3%
	NRVB1	155.72	0.891	0.856	0.814	0.870	158.68	20%
	NRVC1	197.87	0.891	1.001	0.814	0.870	185.46	37%
B	NRVA2	132.47	0.918	0.753	0.814	0.870	143.84	3%
	NRVB2	169.68	0.918	0.856	0.814	0.870	163.55	25%
	NRVC2	206.69	0.918	1.001	0.814	0.870	191.15	38%
C	NRVA3	159.62	0.945	0.753	0.814	0.870	148.12	17%
	NRVB3	179.99	0.945	0.856	0.814	0.870	168.42	27%
	NRVC3	212.24	0.945	1.001	0.814	0.870	196.85	38%
D	NRWA1	147.68	0.891	0.753	0.814	1.000	160.32	3%
	NRWB1	186.85	0.891	0.856	0.814	1.000	182.29	24%
	NRWC1	223.87	0.891	1.001	0.814	1.000	213.05	36%
E	NRWA2	181.67	0.918	0.753	0.814	1.000	165.24	19%
	NRWB2	198.55	0.918	0.856	0.814	1.000	187.89	26%
	NRWC2	236.54	0.918	1.001	0.814	1.000	219.60	38%
F	NRWA3	185.22	0.945	0.753	0.814	1.000	170.16	18%
	NRWB3	208.56	0.945	0.856	0.814	1.000	193.49	27%
	NRWC3	238.47	0.945	1.001	0.814	1.000	226.14	36%

Table 5: Verification of mathematical model results

Group #	Beam #	$P_{ultimate}^{FEM}$ (kN)	R_{Size}	$R_{\mu V}$	R_{Shape}	R_{bp}	$P_{ultimate}^{Calculated}$ (kN)	Calc. Error
Control Beams	NA	238.53	1.0	0.753	1.0	1.0	221.14	17%
	NB	255.73	1.0	0.856	1.0	1.0	251.45	23%
	NC	293.67	1.0	1.001	1.0	1.0	293.89	33%
G	NCVA1	184.55	0.891	0.753	1.000	0.870	171.44	17%
	NCVB1	206.94	0.891	0.856	1.000	0.870	194.94	26%
	NCVC1	231.02	0.891	1.001	1.000	0.870	227.83	34%
H	NCVA2	189.43	0.918	0.753	1.000	0.870	176.70	17%
	NCVB2	211.61	0.918	0.856	1.000	0.870	200.92	26%
	NCVC2	249.00	0.918	1.001	1.000	0.870	234.83	37%
I	NCVA3	193.07	0.945	0.753	1.000	0.870	181.97	16%
	NCVB3	221.49	0.945	0.856	1.000	0.870	206.91	27%
	NCVC3	267.06	0.945	1.001	1.000	0.870	241.83	39%
J	NCWA1	210.74	0.891	0.753	1.000	1.000	196.95	17%
	NCWB1	233.25	0.891	0.856	1.000	1.000	223.94	25%
	NCWC1	255.57	0.891	1.001	1.000	1.000	261.74	31%
K	NCWA2	218.65	0.918	0.753	1.000	1.000	203.00	17%
	NCWB2	243.00	0.918	0.856	1.000	1.000	230.82	26%
	NCWC2	270.00	0.918	1.001	1.000	1.000	269.77	33%
L	NCWA3	225.68	0.945	0.753	1.000	1.000	209.05	18%
	NCWB3	251.65	0.945	0.856	1.000	1.000	237.70	26%
	NCWC3	282.35	0.945	1.001	1.000	1.000	277.81	34%

3.6 Summary

The following equation was formulated to calculate the ultimate load capacity of reinforced concrete beam with multiple transversal openings using ultimate shear strength of solid beam as per ACI 318-11. [18]

$$Vu\ opening = Vu \times R_{Size} \times R\mu_v \times R_{bp} \times R_{shape} \quad \textbf{(Equation 5)}$$

$$Vu = \emptyset\, Vn \quad \text{ACI 318-11 Eq.(11-1)[12]}$$

$$R_{Size} = 1 - 0.22 \times \frac{d_o}{t}$$

$$R\mu_v = 0.67 + 0.33 \times \frac{\mu_v}{6.7 \times \mu_{v\,min}}$$

$$R_{bp} = 0.74 + 0.26 \times \frac{b_p}{200}$$

$$\boldsymbol{R}_{\text{shape}} = \begin{cases} \mathbf{1}, & \boldsymbol{for\ circular\ openings} \\ \mathbf{0.8}, & \boldsymbol{for\ square\ openings} \end{cases}$$

Where:

d_o = Depth of opening.

t = Total depth of beam.

$\mu_v = \frac{n \times Av}{b \times s}$

$\mu_{v\ min}$ =greater of $\begin{cases} \frac{0.062 \times \sqrt{fc'}}{fyt} \\ \frac{0.35}{fyt} \end{cases}$ ACI 318-11 Eq.(11-13)[12]

n = No. of stirrup branches.

Av = Stirrups cross sectional area, mm^2.

b = Beam width, mm.

s = Stirrups spacing, mm.

f_c' = Specified compressive strength of concrete, N/mm^2.

f_{yt} = Specified yield strength of transverse reinforcement, N/mm^2.

b_p = width of post between openings, mm.

4. CONCLUSIONS

Based on the evaluation and analysis of the results obtained from finite element modeling for simply supported reinforced concrete beams with multiple transversal openings, the following can be concluded:

- Beams with multiple transversal openings tend to shift the failure mode of the beam from flexural failure to shear failure of Vierendeel action.
- By increasing the opening size, the ultimate load capacity decreases due to the decrease of concrete depth resisting the shear force.
- By increasing the shear reinforcement (Stirrups), the ultimate load capacity increases as the failure mode of the beam is shear failure.
- By increasing the post width, the ultimate load capacity increases because of the increase in shear capacity.
- Reinforced concrete beams with circular openings have higher load capacities compared with square openings beams. The analysis indicated that the stress concentrations at corners of square openings result in cracking, which leads to the reductions in the flexural rigidities.
- A mathematical model was formulated to calculate the ultimate capacity of reinforced concrete beam with multiple transversal openings using ultimate shear strength of solid beam to simplify the design process taking the effect of studied parameters which are opening size, opening shape, post width, shear RFT.

$$Vu\ opening = Vu \times R_{Size} \times R\mu_v \times R_{bp} \times R_{shape}$$

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