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A Novel Technique to Maximize Fuzzy Net Present Value

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الملخص
فى البحث الحالى يتم استخدام المتغيرات الضبابيه للتعبير عن خطر المسار النقدى المخطط. يتم التعامل مع الأرقام الضبابيه ذات التوزيع المثلثى والعاديه (membership=1) و ذلك لتركيز نطاق هذه الدراسه.تم استخدام ١٠ طرق من طرق الترتيب باستخدام المنطق الضبابى ، حيث ان كل طريقه تعطى ترتيب للمسار النقدى لكل نشاط ، فاذا تم استخدام هذه الترتيبات كاولويات لجدولة الأنشطة ، فان كل طريقه سوف تعطى نموذج مختلف للمسار النقدى و بالتالى قيمه صافيه حاليه ممثله بالمنطق الضبابى.ان الهدف من هذا البحث هو : (١) اقتراح تقنيه لتعظيم القيمه الصافيه الحاليه الممثله بالمنطق الضبابى باستخدام طريقة الوزن للتدفق النقدى. ،(٢) لاكتشاف أفضل طريقه ترتيب تستخدم المنطق الضبابى لتعظيم القيمه الصافيه الحاليه الممثله بالمنطق الضبابى. تم تبنى سيناريوهين فى الحل و هما اهمال معدل التضخم واعتبار معدل التضخم ، تم تطبيق خطوات التقنيه المقترحه على مشروع لتوضيح كيفية عمل التقنيه. تم تقديم ثلاثة حالات دراسيه. تم التوصل للاستنتاجات اعتمادا على نتائج المثال و الثلاث حالات الدراسيه.

ABSTRACT
In the current research, fuzzy variables are used to express the risk of planned cash flow. Triangular normal fuzzy numbers have been only dealt with in this research to focus the scope of is study. Ten fuzzy ranking methods are applied to calculate fuzzy net present value (FNPV) of the project, since each method gives a rank for each activity's cash flow. If these ranks are used as priorities for scheduling activities, thus each method will give different cash flow pattern and intern different FNPV. This paper has two objectives: (1) To develop a technique for maximizing FNPV of the project using cash flow weight technique and (2) To discover which fuzzy ranking method is best for maximizing FNPV. Two scenarios of neglecting and considering inflation rate are adopted. The procedure is implemented to an example project to show how the technique performs. Three case studies are presented. Depending on the results of the example project and the three case studies the conclusions are drawn.

Keywords: Constrained resource allocation; Triangular Fuzzy numbers; Fuzzy net present value; Fuzzy ranking methods.

INTRODUCTION
There are several different algorithms proposed in the literature which maximize NPV of the project. These algorithms could be classified into two categories. The first category assumes crisp values of cash flows. The second category takes the risk of planned cash flows into account by expressing the planned cash flows as random or fuzzy variables. Fuzzy logic will be used in the current study to model uncertain cash flows to enable us to deal with the uncertainty.

NPV is maximized when positive net cash flow is as close as possible to the start of the project. The activities of a project are rearranged in the constrained resource allocation such that it achieves the constraint on the resource units. In the current research, fuzzy variables are used to express the risk of planned cash flow. Ten fuzzy ranking methods are applied to find the importance of the activities with respect to the FNPV of the project, since each method gives a rank for each activity's cash flow. These ranks are used as priorities for scheduling activities, thus each method will give different cash flow pattern and intern different NPV. This paper has two objectives: (1) To develop a technique for maximizing FNPV of the project using cash flow weight technique; (2) To discover which fuzzy ranking method is best for maximizing FNPV. The paper is organized as follows. In section 2, a literature review on project scheduling to maximize NPV of the project in the crisp and probabilistic case in addition to the studies which have fuzzy cash flows are given. In section 3, cash flow weight algorithm is demonstrated. Fuzzy logic and operation of the fuzzy procedures are presented in section 4. Fuzzy ranking methods are presented in section 5. Formulation of the developed technique is described in section 6. In section 7, an example project is solved manually step by step to show how the technique performs. Three case studies and analysis of the results are conducted in section 8. The conclusions are drawn in the last section.

LITERATURE REVIEW

An elementary approach that maintains the essential simplicity of the problem of maximizing NPV of the project was presented by Elmaghraby and Herroelen [1]. Yang et al. [2] presented an integer programming algorithm for determining scheduled start and finish times for the activities of a project subjected to resource limitation. The resource limitation was during each period of the schedule duration to maximize the NPV of the project. Sung and Lim [3] proposed a solution procedure for a general resource-constrained project scheduling problem (RCPSP) with the objective of NPV maximization. An optimization model for the problem of scheduling a project to maximize its NPV when net cash flow magnitudes are dependent of the time of their occurrence was developed by (Etgar et al. [4]).In their work, Baroum and Patterson [5] developed heuristic procedures for obtaining improved solutions to the maximization of NPV in a network problem. Pinder and Marucheck [6] evaluated the performance of seventeen scheduling heuristics separately on maximization of project NPV and minimization of project duration. Neumann and Zimmermann [7] developed exact heuristic procedures for resource leveling and NPV problems. In his work, Vanhoucke et al. [8] developed an exact branch and bound procedure for the unconstrained project scheduling problem with discounted cash flows. A multi-mode resource-constrained project scheduling problem with discounted cash flows was presented by Mika et al. [9].Waligora [10]adopted a discrete–continuous project scheduling problems with positive discounted cash flows for the maximization of the NPV. Publications on various variants and generalizations of the resource-constrained project scheduling problem were summarized and classified (Hartmann and briskorn; [11]) summarized and classified. Tantisuvanichkul[12] proposed a new rule called modified cumulative cash flow (m-CCF) with improved performance from the existing one. It was found that the m-CCF resulted in higher NPV than any other heuristics.

On the other hand, probability theory on project scheduling was adopted in some works to maximize NPV of a project. For instance, Yang et al. [13] evaluated nine stochastic scheduling rules for maximizing NPV of a project with probabilistic cash flows. Eight of these rules are extensions of corresponding single pass heuristic scheduling rules, while the ninth rule uses the process of simulated annealing. Sobel et al.[14] proposed an algorithm to identify an optimal adaptive policy to schedule a project with stochastic activity durations. The objective is to maximize the expected present value of the project cash flow. In their work, Creemers et al. [15] adopted a continuous-time Markov decision chainon project scheduling with NPV objective and exponential activity durations. Wiesemann et al., [16] proposed a model for maximizing the expected NPV of a project under uncertainty. The activity durations and cash flows are described by a discrete set of alternative scenarios with associated occurrence probabilities. The employment of Monte Carlo method in the NPV model in order to achieve reliable cash flows estimation was described by Shaffie and Jaaman[17]. Monte Carlo provides the risk analysis which can be adopted by investors in making capital budgeting decisions.

Another group of studies on RCPSP to maximize NPV which have fuzzy cash flows has been found in the literature review. Examples of these studies are,Kuchta[18] proposed fuzzy equivalents for all the methods of evaluating and comparing investment projects. These fuzzy equivalents evaluate projects whose cash flows and/or duration are given in the form of fuzzy numbers. In their work, Chui and Park [19] presented an engineering economic decision model in which the uncertain cash flows and discount rates are specified as triangular fuzzy numbers.

Melik[20] proposed a realistic, reliable and cost-schedule integrated cash flow modeling technique by using fuzzy set theory for including the uncertainties in project cost and schedule. Ucal and Kuchta[21] developed two different heuristic methods for project scheduling to maximize fuzzy net present value of a project. They found that the schedules resulting from cash flow weight and discounted cash flow weight heuristics are different which make differences on project's FNPV. Accordingly, the ranking method chosen for the ranking could change FNPV, and realization time of the project. Therefore, in the current research the authors will adopt different fuzzy ranking methods in the developed technique to identify which one is best for maximizing fuzzy net present value. Nosratpoura et al. [22] developed a model that considers all expenditures and revenues in terms of triangular fuzzy numbers and based on fuzzy theory. Kumar and Bajaj [23] proposed a new model to calculate NPV under Intuitionistic Fuzzy (IF) environment. Two different methods for project scheduling to maximize IFNPV of investment project were also proposed.

CASH FLOW WEIGHT ALGORITHM

In their work, Ucal and Kuchta[21]assumed a project with activities (A_i,i=1,2,..., N) and resources required by activities denoted by r_{ik}(i=1,2,..., N and k = 1,2,...,m). The durations of the activities denoted by di (i=1, 2,..., N). Net cash flows of activities occur at the beginning or end of the related activity and the value of it is independent of the starting or ending moment of the activity. The sum of all the cash flows from different activities starting or finishing in moment j will be denoted as CF_j (j=1, 2, ..., T^H) where T^H denotes time horizon. Present value (PV) of a single future payment occurred in the end of n^{th} year from now is given in (1) where F stands for amount of the payment and r denotes the interest rate (cost of capital).

$$PV=\frac{F}{(1+r)^n} \quad (1)$$

The goal is to find a schedule with a maximal NPV which is sum of all discounted cash flows formulated as in Eq. 2:

$$NPV=\sum_{j=0}^{n}\frac{CFj}{(1+r)^j} \quad (2)$$

Where CFj denotes cash flow

Cash flow weight (CFW) algorithm is a heuristic which dynamically selects a high priority activity from available activities for the assignment of resources. In this heuristic, the priority of an activity is linked to the cash flows linked to every activity and all the activities which follow it. The priority is measured by means of cash flow weighting (Barroum and Paterson; [5]).

The philosophy behind cash flow weight (CFW) heuristic procedures is to select a high priority activity from a list of available activities for the assignment of resource (Barroum and Paterson; [5]). This is because, the higher NPV is achieved by advancing positive cash flows as close to the start of the project as possible, while delaying negative payment as far back (to the right) as possible. This is accompanied by not moving any activities on the critical path and still satisfying precedence and resource constraint. If resource conflicts arise, the activity that is holding back a greater sum of cash inflows is scheduled first or receives higher priority in the assignment of resources.

However, cash flow weight procedure consists of three steps. In the first step, the cash flow weights of each activity are determined. All activities are included to the list of available activities in an order of i(i=1,2,..., N) without taking into account the predecessors. The activity with the highest CFW is selected from the list of available activities in the next step. The activity with the lowest number is assigned first if a tie exists. In order to assign the selected activity as soon as possible, the predecessors of the selected activity are assigned respectively in the increasing order of their indices i(i=1,2,..., N) and as soon as possible with respect to the resources available. On the other hand, the available resources are updated after assignment of the selected activity. In the third step, if there is any unassigned activity second step is repeated, otherwise the project schedule is completed (Barroum and Paterson; [5])

FUZZY LOGIC AND OPERATION OF THE FUZZY PROCEDURES

Zadeh[24] was the first founded fuzzy set theory. Fuzzy set theory has become an important tool for modeling the uncertainty. There is a general definition of fuzzy numbers but usually their simplest form, triangular fuzzy numbers (*TFN*) are preferred to simplify the calculations. A *TFN* has linear membership (possibility) functions both on the left and right sides. The membership function of *TFN* is given by Eq. (3).

$$\mu(x) = \begin{cases} \frac{x-M_l}{M_m}, & M_l \leq x \leq M_m \\ \frac{M_r-x}{M_r-M_m}, \quad if & M_m \leq x \leq M_r \\ 0, & otherwise \end{cases} \tag{3}$$

Where: M_l *and* $M_r$represents the smallest and greatest values of fuzzy number M. $M_m$is the mean or the mode of the fuzzy number. The difference interval (M_l, M_r) is called support of the fuzzy number *M*. Algebraic operations for *TFN*s are given by Eq.(s) 4-10where all the fuzzy numbers are positive (here it is assumedto mean M_l,≥ 0,N_l≥ 0 (Chen et al.[25]**):**

$(M_l,M_m, M_r) + (N_l,N_m, N_r) \cong (M_l+ N_l,M_m+ N_m, M_r +N_r)$ (4)

$(Ml ,M_m,M_r) - (N_l, N_m, N_r) \cong (M_l- N_r,M_m-N_m,M_r-N_l)$ (5)

$(Ml ,M_m,M_r) \times (N_l, N_m, N_r) \cong (M_lN_l,M_mN_m,M_rN_r)$ (6)

$(M_l,M_m,M_r) \div (N_l,N_m, N_r) \cong (\frac{Ml}{Mr},\frac{Mm}{Mm},\frac{Mr}{Ml})$ (7)

$$\lambda \times (M_l, M_m, M_r) \cong \begin{cases} (\lambda M_l, \lambda M_m, \lambda\, M_r), \lambda \geq 0 \\ \quad if \qquad , \forall \lambda \in R \\ (\lambda M_r, \lambda M_m, \lambda\, M_l), \quad \lambda \leq 0 \end{cases} \tag{8}$$

$$\lambda \div (M_l, M_m, M_r) \cong \begin{cases} (\frac{\lambda}{M_r}, \frac{\lambda}{M_m}, \frac{\lambda}{M_l}), & \lambda \geq 0 \\ & if \qquad , \forall \lambda \in R \\ \left(\frac{\lambda}{M_l}, \frac{\lambda}{M_m}, \frac{\lambda}{M_r}\right), & \lambda \leq 0 \end{cases} \qquad (9)$$

$$(M_l, M_m, M_r)^{\lambda} \cong \begin{cases} (M_l{}^{\lambda}, M_m{}^{\lambda}, M_r{}^{\lambda}), & \lambda \geq 0 \\ & if \qquad , \forall \lambda \in R \\ \left(\frac{1}{M_r{}^{\lambda}}, \frac{1}{M_m{}^{\lambda}}, \frac{1}{M_l{}^{\lambda}}\right), & \lambda \leq 0 \end{cases} \qquad (10)$$

A negative fuzzy number is a positive fuzzy number multiplied by -1.
In the current research, triangular fuzzy numbers are only applied for their simplicity. Also, to focus the scope of the research only normal numbers are dealt with because their membership function is equal to 1 (Thorani et al. [26]).

FUZZY RANKING METHODS

Ranking fuzzy numbers is usually used in decision making, data analysis, artificial intelligence, economic systems and operation research (Kwang and Lee, [27]). In a fuzzy environment, ranking is a very important decision making procedure. Since fuzzy numbers are represented by possibility distributions, they can overlap with each other and, thus, it is difficult to determine clearly whether one fuzzy number is larger or smaller than other (Kwang and Lee, [27]). Chenet.al.[25]reported that many methods have been proposed for ranking different types of fuzzy numbers and can be classified into four major classes.These are: preference relation, fuzzy mean and spread, fuzzy scoring and linguistic expression. However, each method appears to have advantages as well as disadvantages. One of the most commonly used methods under the class of fuzzy scoring is the centroid point method. Thus, in this study the centroid point methods in ranking fuzzy numbers are only applied.

Cheng [28] used a centroid-based distance approach to rank fuzzy numbers. For a trapezoidal fuzzy number A=(a,b,c,d; w), the larger the value of ranking *(R(A)),* the better the ranking will be of A. For trapezoidal fuzzy numbers Eq.(s) 11, 12 and 13 could be used.

$$x_A^{-} = \frac{w(d^2 - 2c^2 + 2b^2 - a^2 + dc - ab) + 3(c^2 - b^2)}{3w(d - c + b - a) + 6(c - b)} \qquad (11)$$

$$y_A^{-} = \frac{w}{3}\left[1 + \frac{(b + c) - (a + d)(1 - w)}{(b + c - a - d) + 2(a + d)w}\right] \qquad (12)$$

$$R(\tilde{A}) = \sqrt{\bar{x}_A^2 + \bar{y}_A^2} \qquad (13)$$

In the case of a triangular fuzzy number A = (a, b, d; w), c=band for normal fuzzy number w=1. Therefore, for a triangular normal fuzzy number A, Eq.(s) 14 and 15 could be used instead of Eq.(s) 11 and 12 respectively.

$$x_A^{-} = \frac{(d^2 - a^2 + db - ab)}{3(d - a)} \qquad (14)$$

$$y_A^{-} = \frac{1}{3}\left[1 + \frac{2b}{2b + a + d}\right] \qquad (15)$$

Chu and Tsao[29] proposed a new ranking index function (S_A).

$$S_A = x_A^{-} \times y_A^{-} \qquad (16)$$

The larger the value is of S_A, the better the ranking will be of A. For triangular normal fuzzy number $\bar{x}_A$ and $\bar{y}_A$ are as given by Eq. (s) 14 and 15, respectively.
Chen and Chen [30] proposed an approach for ranking generalized trapezoidal fuzzy numbers. The ranking value for a generalized trapezoidal fuzzy number A = (a1 ,a2, a3, a4; w) is defined as

$$Rank(A) = x + (w - y)^{S_A}(y + 0.5)^{1-w} \quad (17)$$

Where

$$y = \frac{w}{6}\left(\frac{a_3-a_2}{a_4-a_1} + 2\right) for a_1 \neq a_4; {}^{w}/_{2} \quad (18)$$

$$x = \frac{y(a_3+a_2)+(a_4+a_1)(w-y)}{2w} \quad (19)$$

$$S_A = \sqrt{\frac{\sum_{j=1}^{4}(a_j-\bar{a})^2}{3}} \quad (20)$$

$$\bar{a} = \frac{a_1+a_2+a_3+a_4}{4} \quad (21)$$

The larger the value of rank (A), the better the ranking of A.
In the case of a triangular normal fuzzy number A = (a, b, c; w), a2=a3 =b and w=1. Eq. (s) 22, 23, 24, and 25 are used instead of Eq. (s) 18, 19, 20, and 21 respectively.

$$y = \frac{1}{3} \quad (22)$$

$$x = \frac{2by+(c+a)(1-y)}{2} \quad (23)$$

$$S_A = \sqrt{\frac{\sum_{j=1}^{3}(a_j-\bar{a})^2}{2}} \quad (24)$$

$$\bar{a} = \frac{a+b+c}{3} \quad (25)$$

Wang et. all[31] presented the following formulas for the trapezoidal fuzzy numbers

$$\tilde{x}_0(\tilde{A}) = \frac{1}{3}\left[a + b + C + d - \frac{dc-ab}{(d+c)-(a+b)}\right] \quad (26)$$

$$\tilde{y}_0(\tilde{A}) = w \cdot \frac{1}{3}\left[1 + \frac{c-b}{(d+c)-(a+b)}\right] \quad (27)$$

And the ranking function is

$$R(\tilde{A}) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2} \quad (28)$$

The larger the value of $R(\tilde{A})$ the better the ranking of A
In the case of a triangular normal fuzzy number A = (a, b, d; w), c=b and w=1, Eq. 29 and 30 are applied instead of 26 and 27 respectively.

$$\tilde{x}_0(\tilde{A}) = \frac{1}{3}(a+b+d) \quad (29)$$

$$\tilde{y}_0(\tilde{A}) = \frac{1}{3} \quad (30)$$

Chen and Chen [32] again indicated the shortcomings of existing centroid methods, for Chu and Tsao's [29], Cheng's [28] approaches. He developed Eq. 31 for calculation of $Score(A_i)$ of a fuzzy number.

$$Score(A_i) = \sqrt{(x_{Ai} - x_{Aimin})^2 + (y_{Ai}^{s})^2} \quad (31)$$

Where (xAi) is as defined in Chen and Chen [30] and represented by Eq. 23. y_{Ai}^{s} is given by Eq. 32 andy_{Ai} represented by Eq. 22.

$$y_{Ai}^{s} = \frac{w_{Ai}}{2} - (y_{Ai} \times s_{Ai}) \quad (32)$$

Where

$$S_{Ai} = \sqrt{\frac{\sum_{j=1}^{4} (a_{ji} - \bar{a}_i)^2}{3}} \quad (33)$$

$$\bar{a}_i = \frac{a_{1i} + a_{2i} + a_{3i} + a_{4i}}{4} \quad (34)$$

The higher the value of Score (Ai) is , the better the ranking of the fuzzy number Ai.
In the case of a triangular normal fuzzy number A = (a, b, c; w) where a2=a3 =b and w=1,y_{Ai},x_{Ai}, S_{Ai} and $\bar{a}_i$ are represented by Eq. (s) 35, 36, 37, and 38 respectively.

$$y_{Ai} = \frac{1}{3} \quad (35)$$

$$x_{Ai} = \frac{2by + (c+a)(1-y)}{2} \quad (36)$$

$$S_{Ai} = \sqrt{\frac{\sum_{j=1}^{3} (a_j - \bar{a})^2}{2}} \quad (37)$$

$$\bar{a}_i = \frac{a+b+c}{3} \quad (38)$$

Due to the shortcomings of methods applied by Chen and Chen [30] and Chen and Chen [32], Chen and Chen [33] proposed an approach for ranking generalized fuzzy numbers with different heights and different spreads.The score value of each standardized generalized fuzzy number$Ai = (a_{1i}, a_{2i}, a_{3i}, a_{4i}; w_{Ai})$is defined as

$$Score(A) = \frac{(\bar{a}_{Ai} \times w_{Ai})}{(1 + s_{Ai})} \quad (39)$$

Where

$$s_{Ai} = \sqrt{\frac{\sum_{j=1}^{4} (a_{ij} - \bar{a}_{Ai})^2}{3}} \quad (40)$$

$$\bar{a}_{Ai} = \frac{a_{i1} + a_{i2} + a_{i3} + a_{i4}}{4} \quad (41)$$

The larger the valueof Score (Ai) is, the better the ranking of Ai.
In the case of a triangular normal fuzzy number A = (a, b, c; w), where a2=a3 =b and w=1 Eq. (s) 40 and 41 become as given by Eq.(s) 42 and 43 respectively.

$$s_{Ai} = \sqrt{\frac{\sum_{j=1}^{3} (a_j - \bar{a}_{Ai})^2}{2}} \quad (42)$$

$$\bar{a}_{Ai} = \frac{a+b+c}{3} \quad (43)$$

Dat et al. [34] presented a new ranking method as follows: Suppose $A1, A2, ..., An$ are fuzzy numbers. First, the centroid point of all fuzzy numbers are calculated using Wang et al. [31]. Thus, Eq. (s) 29 and 30 of Wang et al. [31] are used first for calculating $\bar{x}_A$ and $\bar{y}_A$, then the distance between the centroid point, $A_i = (\bar{x}_{Ai}, \bar{y}_{Ai})$, i=1,2,…,n and the minimum point $G = (x_{min}, y_{min})$, is as proposed in Eq. (44)

$$D(A_i, G) = \sqrt{(\bar{x}_{Ai} - x_{\min})^2 + (\bar{y}_{Ai} - \frac{w}{3} y_{\min})^2} \quad (44)$$

Thus, if Ai, Aj are two fuzzy numbers, then their ranking order is defined as follows: (1) $A_i \prec A_j$, if $D(A_i, G) \prec D(A_j, G)$;(2) $A_i \succ A_j$, if $D(A_i, G) \succ D(A_j, G)$, and (3) $A_i \sim A_j$, if $D(A_i, G) = D(A_j, G)$.
In their work, Allahviranloo and Saneifard[35] assumed that if there are n fuzzy numbers A1, A2, ...,An, the ranking method for these numbers A1, A2, ...,An is presented as follows:

Step 1: Using formulas given by Wang et al. [31] to calculate the centroid point ($\bar{x}_{A_j}$, $\bar{y}_{A_j}$) of each fuzzy number A_j, where $1 \le j \le n$.

Step2: Calculating the maximum crisp value τ_{max} of all fuzzy numbers A_j , where $1 \le j \le n$.

Step 3: Using the point ($\bar{x}_{A_j}$, $\bar{y}_{A_j}$) to calculate the ranking value *Dist* (A_j) of fuzzy numbers A_j, where $1 \le j \le n$, as in this Eq.

$$Dist(Aj) = \sqrt{(\bar{x}_{A_j} - \tau_{\max})^2 + (\bar{y}_{A_j} - 0)^2} = \sqrt{(\bar{x}_{A_j} - \tau_{\max})^2 + (\bar{y}_{A_j})^2} \quad (45)$$

Dist(A1) $\prec$ Dist(A2) if and only if A1 $\succ$ A2 ; (2)Dist(A1) $\succ$ Dist(A2) if and only if A1 $\prec$ A2 and (3) Dist (A1) = Dist (A2) if and only if A1 $\sim$ A2.

Thoraniet. all [26]reported that the generalized trapezoidal fuzzy number A = (a,b,c,d;w) is defined as :

$$I_{\tilde{A}}(\bar{x}_0, \bar{y}_0) = \left[\frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+C}{2}\right)+\gamma\left(\frac{2C+d}{3}\right)}{\alpha+\beta+\gamma}, \frac{\alpha\left(\frac{w}{3}\right)+\beta\left(\frac{w}{2}\right)+\gamma\left(\frac{w}{3}\right)}{\alpha+\beta+\gamma}\right) \quad (46)$$

Where

$$\alpha = \frac{\sqrt{(c-3b+2d)^2+w^2}}{6} \quad (47)$$

$$\beta = \frac{\sqrt{(2c+d-a-2b)^2}}{3} \quad (48)$$

$$\gamma = \frac{\sqrt{(3c-2a-b)^2+w^2}}{6} \quad (49)$$

As a special case, for triangular normal fuzzy number $\tilde{A} = (a, b, d; 1)$ i.e., c = b the centre of centroids is given by Eq. 50

$$I_{\tilde{A}}(\bar{x}_0, \bar{y}_0) = \left[\frac{x\left(\frac{a+2b}{3}\right)+y\mathrm{b}+z\left(\frac{2b+d}{3}\right)}{\alpha+\beta+\gamma}, \frac{x\left(\frac{1}{3}\right)+y\left(\frac{1}{2}\right)+z\left(\frac{1}{3}\right)}{x+y+z}\right) \quad (50)$$

Where

$$x = \frac{\sqrt{(2d-2b)^2+1^2}}{6} \quad (51)$$

$$y = \frac{\sqrt{(d-a)^2}}{3} \quad (52)$$

$$z = \frac{\sqrt{(2b-2a)^2+1^2}}{6} \quad (53)$$

The ranking function of fuzzy number is

$$R(\tilde{A}) = \bar{x}_0 \times \bar{y}_0 \quad (54)$$

The larger the value is of $R(\tilde{A})$ the better the ranking will be of A.

Gani and Mohamed [36] reported that, the centroid point of a trapezoid is considered it is balancing point.The trapezoid could be divided into three plane figures (two triangles and a rectangle). Consider a generalized trapezoidal fuzzy number A= (a,b,c,d:w). The centroid of the first triangle, the centroid of the rectangle is and the centroid of the second triangle are $G_1 = ((a+2b)/3, w/3)$, $G_2 = ((b+c)/2, w/2)$ and $G_3 = ((2c+d)/3, w/3)$ respectively. Accordingly, they concluded that the centroid

$G_A(\bar{x}_0, \bar{y}_0)$ of the triangle with vertices G1, G2, andG3 of the generalized trapezoidal fuzzy number $A = (a,b,c,d{:}w)$ as $G_A(\bar{x}_0, \bar{y}_0) = \left(\frac{2a+7b+7c+2d}{18}\right), \left(\frac{7w}{18}\right)$.

The ranking function of the generalized trapezoidal fuzzy number A=(a,b,c,d:w) which maps the set of all fuzzy numbers to a set of real numbers is defined as in Eq. 55.

$$R(A) = \bar{x}_0 . \bar{y}_0 = \left(\frac{2a+7b+7c+2d}{18}\right)\left(\frac{7w}{18}\right) \quad (55)$$

The larger the value is of R_A, the better the ranking will be of A.
In the case of a triangular normal fuzzy number A = (a, b, d; 1), c=b

$$x = \frac{2a+14b+2d}{18} \quad (56)$$

$$Y = \frac{7}{18} \quad (57)$$

STEPS OF DEVELOPING THETECHNIQUE

The developed technique consists of four steps as follows:

Step 1: fuzzy cash flow weight is calculated

Two cases are applied in calculating fuzzy cash flow weight. The first case when fuzzy cash flows are given directly. In this case the following procedure are adopted.If for example we have the network shown in Fig.1. Cash flow weight of activities 1 and 4 for instance could be calculated as follows:

$CFW_1 = CF_1 + CF_4 + CF_5 + CF_6 + CF_7$ (58)

$CFW_4 = CF_4 + CF_5 + CF_6 + CF_7$ (59)

In the real projects with large number of activities the calculations of cash flow weight will be complicated. Therefore,an equation will be established for calculating cash flow weight using software program (We use Microsoft Excel).

If Eq. 59 rewritten in the format of Eq. 60

$CFW_4 = CF_4 + CF_5 + CF_6 + CF_7 + CF_7 - CF_7$ (60)

Since,

$CFW_5 = CF_5 + CFW_7 = CF_5 + CF_7$ (61)

$CFW_6 = CF_6 + CFW_7 = CF_6 + CF_7$ (62)

Substitute from Eq. (s) 61 and 62 in Eq. 60 result in Eq. 63.

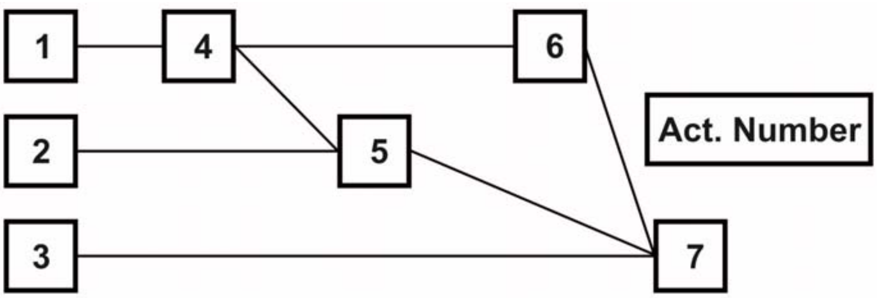

Fig. 1: Assumed logic to show how to calculate fuzzy cash flow weight

$CFW_4 = CF_4 + CFW_5 + CFW_6 - CF7$ (63)

Activities 5 and 6 are immediate successors to activity 4.Accordingly, for any activity i with successor j CFWi could be substituted with its equivalent value as in Eq. 64.

$CFW_i = CF_i + \sum_{j=1}^{n} CFW_j - CF_D$ (64)

Where: n is the number of successor activities to activity i and CFD is the duplicated cash flow.

If fuzzy cash flows are not given directly (second case), i.e. the direct cost is estimated and given in crisp value. In this case, the direct cost could be fuzzified using a random function applied from Excel software. The limits of this function could be determined according to the reported in Guide to Cost Predictability in Construction [37] for accuracy of direct cost. In this Guide, for 100% complete tender documents (at tender stage) the direct cost estimate variance ranges from ±5% to ± 10. In the current research, the maximum limits of variation for direct cost adopted ranges from 90% to 110%. On the other hand, indirect cost consists of site overhead and head office overhead. El-Dosouky [38] reported that site overhead may vary from 5% to 15% of project's direct cost. Also, he gave that head office overhead may vary from 2% to 5% of project's direct cost. This is implies that, the range of variation in indirect cost ranges from 7% to 20%. Accordingly, a fuzzy indirect cost (7%, 13.5%, 20%) of direct cost is adopted in the current research.

On the other hand, markup consists of profit margin and risk allowance. The margin of profit includes both the contractor's profit and the cost of finance for his investment in the contract. Park [39] declared that a typical construction contractor operates with (2-4%) of project's direct cost as a margin of profit. Therefore, the maximum expected variation in the margin of profit between contractors for a specific project is 2% of it's direct cost. A real assessment of risk allowance for a specific project requires both sensitivity and probability analyses. For the purpose of this study the authors assume that risk allowance ranges from 1% to 3% of project's direct cost. Thus, the range of variation in markup is from 3% to 7% of project's direct cost will be adopted. Accordingly, the applied fuzzy markup will be (3%, 5%, 7%) of direct cost.

Step 2: Applying fuzzy ranking methods

In each method each activity is ranked according to cash flow weight. According to this rank, the priority for each activity is determined, such that the activity with the highest rank will be assigned the highest priority for all ranking methods except Allahviranloo and Saneifard [35] method. In this method the activity with the least distance will be assigned the highest priority.

Step 3: A resource scheduling procedure is performed based on activities priorities resulted from step 2 for each ranking method using software program (Microsoft project in our research.

Step 4: Fuzzy net present value is calculated for all the activities in the project according to each schedule resulted from step 3.

In this research two scenarios are adopted. In the first scenario the inflation has been neglected, while in the second one the inflation has been considered. Eq. (s) 65 and 66 are applied for calculating FNPVS1 and FNPVS2for the first and second scenarios respectively. Best ranking method is then determined according to the highest value of FNPV in each scenario.

$FNPV_{S1} = \frac{F}{(1+r)^n}$ (65)

$FNPV_{S2} = \frac{F}{(1+m)^n}$ (66)

Where: F stands for amount of the future payment occurred in the end of nth year from now, r denotes the interest rate, and m denotes inflated interest rate and represented by Eq. 67, where t denotes the inflation rate.

$m = \tilde{r} + \tilde{t} + \tilde{r} \times \tilde{t}$ (67)

In Egypt the inflation rate was 14% in 2014 and it is anticipated that it is value will be about 7.5%in2020 [40]. Thus, we adopt a range of variation for inflation from 7.5% to 14%. To express the inflation rate as a fuzzy number $\tilde{t}$= (0.075, 0.105, 0.14) will be adopted.

These steps are explained in details through the example project given in the next section.

IMPLEMENTATION OF THE DEVELOPED TECHNIQUE

The developed technique is implemented in this section on example project. The data of this example were obtained from Ucal and Kuchta [18]. We have modified the data reported inUcal and Kuchta[18] to the the logic shown in Fig.2. In this Fig., the fuzzy cash flows occurred at the beginning of each activity. Activities, fuzzy cash flow, immediate predecessors, durations and resource requirements for each activity are shown in this Fig. The project has just one type of resource which is limited to 5 over the project realization time. In this example, an annual fuzzy interest rate r = (0.08, 0.10, 0.12) was used and an annual inflation rate t= (0.07, 0.105, 0.14) was applied.

Step 1: Fuzzy cash flow weight calculation

The calculations of CFW for activities 1 and 4 shown in Fig. 2 are given below as examples:

$CFW_1 = CF_1 + CF_4 + CF_5 + CF_6 + CF_7$

= (40, 50, 60) + (-36,-30,-24) + (37, 45, 53) + (35, 50, 65) + (2, 10, 18)

= (78,125,172)

$CFW_4 = CF_4 + CF_5 + CF_6 + CF_7$

= (-36,-30,-24) + (37, 45, 53) + (35, 50, 65) + (2, 10, 18) = (38, 75,112)

Applying Eq. 63 or Eq. 64 for calculating CFW4 since CF7 is duplicated in CFW5 and CFW6. Therefore, when we calculate CFW4 we must subtract CF7.

$CFW_4 = CF_4 + CFW_5 + CFW_6 - CF7$

= (-36,-30,-24) + (39, 55, 71) + (37, 60, 83) - (2, 10, 18) = (38, 75, 112)

Using Microsoft Excel, fuzzy cash flow weights were calculated for all activities as shown in Table 1. From this table it can be shown that Eq. 64 is applied for activities have two successors.

Step 2: Fuzzy ranking methods

The ranking is calculated for each activity in the example by each fuzzy ranking method. Calculation the ranking of activity 1 is shown for example for two ranking methods. Fuzzy cash flow weight of activity 1 as calculated in step 1 is (78, 125, 172).

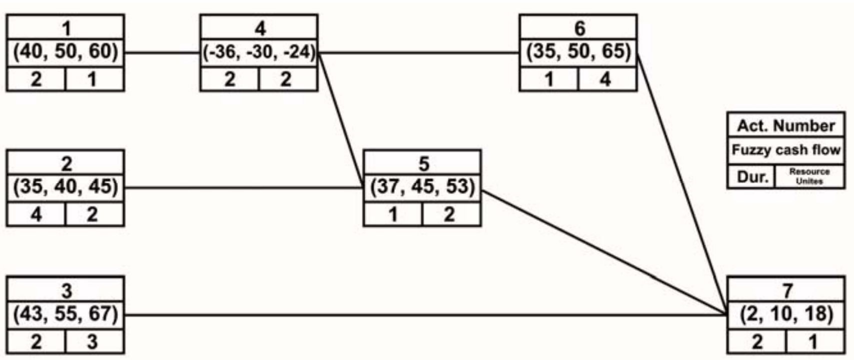

Fig. 2: Developed logic and data of example project

Table 1: Calculated fuzzy cash flow weights for example project's activities

Activity	Cash			Successor 1	Successor 2	Duplication	Cash flow weight		
	Low	Medium	High				Low	Medium	High
1	40	50	60	4			78	125	172
2	35	40	45	5			74	95	116
3	43	55	67	7			45	65	85
4	-36	-30	-24	5	6	7	38	75	112
5	37	45	53	7			39	55	71
6	35	50	65	7			37	60	83
7	2	10	18				2	10	18

1. For Cheng [28] ranking method, the rank value for cash flow weight is calculated as follows: apply Eq.(s) 14, 15 and 13 to calculate $\bar{x}_A$, $\bar{y}_A$ and $R(\tilde{A})$, respectively.

$$\bar{x}_A = \frac{(d^2-a^2+db-ab)}{3(d-a)} = \frac{(172^2-78^2+172*125-78*125)}{3(172-78)} = 125$$

$$\bar{y}_A = \frac{1}{3}\left[1+\frac{2b}{2b+a+d}\right] = \frac{1}{3}(1+\frac{2*125}{2*125+78+172}) = \frac{1}{2}$$

$$R(\tilde{A}) = \sqrt{\bar{x}_0^2+\bar{y}_0^2} = R(\tilde{A}) = \sqrt{125^2+(1/2)^2} = 125$$

According to the ranks, the priorities of activities have been established such that the activity with the highest rank is assigned the highest priority (see Table 2).

For Chu and Tsao[29] ranking method, the rank is calculated as follows: apply Eq.(s) 14 and 15 to calculate $\bar{x}_A$ and $\bar{y}_A$ as previously in Cheng [28]. These values are 125 and 0.5 as previously determined.. Then determine $R(\tilde{1})$ using Eq. 16.

$R(\tilde{1})$=125×0.5=62.5

Similarly, the ranks for all other activities adopting previous ranking methods are calculated and the priorities for all activities are established according to these ranks. Table 3 shows the priorities for all activities.

It can be shown that all methods gave the same priorities except Chen and Chen [33]. This result could be interpreted as the project is very small, it has just seven activities. For medium and large size projects a large variation between these methods of ranking is anticipated. This issue will be dealt with later in section of case studies.

Table 2: Calculated ranks and priorities for activities of example project Using Cheng [28] method

Activity	1	2	3	4	5	6	7
Rank	125	95.001	65.002	75.002	55.002	60.002	10.012
Priority	7	6	4	5	2	3	1

Step 3: Performing resource scheduling based on activity priority.

The priorities of activities established from previous step for each ranking method were applied for performing a resource scheduling procedure using a software program (Microsoft Project). Fig. 3 shows finish time for each activity for all methods except Chen and Chen [33] for example.

Table 3: Priorities of example project's activities for used ranking methods

Act.	Cash flow weight			Fuzzy ranking method	
	L	M	H	All methods of ranking except Chen and Chen [33]	Chen and Chen [33]
1	78	125	172	7	4
2	74	95	116	6	7
3	45	65	85	4	5
4	38	75	112	5	2
5	39	55	71	2	6
6	37	60	83	3	3
7	2	10	18	1	1

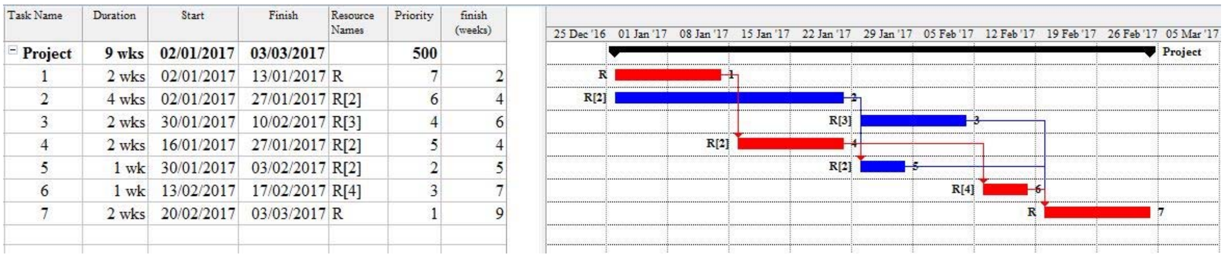

Fig.3: Resource scheduling for example project depending on activities priorities (For all methods except Chen and Chen 2009)

Step 4: Calculating Fuzzy net present value

Two scenarios are adopted for calculating FNPV. The first scenario (scenario 1) represents the case of neglecting inflation, while the second one (scenario 2) shows the case of considering inflation. The calculation of FNPV for activity 1 for all methods except (Chen and Chen [33]) is given below for the two scenarios as an example. Knowing cash flow of activity 1, CF1= (40, 50, 60) and finish time=2, fuzzy net present value of activity 1 for scenarios 1 and 2 (FNPVS1) and (FNPVS2) respectively are calculated as follows:

$$FNPV_{S1}=\frac{F}{(1+r)^n}=\frac{(40,50,60)}{(1+\frac{(0.08,0.10,0.12)}{12})^{2/4}}=\frac{(40,50,60)}{(1+\frac{0.08}{12},1+\frac{0.10}{12},1+\frac{0.12}{12})^{1/2}}=$$

$$\frac{(40,50,60)}{(1.0067,1.0083,1.01)^{1/2}}=(\frac{40}{1.01^{1/2}},\frac{50}{1.0083^{1/2}},\frac{60}{1.0067^{1/2}})=\mathbf{(39.80, 49.79, 59.80)}$$

$$FNPV_{S2}=\frac{F}{(1+m)^n}=\frac{(40,50,60)}{(1+\frac{(0.215,0.2155,0.2768)}{12})^{2/4}}=\frac{(40,50,60)}{(1+\frac{0.215}{12},1+\frac{0.2155}{12},1+\frac{0.2768}{12})^{1/2}}=$$

$$\frac{(40,50,60)}{(1.0179,1.018,1.0231)^{1/2}}=(\frac{40}{1.0231^{1/2}},\frac{50}{1.018^{1/2}},\frac{60}{1.0179^{1/2}})=\mathbf{(39.546, 49.556, 59.470)}$$

For Chen and Chen [33] and knowingcash flow of activity 1, CF1= (40, 50, 60) and finish time= 4.In this case FNPVS1= (39.6, 49.59, 59.6) and FNPVS2= (39.1, 49.12, 58.94).
The total FNPV for all activities of all cases for the two scenarios are shown in Table 4. However,the same results obtained for scenario 1 or the very small differences resulted in scenario 2 in this example may be due to the small number of activities for this project. Accordingly, three case studies are adopted in the following section to draw a final conclusion about the best ranking method for maximizing FNPV.

CASE STUDIES

Three case studies are applied in this section. The data of first case study was obtained from Melik[20], whereas the data of the second and third case studies were obtained by graduation project students supervised by the first author in 2014. The direct cost for case
Studies2 and 3 estimated by graduation project students and revised by the authors. A fuzzy interest rate= (0.08, 0.10, 0.12) is assumed for these case studies. Also, a fuzzy inflation rate (0.075, 0.105, 0.14) is adopted.

Case Study 1

The project of this case study is a warehouse project constructed in Ankara and consists of 18 activities. Some of the activities are assumed to be subcontracted. No resource is assigned to the activities planned to be subcontracted. The data were obtained from Melik[20].
Applying Eq. 64 fuzzy cash flow weight was calculated for each activity. Also, ranking methods were applied. The results for ranking methods revealed that methods (Chen and Chen [30], Chen and Chen [32], Wang et al. [31], Cheng [28], Dat et al.[34], and Sanifard [35] give the same ranking for all activities. The two methods: Chu and Tsao [29] and Gani and Mohamed [36] give the same ranking for all activities. Thoraniet all[26] gives different ranking for some activities. Chen and Chen [33] give different ranking for majority of activities. The ranking methods could be categorized into two categories (see Table 5). A resource scheduling based on activities priorities was performed using Microsoft Project software. According to the resulted finish time in each category, FNPV was calculated for the two adopted scenarios. The results are shown in Table 5.

Table 4: FNPV, distance, finish time for all activities for applied ranking methods

Category	Ranking methods	Scenario	FNPV			Distance	Finish time	Assigned score
			L	M	H			
1	All methods of ranking except Chen and Chen [33]	1	153.95	217.66	218.62	62.92	9	1
		2	151.4	215	277.6	62.92		1
2	Chen and Chen [33]	1	153.95	217.66	218.62	62.77	10	2
		2	151.6	215.2	277.7	62.77		2

Table 5: FNPV, distance, finish time for different ranking methods (case study 1)

Category	Ranking methods	Scenario	FNPV			Distance	Finish time	Assigned score
			L	M	H			
1	All methods of ranking except Chen and Chen [33]	1	5521.4	36992	55287	22687	120	2
		2	5493.6	36244.8	53931.2	22041.3		2
2	Chen and Chen [33]	1	5521.8	36990	55284	22688	120	1
		2	5494.4	36241.7	53924.4	22044.4		1

Case Study 2

This project is a social housing project, model no.1 in Fashn city at Beni- Suef governorate, Egypt and consists of 187 activities and has a direct cost of3378483 L.E as planned and estimated by the graduation project students. The number of available resource units (normal labor) for this project was assumed to be limited to 5 over the project realization time.

The direct cost for each activity was fuzzified using the random function applied from Excel software with limits ranges from 90 to 110%. The proposed fuzzy indirect cost (7%, 13.5%, 20%) of direct cost anda proposed fuzzy markup (3%, 5%, 7%) of direct cost were also adopted as previously explained in the steps of developing the technique. The same procedure of solution was adopted as in case study 1.Allahviranloo and Sanifard [35] has been chosen, and the distances for FNPV have been calculated, show Table 6.

Case Study 3

The project is a Water Treatment Plant and it's networks in Mit-Ghamr city, at Dakhlia governorate, Egypt and consists of 15 buildings with 724 activities and has a direct cost of 106868658 L.E. as planned and estimated by the graduation project students. The number of available resource units (normal labor) for this project was assumed to be limited to 8 over the project realization time.

Table 6: FNPV, finish time and assigned scores (case studies 2 and 3)

Category	Ranking methods	Case study 2			Case study 3		
		Scenario	Finish time	Assigned score	Scenario	Finish time	Assigned score
1	Chen and Chen [30], Wang et al. [31], Cheng [28], Dat et al. [34], and Allahviranloo and Sanifard [35]	1	451	3	1	2078	4
		2		4	2		4
2	Chen and Chen [32]	1	450	1	1	2140	2
		2		2	2		2
3	Chen and Chen [33]	1	471	6	1	2325	**1**
		2		1	2		1
4	Chu and Tsao [29]	1	447	5	1	2068	5

		2		6	2		**5**
5	Thoranie et al. [26]	1	453	2	1	2063	6
		2		3	2		6
6	Gani and Mohamed [36]	1	447	4	1	2073	3
		2		5	2		3

Similarly, the direct cost for each activity was fuzzified as in case study 2. Also, the same percentage of fuzzy indirect cost and fuzzy markup were applied. The results of this case study are presented in Table 6.

ANALYSIS AND DISCUSSION OF THE RESULTS

Depending on the results obtained from the example project and the three case studies, five ranking methods are among category 1. These methods are: Chen and Chen [30], Wang et al. [31], Cheng [28], Dat et al. [34], and Allahviranloo and Sanifard [35]. Therefore, it has been decided to choose one of them to rank the FNPV results for all methods for the adopted scenarios.Allahviranloo and Sanifard [35] has been chosen, and the distances have been calculated, show Tables 4, 5, 6. Table 7 shows a summary of the results for the example project and the three case studies. In this table a score is established for each ranking method in each case study according to the distance calculated, such that the method with the least distance is assigned the highest rank as declared previously in this method and the vice versa. A final score is calculated for each method by summing up the rank assigned to each method according to the example project and the three case studies. Now we can answer the question about the best fuzzy ranking method

No	Fuzzy ranking method	Scenario 1					Scenario 2				
		Assigned rank				F.S.	Assigned rank				F.S.
		E.P.	C.S. (1)	C.S. (2)	C.S. (3)		E.P.	C.S. (1)	C.S. (2)	C.S. (3)	
1	Chen and Chen [30]	1	2	3	4	10	1	2	4	4	11
2	Chen and Chen [32]	1	2	1	2	6	1	2	2	2	7
3	Chen and Chen [33]	2	1	6	1	10	2	1	1	1	5
4	Wang et al. [31]	1	2	3	4	10	1	2	4	4	11
5	Cheng [28]	1	2	3	4	10	1	2	4	4	11
6	Chu and	1	2	5	5	13	1	2	6	5	14

	Tsao [29]										
7	Dat et al. (2012)	1	2	3	4	10	1	2	4	4	11
8	Allahviranloo and Saneifard [35]	1	2	3	4	10	1	2	4	4	11
9	Thoranie et al.[26]	1	2	2	6	11	1	2	3	6	12
10	Gani and Mohamed [36]	1	2	4	3	10	1	2	5	3	11
Where: E.P. = Example project C.S.=Case study F.S.=Final score											

Table 7: Developed final score for the applied ranking methods

for maximizing FNPV. It is clear from table 7, that Chu and Tsao[29] is the best ranking method for maximizing FNPV depending on the highest associated final scores 13 and 14 in case of neglecting and considering inflation respectively. The second method is Thorani et al. [26] for the two adopted scenarios. The worst methods are Chen and Chen [32] and Chen and Chen [33]in case of neglecting and considering inflation respectively.

SUMMARY AND CONCLUSIONS

In this papera novel technique for maximizing FNPV has been proposed with different fuzzy ranking methods. The desirable characteristics of the developed technique were given. An equation was extracted for calculating cash flow weight to be used for large projects. A random function applied from Excel software was proposed to fuzzyfy crisp direct cost. Two scenarios was adopted in dealing with inflation, these are: neglecting and considering inflation. An example project was solved manually step by step to show how the technique performs. Analysis of the results for the example project and three case studies revealed that among the ten fuzzy ranking methods presented Chu and Tsao [29] is the best ranking method for maximizing FNPV in case of neglecting and considering inflation. Thoranie et al.[26] method comes after Chu and Tsao[29] for the two adopted scenarios. The worst methods are Chen and Chen [32] and Chen and Chen [33]in case of neglecting and considering inflation, respectively. This is implies that the result of the best method for maximizing fuzzy net present value in case of neglecting or considering inflation could be generalized in other countries have inflation rate up to 14%, since the inflation has no effect up to 14%.It must be noted that the results are limited to triangular normal fuzzy numbers and when the resource scheduling problem is performed for only one type of resources. As a future research the proposed technique could be expanded for other types of fuzzy numbers such as trapezoidal and for non normal fuzzy number. Also, more than one type of resources could be applied in resource scheduling problem. Moreover, the effect of higher percentages of inflation requires study. As a further step, the developed model could be computerized.

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