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NUMERICAL INVESTIGATION ON THE BEHAVIOR OF REINFORCED CONCRETE WIDE BEAMS- COLUMN JOINTS UNDER CYCLIC LOAD

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ملخص البحث :

يهدف هذا البحث الي دراسة سلوك وصلات الكمرات العريضة والاعمدة الخرسانية المسلحة والمعرضه لأحمال دوريه باستخدام برنامج التحليل العددى (ELS) مع مقارنة بين النتائج العملية لثلاث عينات ونتائج برنامج التحليل العددى وأظهرت النتائج ان التحليل باستخدام برنامج (ELS) كان فعالا. تم دراسة نتائج التحليل العددى تحت تاثير العديد من المتغيرات هي عرض الكمرة، توزيع الكانات بالكمرة، الحمل الرأسى على العمود، مقدار الازاحة بالدورة، ورتبة الخرسانة. وقد توصلت الدراسة الي أن وصلات الكمرات العريضة والاعمدة الخرسانية المسلحة التى صممت طبقا لاشتراطات الزلازل بالكود المصري توفر أداء زلزالي جيد حتى عندما كانت نسبة عرض الكمرة لعرض العمود أكبر من اثنين، وعندما كان ثلثي الحديد الرئيسى خارج قطاع العمود. وعلي ذلك يجب تغيير الحدود المسموح بها لعرض الكمرة في الكود المصرى. كما وجد أن زيادة الحمل المحوري لا يؤثر على قدرة الوصلة، حيث ان الكمرات الضعيفة نسبيا والعمود يلغي تأثير الضغط المحوري.

1. A b s t r a c t

This study aims to evaluate numerically the hysteretic behavior of reinforced concrete wide beam –column joints when subjected to lateral cyclic loading. In the current investigation, the Applied Element Method was adopted. The extreme loading for structure ELS software is used for the analysis. Three experiments for wide beam column – joints specimens were analyzed. The results showed that the ELS investigation was effective and successfully predicted the local behavior of the wide beam- column joints. A numerical parametric study was performed to investigate the behavior of reinforced concrete wide beam –column joints due to the variation of beam width, stirrups configuration, column axial load, loading history and compressive strength. Based on this study, wide beam – column joints designed according to seismic provision of the Egyptian code provide ductile seismic performance even when beam width to column width is greater than two and when two – thirds of the wide beam flexure reinforcement is anchored outside column core. Therefore, beam width limit in the Egyptian code can be changed. Increasing axial load does not affect the capacity of the joint, a weak beam-strong column fact eliminates the effect of the axial compression.

Keywords: wide beam- column joints, cyclic response, earthquake resistant structure, seismic design, seismic behavior, Applied Element Method.

2. Introduction

Wide beams column joints is an important part of reinforced concrete building structural systems in Egypt. In this system, a one-way ribbed slab frames into wide and shallow beams having a depth equal to that of the slab. For most buildings, these joints are not only designed and detailed with no account for earthquake loads, but because of the architectural need for space and construction limitations, the dimensions of these joints often violate follow design codes.

Design codes [1, 2] place restrictions on the use of wide-beam framing systems in seismic regions because of the little or insufficient information about their behavior under the effects of earthquake loads. Several drawbacks exist in the wide beams, an insufficient transfer of the bending moment from the wide beam to the column, a low lateral stiffness and a low energy dissipation. The maximum effective wide beam width according to ECP [1] is the smaller of $b_c + h_b$ and $2b_c$, where b_c is the column width and h_b is the beam depth, which depends principally on the column dimension than it is to the depth of the wide beam. On the other hand the maximum effective beam width allowed by ACI [2] is the smaller of bc + $1.5h_c$ and $3b_c$, where h_c is the column depth, which is more related to the wide beam depth than it is to the column dimension.

Elsouri and Harajli [3] found that when subjecting as-built joints that were detailed without taking earthquake loads into account to cyclic load, wide beam -column joints develop sizable diagonal shear cracks and, then, joint shear failure at a small drift ratios. The specimens didn’t reach the estimated lateral load capacities. Elsouri et al. [4, 5, and 6] studied the seismic performance of the joints by improving the reinforcement details. All specimens performed well in spite of beam to column width ratio was higher than three and when more than two thirds of the wide beam main reinforcement was outside the column core. While other studies, Abdel –Rahman, et al [7, 8], found that the effective beam width is less than the beam width when the beam to column width ratio is higher than three.

Experimental results of Masi, et al [9] showed that the axial load value significantly affect the damage development and the deformation capacity of joints, while other studies, Li et al [10] showed that a weak beam-strong column eliminated the effect of the axial loading.

Studies by Takemura, et al [11] showed that if similar specimens are loaded with different loading protocols, their ductility will change depending on the number of cycles used in the loading protocol, the amplitude of each cycle, and the sequence of the loading cycles.

Li and Kulkarni [8] found that concrete grades did not improve the performance of the specimens, except a minor influence in the joint performance.

In this study a numerical analysis were carried out to investigate the behavior of wide beam –column joints.

3. Applied Element Method

Different methods and different software packages can be used for beam column connection analysis. The Applied Element Method (AEM) was proven to track the structural collapse behavior passing through all stages of application of loads: elastic stage, crack initiation and propagation in tension-weak materials, reinforcement yielding, element separation, and element collision (contact) [12, 13]. AEM is based on discrete crack approach and it is capable of predicting the discrete behavior of the structure to a high degree of accuracy.

The AEM is a numerical method based on stiffness according to the concept of discrete cracking, it deals with the structures as assembly of small elements and this were made by dividing the structure virtually, as shown in Figure 1. The elements are connected with spring group to transfer normal and shear stresses among adjacent elements. Each spring works as stresses and deformations for a certain volume of material. When the springs connected two adjacent elements are fractured, these elements will be completely separated.

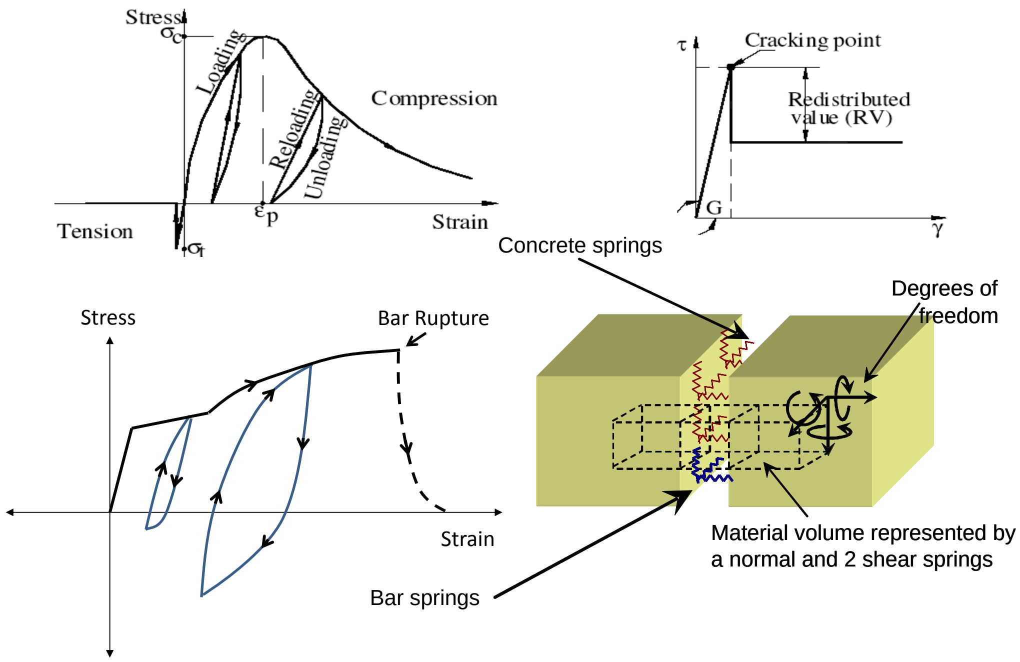


Figure 1: Modeling of a structure with the AEM

AEM uses nonlinear path-dependent constitutive models. The elasto-plastic and fracture model of Maekawa and Okamura [14] is adopted, while the linear stress – strain relationship is adopted till cracking for concrete in tension where the stresses are released to zero. The reinforcing bars are modeled as bare bars for the envelope Okamura and Maekawa, [15] because it is a discrete crack approach, while the model of Ristic et al. [16] is used for the interior cyclic loops.

In this method, the stiffness matrix is compared and the equilibrium equations for stiffness, mass and damping matrices are solved non-linearly for the structural deformations. The equilibrium equations solution is an implicit one which adopts a dynamic step-by-step Newmark-beta time integration procedure Bathe, [17] and Chopra, [18].

As mentioned before, two adjacent elements can separate if the matrix springs between them failed therefore, Elements can separate, re-contact or contact automatically with each other. When two elements contact each other, contact springs are created at the contact points as shown in Figure 2. AEM was validated for structure deformations subjected to extreme loading up to its total failure. As the aim of the present study is to investigate the behavior of reinforced concrete wide beam –column joints when subjected to lateral earthquake loading, AEM was selected as the most suitable numerical tool for such investigation. The Extreme Loading of Structures software (ELS) www.appliedscienceint.com [19], which is AEM-based is used for the investigation.

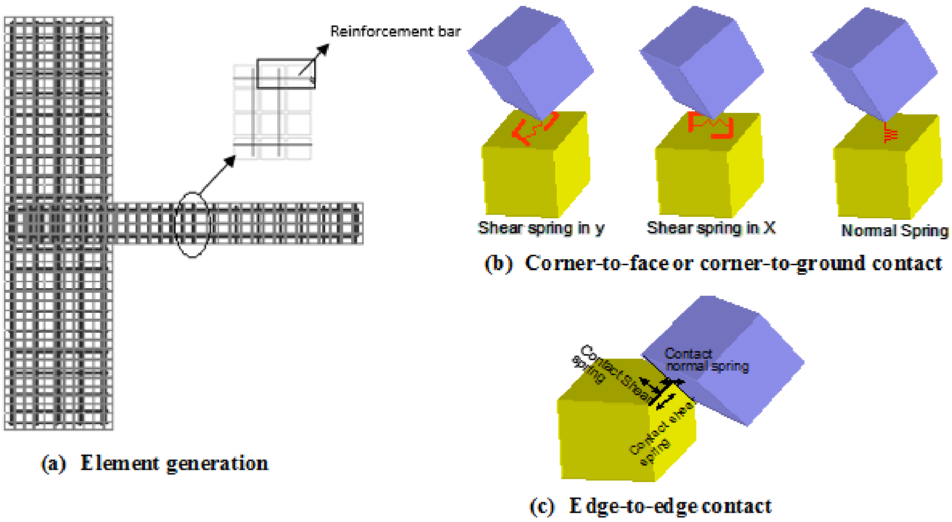


Figure 2: Element generation and different types of element contact

4. Specimens modeling

Three experiments for wide beam – column joints were analyzed using ELS. Figure 3 shows ELS model, where all reinforcement details were precisely modeled. The reinforcement details of specimens are explained later when describing the parametric study. The total number of solid elements in the analytical model was 3216 elements for J1, 3636 elements for J2 and 4156 elements for J5.

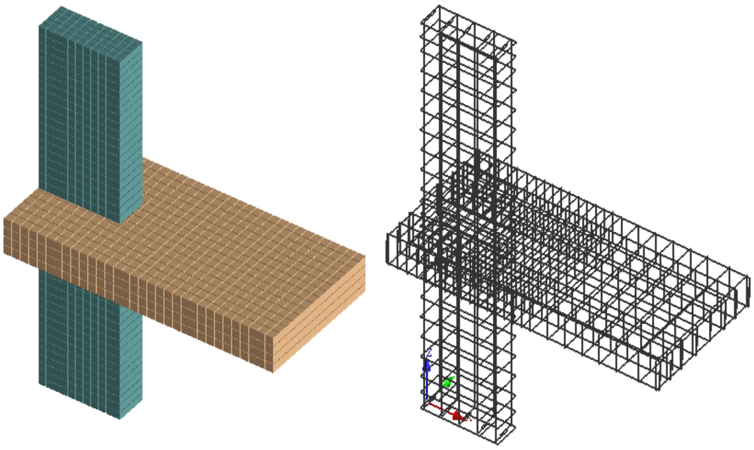


Figure 3: ELS model

All reinforcement properties, for each reinforcement bar (bar area, tensile yield strength, ultimate strength, ultimate strain and concrete cover), and also concrete properties (concrete compressive strength, tensile strength, young's modules, and shear modules) are considered in the model. Table 1 shows concrete compressive strength and Table 2 shows properties of reinforcing bars.

Table 1: Concrete compressive strength

Specimens	f_{cu} after 28 days (N/mm^2)
J1	33
J2	34
J5	32

Table 2: Properties of Steel used

	Ø 6	Ø 8	Φ 10	Φ 12
Grade	24/35	24/35	40/60	40/60
Shape	Plain bars	Plain bars	Deformed bars	Deformed bars
Yield stress (N/mm^2)	354	338	599	526
Ultimate stress (N/mm^2)	495	468	688	650
Ultimate stress/ Yield stress	1.4	1.38	1.15	1.24
Ultimate strain	0.25	0.23	0.16	0.15

5. Loading Scheme

A cyclic loading is applied to the specimens beam tip. Typical quasi – static test is selected for beam-column joints testing. The column was supported laterally at two points, to prevent out – of – plane rotation of column. Axial load approximately $0.15\,f_c{}'A_g$ was applied on column top during the application of the cyclic load. First, the column axial load was applied, then the cyclic displacement was applied at the beam tip. The point of loading for all specimens was 1020 mm away from column face. All specimens were subjected to the same loading protocol shown in Figure 4. All the cycles of the test were carried out in the displacement control mode. The load is composed of two cycles at each displacement.

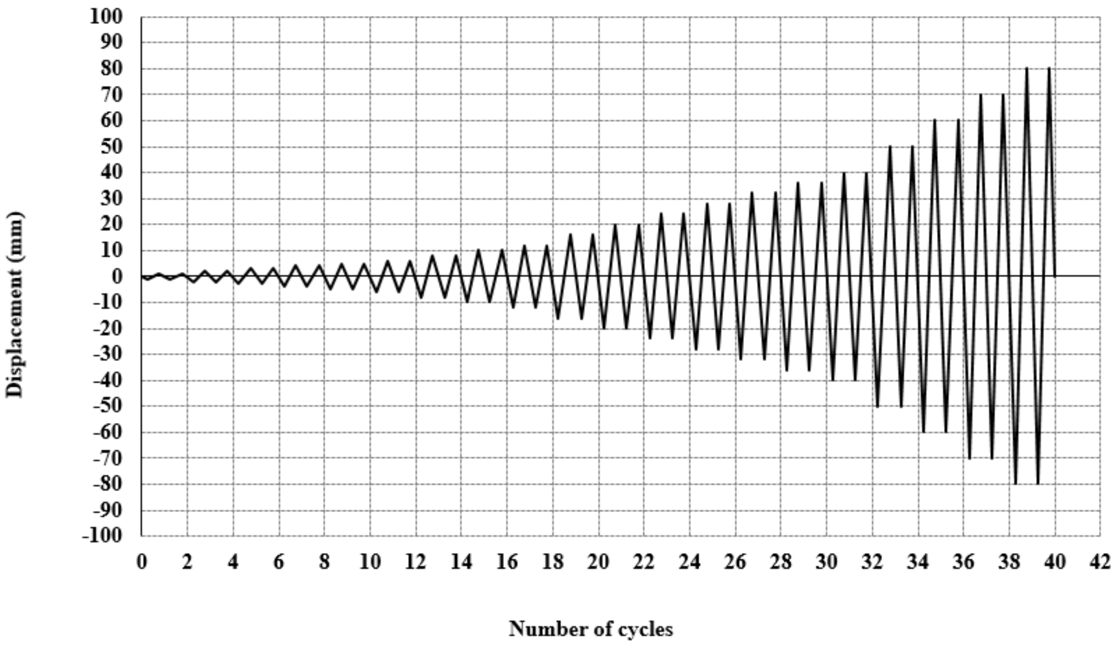


Figure 4: Loading Pattern 1

6. Numerical Results Validation

The numerical results are compared to the experimental ones in terms of load displacement hysteresis and cracking pattern.

6.1 Load – Displacement Hysteresis loops

Figure 5 Shows comparison of hysteresis behavior for the tested specimens and the numerical results. In general, the numerical results shows somehow higher ultimate capacities than the experimental results. Table 3 shows a comparison between the numerical and experimental capacities where a maximum variation of 13% was observed.

Table 3: comparison between numerical and experimental capacities

Specimens	Experimental ultimate capacity (kN)		Numerical ultimate capacity (kN)		Difference (%)	
	Positive loading	Negative loading	Positive loading	Negative loading	Positive loading	Negative loading
J1	66	54	59	57	12	4.4
J2	73	61	65	62	13	1.6
J5	70	63	67	63	5	0.6

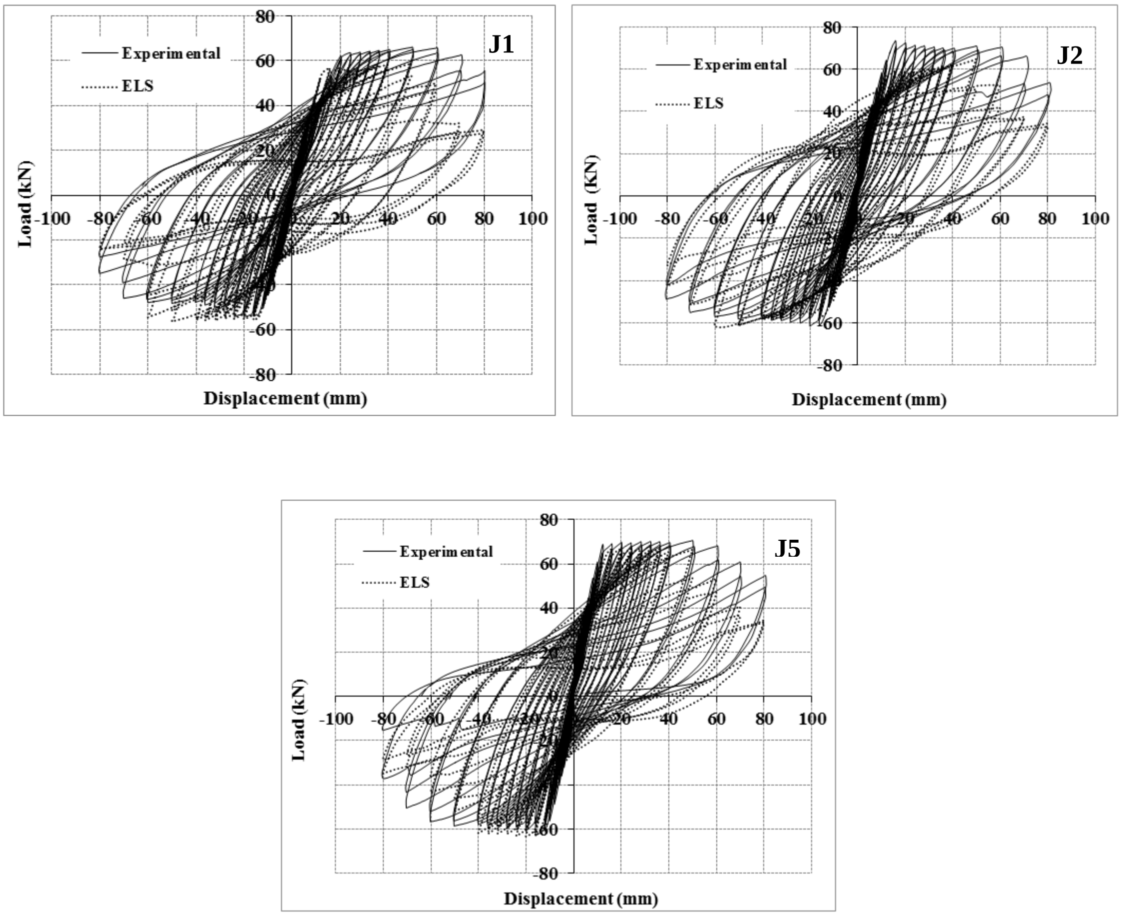
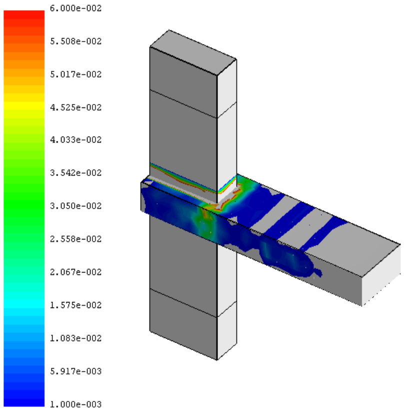
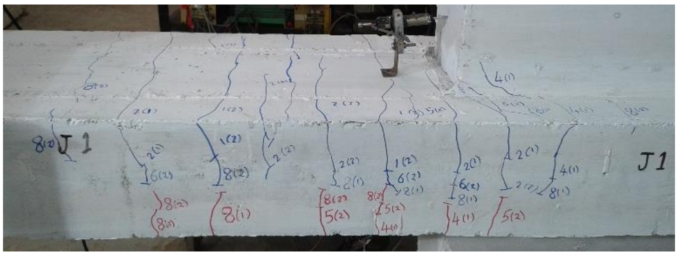


Figure 5: Comparison of hysteresis behavior for specimens J1, J2, and J5

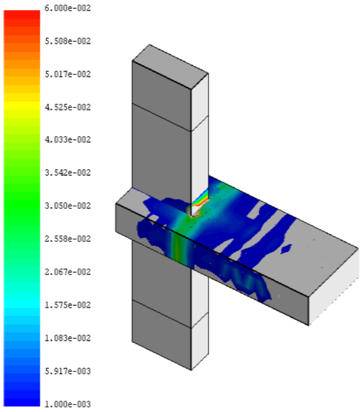
6.2 Concrete strain

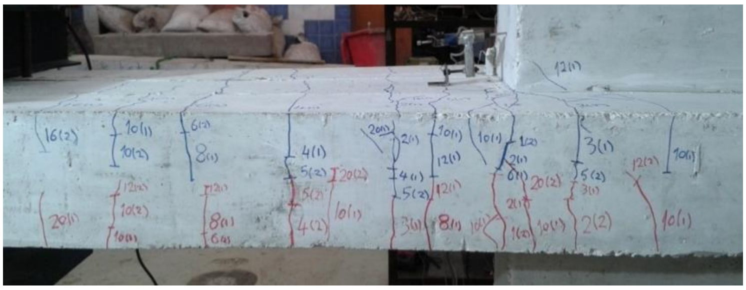
Figure 6 shows a comparison between the experimentally obtained cracking pattern and the principal strains contours for specimens J1, J2, and J5, respectively. The principal strain contours represent a good indicator for the cracking pattern and crack localization. The numerical results showed a good indicator for the cracking pattern compared to the observed cracks.



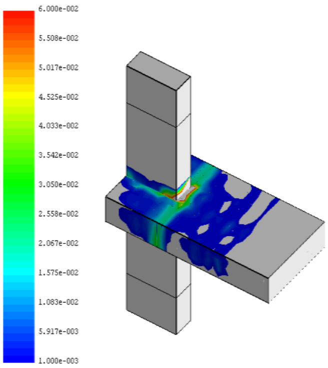


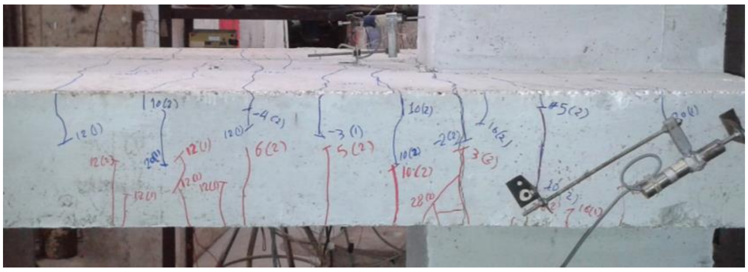
a. **Principal Strain contours for J1** b. **Observed cracking pattern for J1**





a. **Principal Strain contours for J2** b. **Observed cracking pattern for J2**





a. **Principal Strain contours for J5** b. **Observed cracking pattern for J5**

Figure 6: A comparison between observed experimental cracking patterns and numerically – obtained principal strain contours for J1, J2 and J5

7. Parametric Study

Comparing the numerical and experimental results showed reliability and validity of the numerical results. A parametric study was carried out to evaluate the effect of beam width, stirrups configuration, column axial load, loading history, and compressive strength, on the behavior of reinforced concrete wide beam –column joints. Columns have rectangular cross section with dimension 200 × 500 mm and main reinforcement of 10 Φ 12. Dimensions and reinforcement of column were kept constant as well as concrete compressive strength (35 N/mm^2) and steel properties, as shown in Table 4. Table 4-aand 4-b shows all details of specimens and variables through the parametric study.

7.1 Effect of beam width

To study the effect of beam width on the behavior of reinforced concrete wide beam – column joints, six specimens with the same steel area but different in beam width (400, 600, 800, 1000, 1200, and 1400 mm) as shown in Table 4-a. Figure 7 shows the envelope of the load – displacement hysteresis for different beam width. Specimen J1, with the smallest beam width, showed the lowest ultimate capacity. Relative to J1, increasing the beam width causes an increase in lateral load capacity of 5.97%, 18.42 %, 23.45 %, 23.58%, and 24.27 % for beam width of 600, 800, 1000, 1200 and 1400 mm, respectively and an increase in ductility of 18.9 %, 24.5 %, 2.8 %, 17.4%, and 30.7 % for beam width of 600, 800, 1000, 1200 and 1400 mm, respectively. The ductility is calculated in terms of the strain energy, i.e., the area under the load displacement curve.

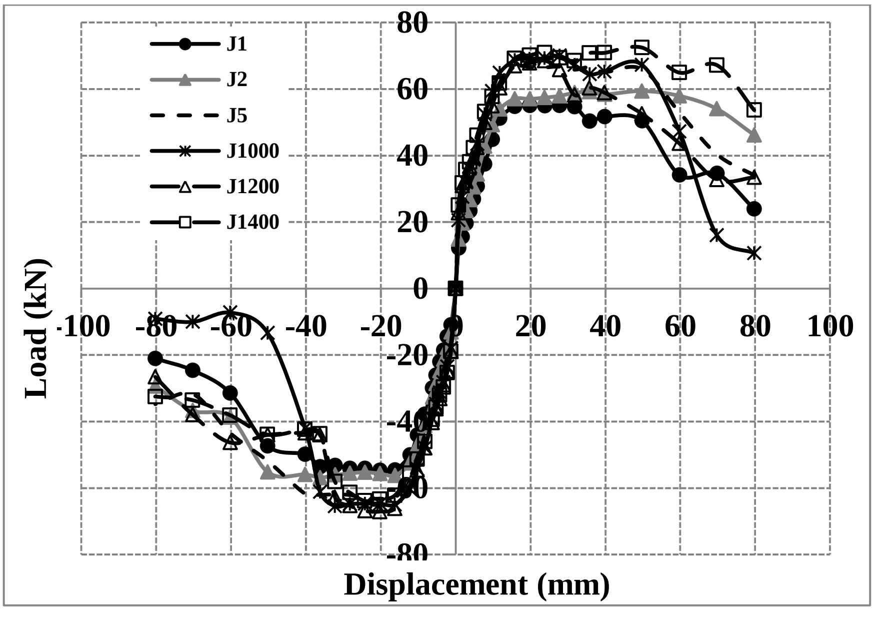


Figure7: Load –displacement envelope for different beam width

Table 4- a: Detail of Specimens and variables in parametric study

Group	Specimens	Beam dimension	Beam reinforcement (top= bottom)	Loading history	Stirrups			Column axial load	Compressive strength
					Plastic hinge zone	Other zone	No. of branches		
Reference	J1	400 × 200	2 Φ 12+ 4 Φ 12*	P 1**	20Ø 6/m	10Ø 6/m	4	$0.15\, f'_c A_g$	35
	J2	600 × 200	2 Φ 12+ 4 Φ 12*	P 1**	20Ø 6/m	10Ø 6/m	6	$0.15\, f'_c A_g$	35
	J5	800 × 200	2 Φ 12 + 6 Φ 10*	P 1**	20Ø 6/m	10Ø 6/m	8	$0.15\, f'_c A_g$	35
Beam width	J1000	1000 × 200	2 Φ 12 + 6 Φ 10*	P 1**	20Ø 8/m	10Ø 8/m	8	$0.15\, f'_c A_g$	35
	J1200	1200 × 200	2 Φ 12 + 6 Φ 10*	P 1**	20Ø 8/m	10Ø 8/m	8	$0.15\, f'_c A_g$	35
	J1400	1400 × 200	2 Φ 12 + 6 Φ 10*	P 1**	20Ø 8/m	10Ø 8/m	8	$0.15\, f'_c A_g$	35
Stirrups configuration	J1-10.6	600 × 200	2 Φ 12+ 4 Φ 12*	P 1**	10Ø 6/m	6Ø 6/m	4	$0.15\, f'_c A_g$	35
	J1-5.3			P 1**	5Ø 6/m	3Ø 6/m	4	$0.15\, f'_c A_g$	35
	J1-without stirrup			P 1**	----	----	----	$0.15\, f'_c A_g$	35

* Outside column core

** Loading Pattern

Table 4- b: Detail of Specimens and variables in parametric study

Group	Specimens	Beam dimension	Beam reinforcement (top= bottom)	Loading history	Stirrups			Column axial load	Compressive strength
					Plastic hinge zone	Other zone	No. of branches		
Column axial load	J2	800 × 200	2 Φ 12+ 4 Φ 12*	P 1**	20Ø 6/m	10Ø 6/m	6	$0.3\,f'_c A_g$	35
			2 Φ 12+ 4 Φ 12*	P 1**	20Ø 6/m	10Ø 6/m	6	$0.45\,f'_c A_g$	35
			2 Φ 12+ 4 Φ 12*	P 1**	20Ø 6/m	10Ø 6/m	6	$0.6\,f'_c A_g$	35
Loading history	J5	400 × 200	2 Φ 12 + 6 Φ 10*	P 2**	20Ø 6/m	10Ø 6/m	8	$0.15\,f'_c A_g$	35
			2 Φ 12 + 6 Φ 10*	P 3**	20Ø 6/m	10Ø 6/m	8	$0.15\,f'_c A_g$	35
Compressive strength	J1-25	400 × 200	2 Φ 12+ 4 Φ 12*	P 1**	20Ø 6/m	10Ø 6/m	4	$0.15\,f'_c A_g$	25
	J2-25	600 × 200	2 Φ 12+ 4 Φ 12*	P 1**	20Ø 6/m	10Ø 6/m	6	$0.15\,f'_c A_g$	25
	J5-25	800 × 200	2 Φ 12 + 6 Φ 10*	P 1**	20Ø 6/m	10Ø 6/m	8	$0.15\,f'_c A_g$	25

* Outside column core

** Loading Pattern

7.2 Effect of stirrup configuration

The effect of stirrups configuration is investigated throughout the three specimens (J1-10.6, J1-5.3, J1- without stirrup) as shown in Table 4-a, were compared with J1, where stirrup detail of J1-10.6, J1-5.3 is shown in Table 4. Figure 8 illustrate the envelope of the load – displacement hysteresis for different stirrups configuration for specimen J1. As seen in Figure 8, relative to J1, increasing spacing between stirrups caused a decrease in lateral load capacity of 1.2 %, 6 %, and 30.3% for specimens J1-10.6, J1-5.3, and J1-without stirrup, respectively. Relative to J1, increasing spacing between stirrups caused a decrease in ductility of 16.4 %, 32.8 %, and 43.4 % for specimens J1-10.6, J1-5.3, and J1-without stirrup, respectively. The reduction in both the capacity and ductility with the increase in stirrups spacing is attributed to the reduction in shear capacity of the beams.

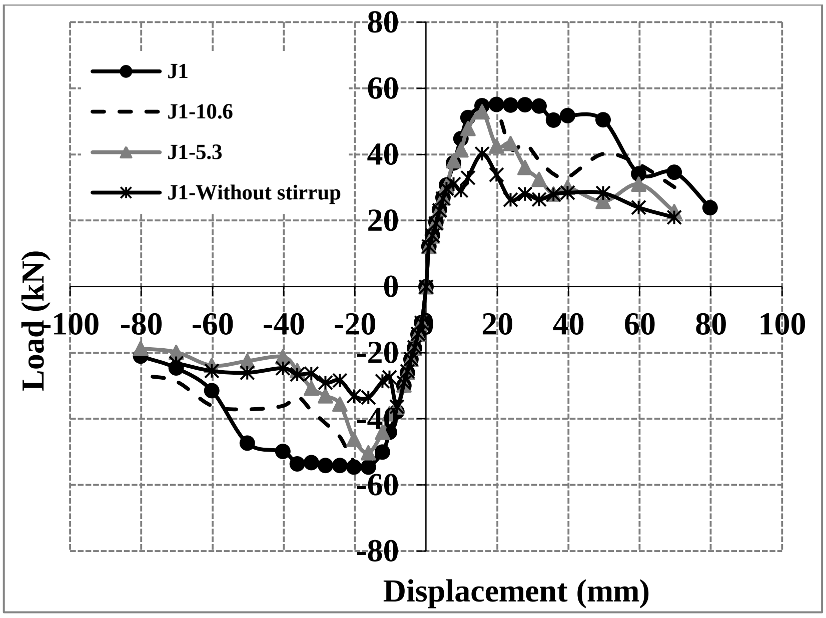


Figure 8: Load –displacement envelope for decreasing stirrup for J1

7.3 Effect of column axial load

In order to study the effect of column axial load, on the behavior of reinforced concrete wide beam –column joints, the same loading history as used in the experiment applied for specimen J2 using different axial loads ($0.15\, f'_c A_g$, $0.3\, f_c' A_g$, $0.45\, f_c' A_g$ and $0.6\, f_c' A_g$). Figure 9 shows the envelope of the load – displacement hysteresis for specimen J2 under different axial loads. As seen in Figure 9, the axial load has very little effect on the joint behavior. This could attributed to the fact that the behavior is governed by the column rather than the beam (strong column – weak beam joints).

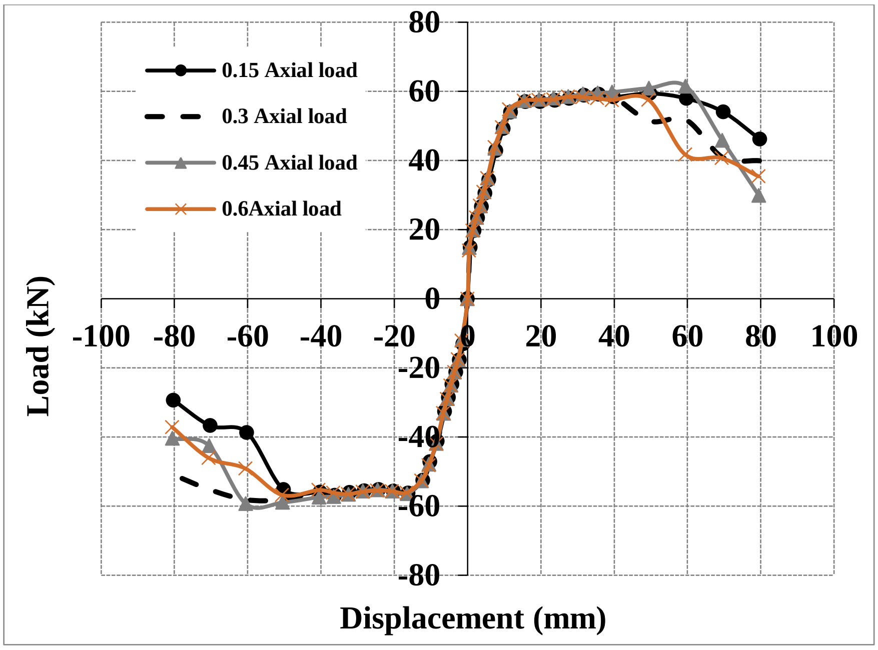


Figure 9: Load –displacement envelope for different axial load for J2

7.4 Effect of loading history

The effect of the loading history is studied by changing the loading pattern. Three loading patterns were used. The first loading pattern is shown in Figure 4. The two other loading pattern had 10 and 20 mm displacement per cycle. The load is composed of two cycles at each displacement. Figure 10 shows the envelope of the load – displacement hysteresis for different loading pattern for specimen J5. For specimen J5, increasing displacement per cycle caused an increase in load capacity of 4.8 % for loading pattern 2 and 8.7 % for loading pattern 3, while caused an increase in ductility of 1.7 % for loading pattern 2 and 6.5 % for loading pattern 3.

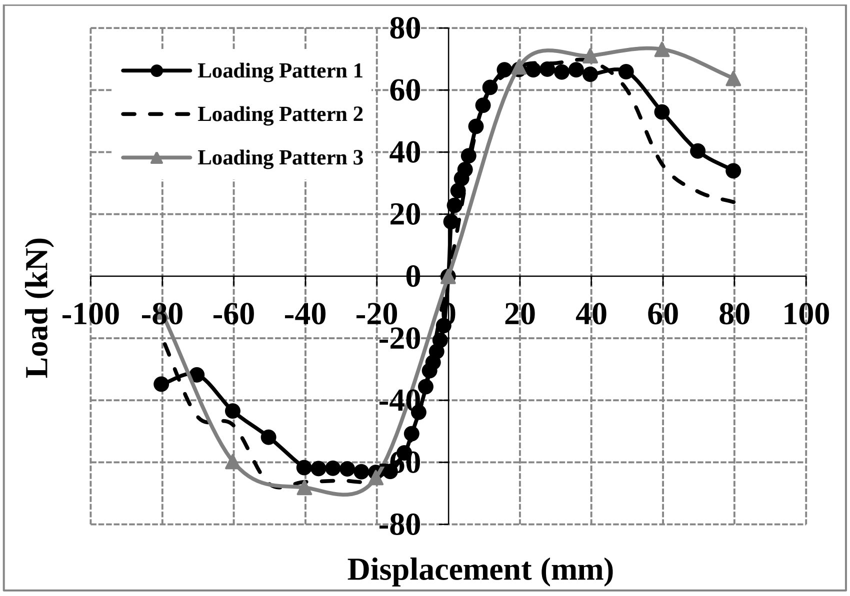


Figure 10: Load –displacement envelope for diff. loading pattern for J5

7.5 Effect of compressive strength

The effect of the compressive strength of concrete was investigated by the three specimens J1- 25, J2- 25and J5- 25 with compressive strength 25 N/mm^2 were compared with J1, J2, and J5, respectively. Figure 11 shows the envelope of the load – displacement hysteresis for specimens. Decreasing concrete compressive strength, caused a decrease in lateral load capacity of 1.87%, 0.29% and 0.74% for specimen J1, J2, and J5, respectively, and a decrease in ductility of 4.03 %, 0.22%, and 8.15% for specimen J1, J2, and J5, respectively.

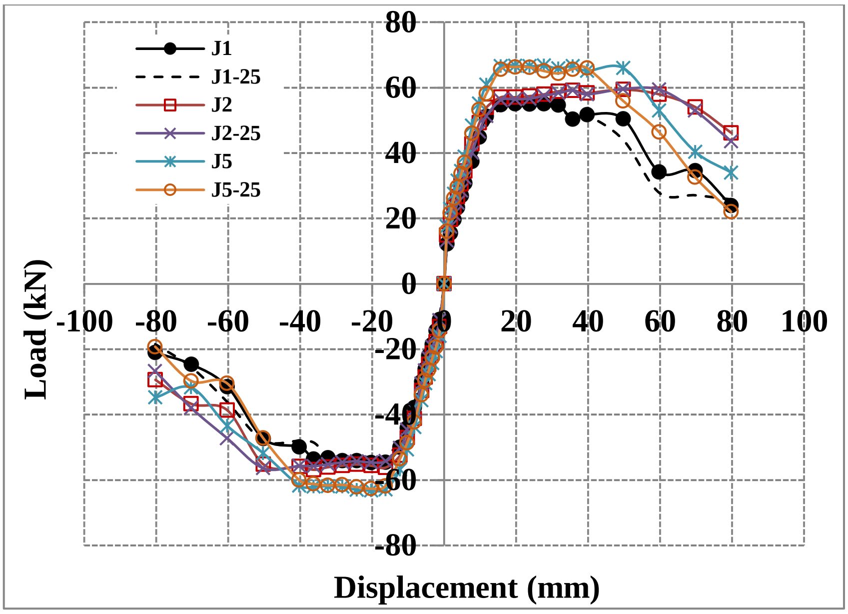


Fig. 11: Load –displacement envelope for specimens

8. CONCLUSIONS

The numerical results obtained by AEM for the experiments carried out to the wide beam – column joints under cyclic loading proved the accuracy of the AEM analysis and hence a parametric study was carried to evaluate the effect of the parameters; beam width, reinforcement configuration, stirrups configuration, column axial load, loading history, compressive strength, and yield strain, on the behavior of reinforced concrete wide beam –column joints.

The following conclusions were drawn from the parametric study:

1. Numerical investigation showed that the wide beam has well performance if they are properly designed and detailed to dissipate the seismic input energy through deformations in the inelastic range although beam to column width ratio was greater than two. It is also performed well even when two thirds of the wide beam flexural reinforcement was anchored outside the column core. Increasing beam width lead to an increase in lateral load capacity of 5.97%, 18.42 %, 23.45 %, 23.58%, and 24.27 % when increasing the beam width with 150%, 200%, 250%, 300% and 350%, respectively. Increasing beam width leads to an increase in ductility of 18.9 %, 24.5 %, 2.8 %, 17.4%, and 30.7 % when increasing the beam width with 150%, 200%, 250%, 300% and 350%, respectively.

2. Stirrups significantly affect the shear behavior of wide beams. Increasing stirrups spacing to 100 mm and 200 mm caused a decrease in ultimate capacity of 1.2 % and 6% and a decrease in ductility of 16.4 % and 32.8 %, respectively. The absence of stirrup decrease lateral load capacity about 30.3 % and decrease ductility with 43.4 %.
3. Increasing axial load does not affect the capacity of the joint, a weak beam-strong column fact eliminates the effect of the axial compression.
4. Increasing displacement per cycle caused an increase in ultimate capacity up to 8.7 % and an increase in ductility up to 6.5 %.
5. Decreasing concrete compressive strength causes slight decrease in lateral load capacity with 1.87%, 0.29% and 0.74% and decreases the ductility with 4.03 %, 0.22%, and 8.15% for specimen J1, J2, and J5, respectively.

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