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Numerical Analysis of DYNAMIC SOIL-STRUCTURE INTERACTION FOR LOW AND HIGH RISE BUILDINGS

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الملخص العربي

فى الآونة الأخيرة وخاصة بعد حدوث العديد من الهزات الأرضية أصبح اعداد دراسة تأثير الزلازل على المباني منخفضة و عالية الارتفاع من الملفات الهامة التى يجب أن يقوم بها المصمم لتحديد الاجهادات الزائدة والمؤثرة على العناصر الانشائية لاخذها فى الاعتبار عند تصميم المبني. يعتبر الفعل الديناميكى المتبادل بين المنشأ و التربة المحيطة (DSSI) من العوامل الهامة التي تؤثر على السلوك الفعلي للمبنى اثناء التعرض للهزات الأرضية وبالتالى ينعكس ذلك على تصميم العناصر الانشائية المقاومة لتلك الحركات. تعتمد الاجهادات المؤثرة على المنشأ نتيجة الاحمال الديناميكية على مدى جساءة او ضعف التربة التى يرتكز عليها المبني وبالتالي يلزم اثناء اعداد الملف العددى لتمثيل المبنى أفتراض نوع ركيزة يتناسب مع جساءة التربة فى كل من الحالة الاستاتيكية و الحالة الديناميكية.

لذلك في البحث الحالي يتم دراسة التداخل الديناميكي المتبادل بين المنشأت و التربة المحيطة اثناء التعرض للهزات الارضية و ذلك باستخدام برنامج التحليل العددي (MIDAS GTS) و هو برنامج تحليل عددي يمكن عن طريقة تمثيل كلا من المنشأت بابعادها الحقيقية و خواصها الفبزيائية و التربة المحيطة بخصائصها الفيزيائية, تم الاخذ فى الاعتبار السلوك اللاخطي للتربة حول المنشأت عن طريق نموذج (Mohr – Coulomb model) و تم اتباع طريقة الاستجابة المركبة و التحليل اللاخطي باستخدام السجلات الزمنية للهزات الارضية (The 1940 El Centro earthquake of magnitude 6.9 with a duration of 30 seconds.) لمنشأت اطارية مرتكزة على قواعد سطحية.

ABSTRACT

This paper presents a direct finite element analysis of dynamic soil-structure interaction. Many problems in civil engineering involve some type of structural element in direct contact with the ground, such as building and bridge foundations and tunnels. When Earthquake forces are applied externally to the structural element and/or develop internally within the ground, both problem components (structural element and ground) must deform and move in a compatible manner. This is because neither the structural-element displacements nor the ground displacements are independent of each other as a result of their intimate physical contact. Therefore, these types of problems are broadly referred to as dynamic soil-structure interactional (DSSI) problems, where The effect of soil on the dynamic behavior of structures is taken into consideration. Full interaction between soil, substructure and superstructure are represented as a dynamic soil-structure interaction problem using the finite element method (In the present research, the MIDAS GTS program) was used. Nonlinear behavior of soil is taken according to Mohr Coulomb theory with a damping ration. El Centro earthquake was used as time history to predict the effects of dynamic soil-structure interaction. The study compares the

results of three cases: modeling of soil-foundation-super-structure, modeling of foundation as springs with the super-structure and modeling of foundation as hinged support with the super-structure

Keywords: Dynamic Soil Structure Interaction (DSSI),- Finite Element,- High Rise Building,- Low Rise Building,- Mohr-Coulomb Theory,- El Centro earthquake.

1. INTRODUCTION

When structures in direct contact with ground, compliance of the soil can induce two distinct effects on the response of the structure, first, modification of the free field motion at the base of the structure, and second, the introduction of deformation from dynamic response of the structure in to the supporting soil. The former is referred to as kinematic interaction, while the latter is known as inertial interaction and the whole process is commonly referred to as dynamic soil-structure interaction (J.P. Wolf and P. Obernhuber). The main concept of site response analysis is that the free field motion is dependent on the properties of the soil In Earthquake conditions the relationship "subsoil - structure - superstructure" ideally should be analyzed as structural continuum When analyzing the seismic response of structures it is common in practice to assume the base of the structure to be rigid, which is a gross assumption since in most situations the foundation soil is flexible. This assumption is realistic only when the structure is founded on solid rock or when the relative stiffness of the foundation soil compared to the superstructure is high. In profile including stiffness of soil layers. The stiffness of the deposit can change the frequency content and amplitude of the ground motion. Likewise, on the path to the structure, wave properties might be changed due to the stiffness of the foundation. In fact, kinematic interaction is the inability of the foundation to conform to the deformations of the free field ground. This study aims to quantify the effect of Soil-Structure Interaction and foundation flexibility on the structural response demands of moment resisting frame MRF buildings so that designers can be aware of the likely impact of their decisions. This investigation is aimed to better understand of the seismic performance of a typical moment resisting frame MRF buildings incorporating soil-structure effect. The seismic response of the structure in terms of the base shear is selected as response parameters of interest as these are generally considered the most important response parameters in seismic design practice. Three different analysis methods for (Ten story MRF buildings) compared to those obtained from fixed base model. the base shear due to seismic excitation of the structures modeled with the soil deposit are always less than the base shear of the structures modeled as fixed-base as expected.

1.2 Statement of the problem

Structural engineers generally assume that structures subjected to dynamic loading are fixed at their bases. This assumption ignores the important effects of the dynamic interaction between structure and soil if the structure is not founded on rock or if the supporting soil does not have high stiffness in comparison to the superstructure. Consequently, accounting for the actual support conditions may decrease the overall stiffness of a structure and lead to a more flexible structure. Damping of the supporting soil, as well as its periods of vibration in relation to that of the structure are also important aspects that affect the overall structural response.

where the base shear and although the internal forces due to seismic excitation of the structures modeled with the soil deposit less than the base shear and internal forces of the structures modeled as fixed-base as expected also axial force Bending moment and the shear force diagrams of the flexible base model are more realistic than of fixed-base models.

Whereas the increase in the natural period may cause a significant change in the seismic response of the structure. Therefore, it is important to incorporate the dynamic soil-structure interaction (DSSI) effects in the analysis of the dynamic behavior of structures.

1.3 RESEARCH OBJECTIVES

The main objective of this research study is: investigating the effect of soil-structure interaction (Rigid and Flexible base) on dynamic response of low and high-rise buildings; such as (Base shear, Lateral displacement, and straining actions on the structural supporting elements.

2 – METHODOLOGY

2.1 INTRODUCTION

When structures in direct contact with ground, like building, tunnels and bridge foundations exposed to Earthquake forces this forces applied externally to the elements of the structures or develop internally in the ground, both of the structure and the ground in fact deform and move in a compatible manner, and this because of the dependency of each other as a result of their intimate physical contact. Problems like these are referred to as dynamic soil structure interaction (DSSI) problems

Three-dimensional finite element analysis on a practical engineering considering dynamic Soil-Structure Interaction DSSI is carried out in this paper. In the computer simulation on dynamic Soil-Structure Interaction DSSI system, the nonlinear behavior of sub grade materials can be characterized by the "Mohr-Coulomb" model, and the viscous boundary is adopted as boundary for the soil. A computationally methods of investigation on practical engineering considering dynamic Soil-Structure Interaction DSSI by general-purpose finite element program MIDAS GTS is explored in this paper, which is of great advantage to the popularization of dynamic soil structure interaction study and promote the study outcomes to guide practical engineering.

Midas GTS is Fully Integrated 2D/3D Finite Element Analysis Software dedicated to geotechnical engineering applications including Tunneling, Mining, Foundations, Excavations, Soil-Structure Interaction, Settlement Analysis, Seepage (groundwater flow) Analysis, Consolidation Analysis and Dynamic Analysis. The Pre/Post-processors and Solvers are Fully Integrated, which means that the user need not acquire different modules for performing analyses such as for Foundation, Tunneling, Excavation, Ground Water Flow Analysis and etc. The Midas GTS technology balances power and simplicity to empower geotechnical engineers who are seeking a reliable platform which can revolutionizes numerical simulation and incorporate actual projects into finite element software. Midas GTS is designed to become an integral part of your professional services which will ultimately add significant value to your engineering innovations and make a positive impact on your organizations.

Various types of interface elements are implemented to simulate Soil-Structure Interaction regardless of geometric complexity and interface positions.

3 SOIL STRUCTURE MODELLING

3.1 Structural Properties

In our study, a two concrete frame resting on a shallow foundation (assuming as moment resisting building) used for analysis are as follow.
First model 2 storey building (This group is supposed to simulate low rise buildings.)
Second model 30 storey (This group is supposed to simulate high rise buildings.)
3.1.1 First model 2 storey building
In this study the building features four bays in east-west and north-south direction, respectively. The height of the storey is 3.0 m and width of the bay is 4.0 m. The total height of the building is 8m. gravity loads only are considered, steel (Fy = 360 MPa) and ordinary Portland cement (fcu = 25 MPa), was employed.
The plan of the typical floor and elevation of the structure are showing in Figure 3.1 : 3.3.
The parameters used for analysis are enlisted below in Table 3-1

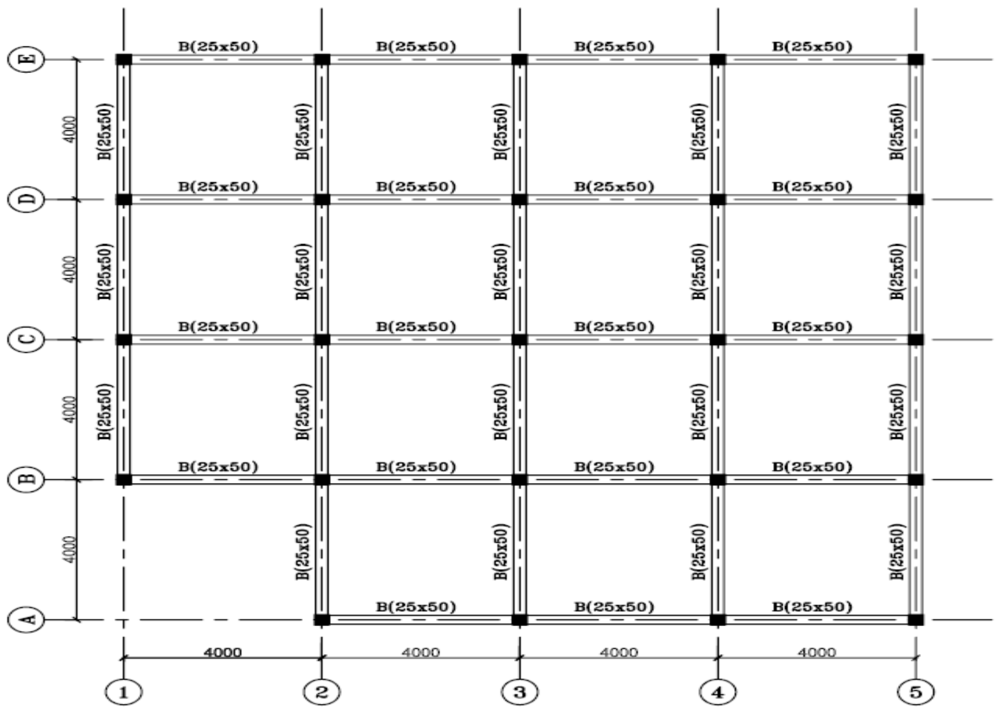

Figure 3.1 plan layouts of the structure used in analyses

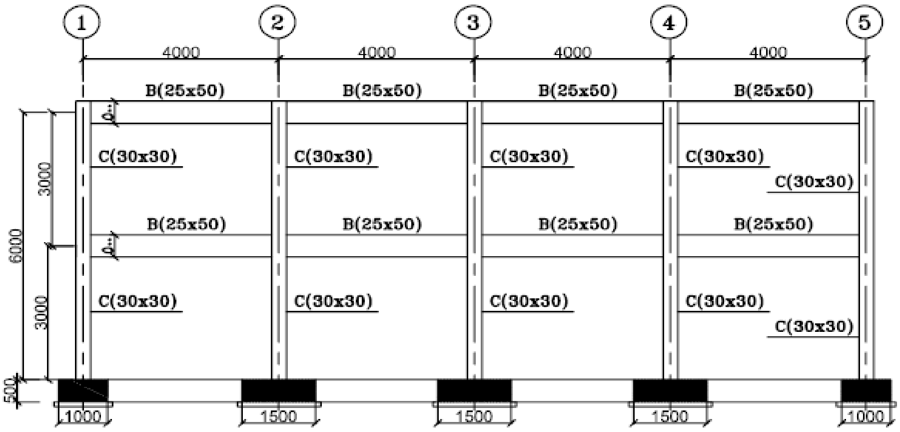

Figure 3.2 elevation layouts of the structure used in analyses.

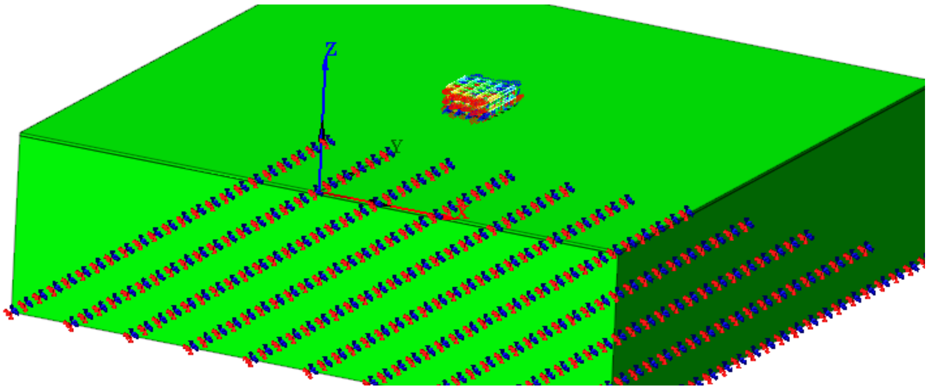

Figure 3.3 3D view of the soil and structure used in analyses.

ID	1
Name	Concrete
Type	EL
Modulus of Elasticity (E) [MPa]	22000
Poisson's Ratio (ν)	0.2
Unit Weight (□) [tonf/m3]	2.5
Damping Ratio	0.05

* EL : Elastic

Table 3.1: The parameters of the building used for analysis

3.1.2 Second model 30 storey building

In this study the building features four bays in east-west and north-south direction, respectively. The height of the storey is 3.0 m and width of the bay is 4.0 m. The total height of the building is 8m. gravity loads only are considered, steel (Fy = 360 MPa) and ordinary Portland cement (fcu = 35 MPa), was employed.

The plan of the typical floor and elevation of the structure are showing in Figure 3.4 : 3.6.

The parameters used for analysis are enlisted below in Table 3-2

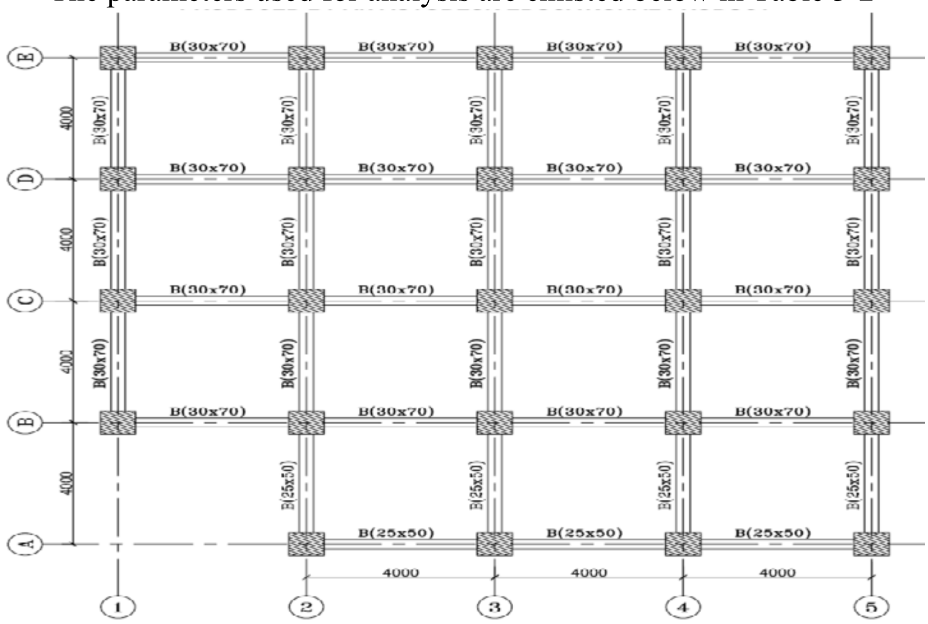

Figure 3.4 plan layouts of the structure used in analyses.

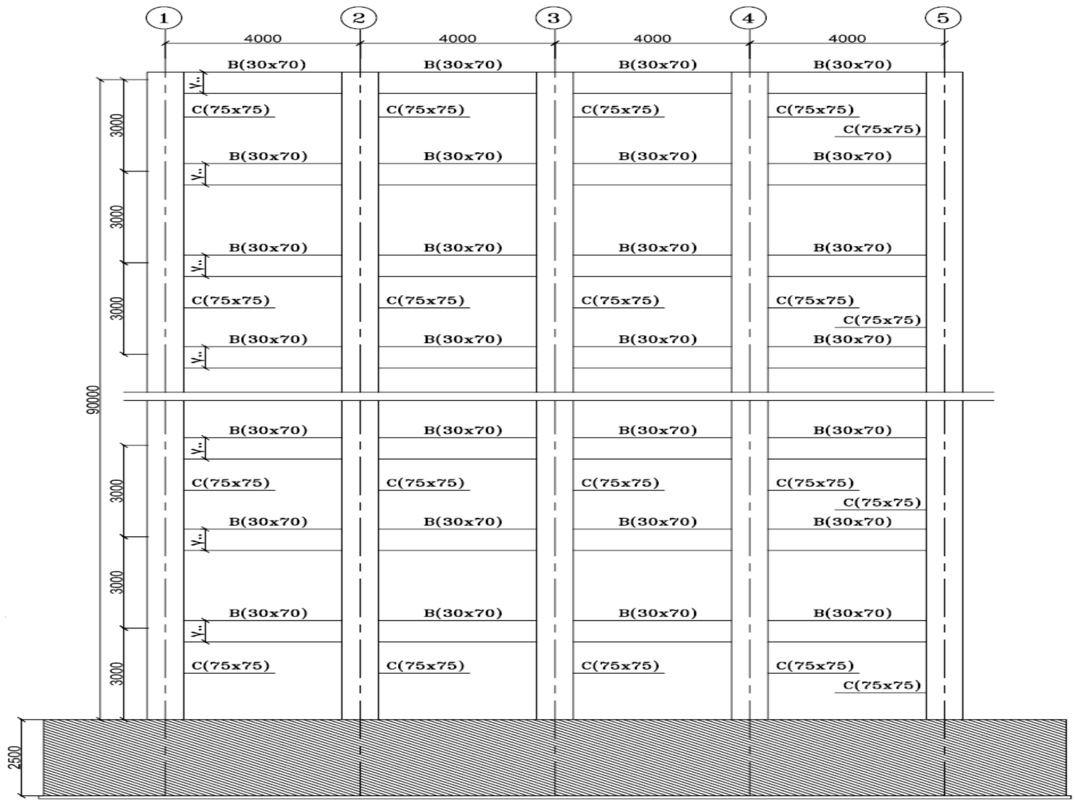

Figure 3.5 elevation layouts of the structure used in analyses.

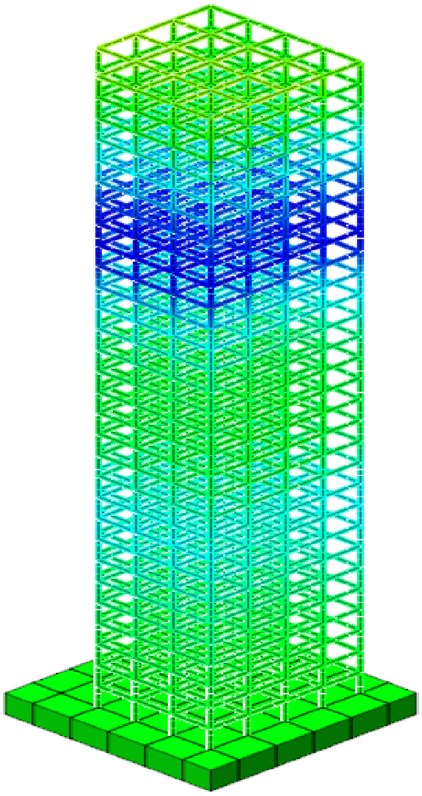

Figure 3.6 3D view of structure used in analyses.

ID	1
Name	Concrete
Type	EL
Modulus of Elasticity (E) [MPa]	26000
Poisson's Ratio (ν)	0.2
Unit Weight (□) [tonf/m3]	2.5
Damping Ratio	0.05

* EL: Elastic

Table 3.2: The parameters of the building used for analysis

3.2 Second model (modeling of foundation as springs and super-structure)

A linear Elastic time-history analysis is carried out using the real ground motion NS direction records from the 18.may, The 1940 El Centro earthquake of magnitude 6.9 with a duration of 30 seconds. The damping is assumed to be 5%, which is a composite damping of Concrete structure and the soil model together See Figure 3.7

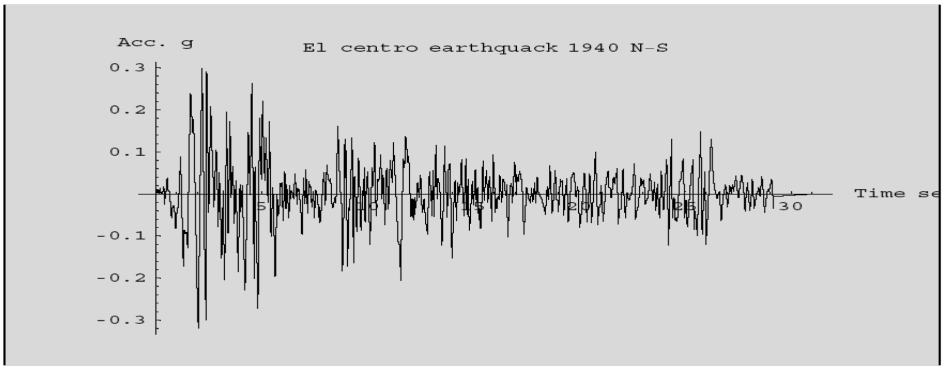

Figure 3.7: The 1940 El Centro earthquake, NS horizontal acceleration component

3.2.1 Structural Properties:

In our study, a three concrete frame resting on a shallow foundation (assuming as moment resisting building) used for analysis are as follow.

First model 2 storey building (This group is supposed to simulate low rise buildings.)

Second model 10 storey (This group is supposed to simulate middle rise buildings.)

Third model 30 storey (This group is supposed to simulate high rise buildings.)

3.2.1.1 First model 2 storey building

In this study the building features four bays in east-west and north-south direction, respectively. The height of the storey is 3.0 m and width of the bay is 4.0 m. The total height of the building is 8m. gravity loads only are considered, steel (Fy = 360 MPa) and ordinary Portland cement (fcu = 25 MPa), was employed.

The plan of the typical floor and elevation of the structure are showing in Figure 3.8.

The parameters used for analysis are enlisted below in Table 3-1

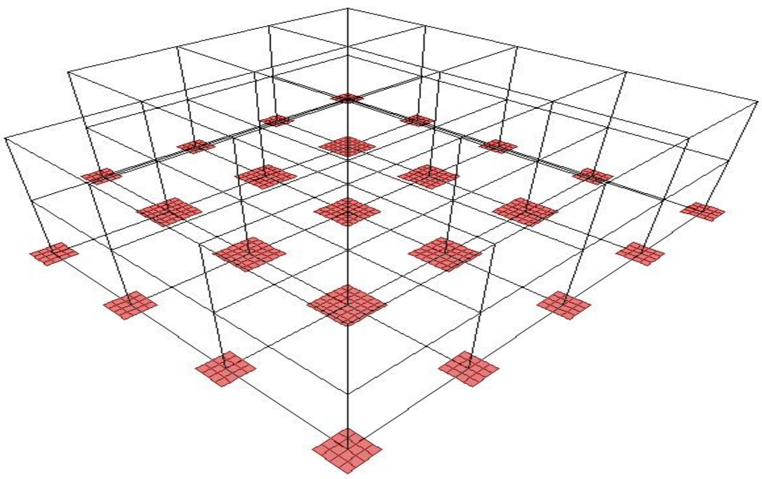

Figure 3.8- 3D view of the soil and structure used in analyses

3.2.1.2 Third model 30 storey building

In this study the building features four bays in east-west and north-south direction, respectively. The height of the storey is 3.0 m and width of the bay is 4.0 m. The total height of the building is 8m. gravity loads only are considered, steel (Fy = 360 MPa) and ordinary Portland cement (fcu = 35 MPa), was employed.

The plan of the typical floor and elevation of the structure are showing in Figure 3.9

The parameters used for analysis are enlisted below in Table 3-2

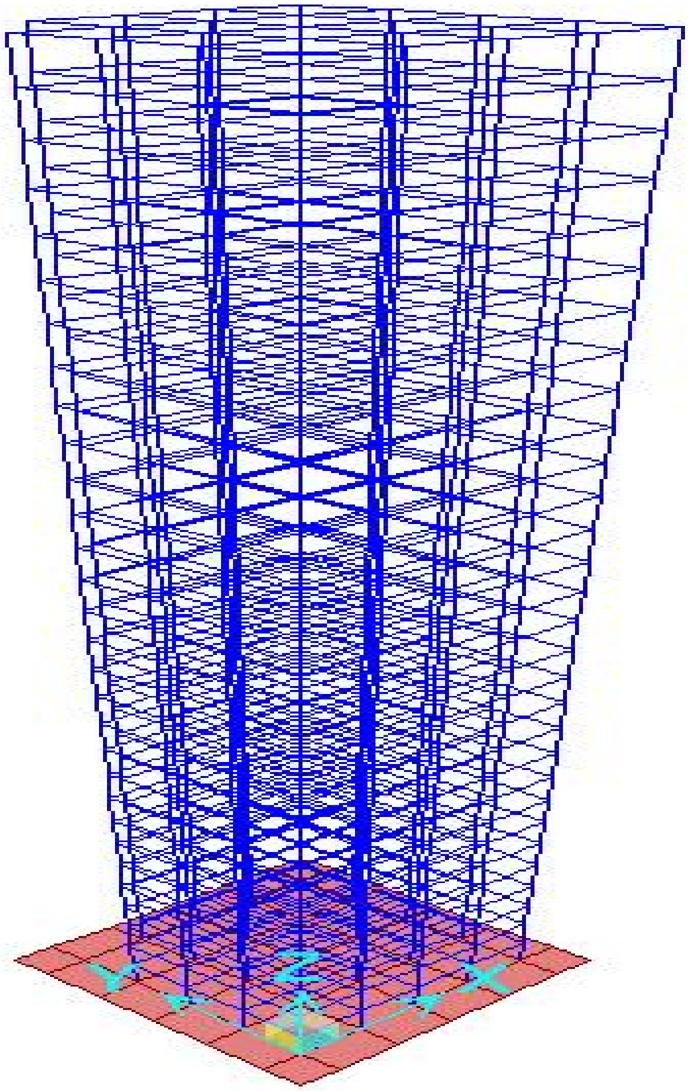

Figure 3.9 - 3D view of structure used in analyses.

3.3 Third model (modeling of foundation as hinged support and super-structure)

A linear Elastic time-history analysis is carried out using the real ground motion NS direction records from the 18.may, The 1940 El Centro earthquake of magnitude 6.9 with a duration of 30 seconds. The damping is assumed to be 5%, which is a composite damping of Concrete structure and the soil model together See Figure 2.10

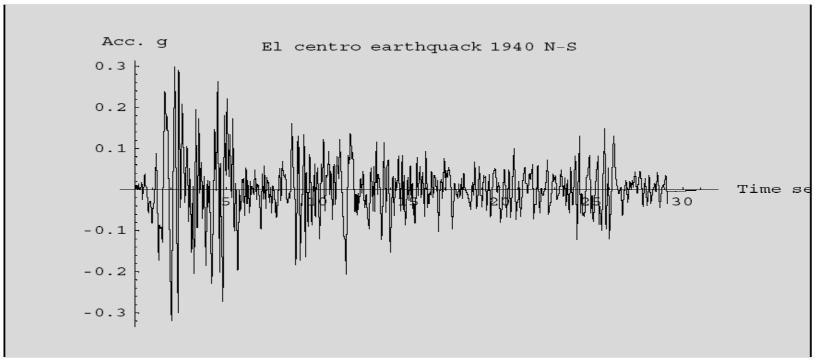

Figure 2.10: The 1940 El Centro earthquake, NS horizontal acceleration component

3.3.1 Structural Properties:

In our study, a three concrete frame resting on a shallow foundation (assuming as moment resisting building) used for analysis are as follow
First model 2 storey building (This group is supposed to simulate low rise buildings.)
Second model 10 storey (This group is supposed to simulate middle rise buildings.)
Third model 30 storey (This group is supposed to simulate high rise buildings.)

3.3.2 First model 2 storey building

In this study the building features four bays in east-west and north-south direction, respectively. The height of the storey is 3.0 m and width of the bay is 4.0 m. The total height of the building is 8m. gravity loads only are considered, steel (Fy = 360 MPa) and ordinary Portland cement (fcu = 25 MPa), was employed.
The plan of the typical floor and elevation of the structure are showing in Figure 3.11
The parameters used for analysis are enlisted below in Table 3-1

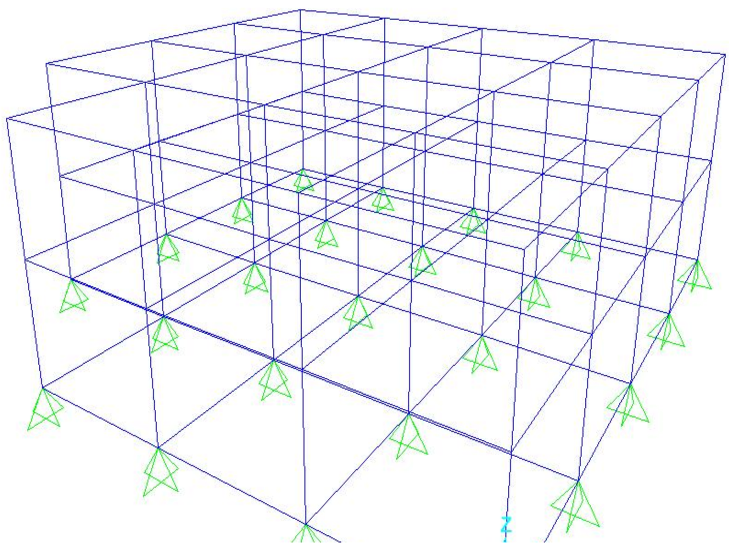

Figure 3.11- 3D view of the soil and structure used in analyses.

3.3.4 Third model 30 storey building

In this study the building features four bays in east-west and north-south direction, respectively. The height of the storey is 3.0 m and width of the bay is 4.0 m. The total height of the building is 8m. gravity loads only are considered, steel (Fy = 360 MPa) and ordinary Portland cement (fcu = 35 MPa), was employed.

The plan of the typical floor and elevation of the structure are showing in Figure 3.12

The parameters used for analysis are enlisted below in Table 3-2

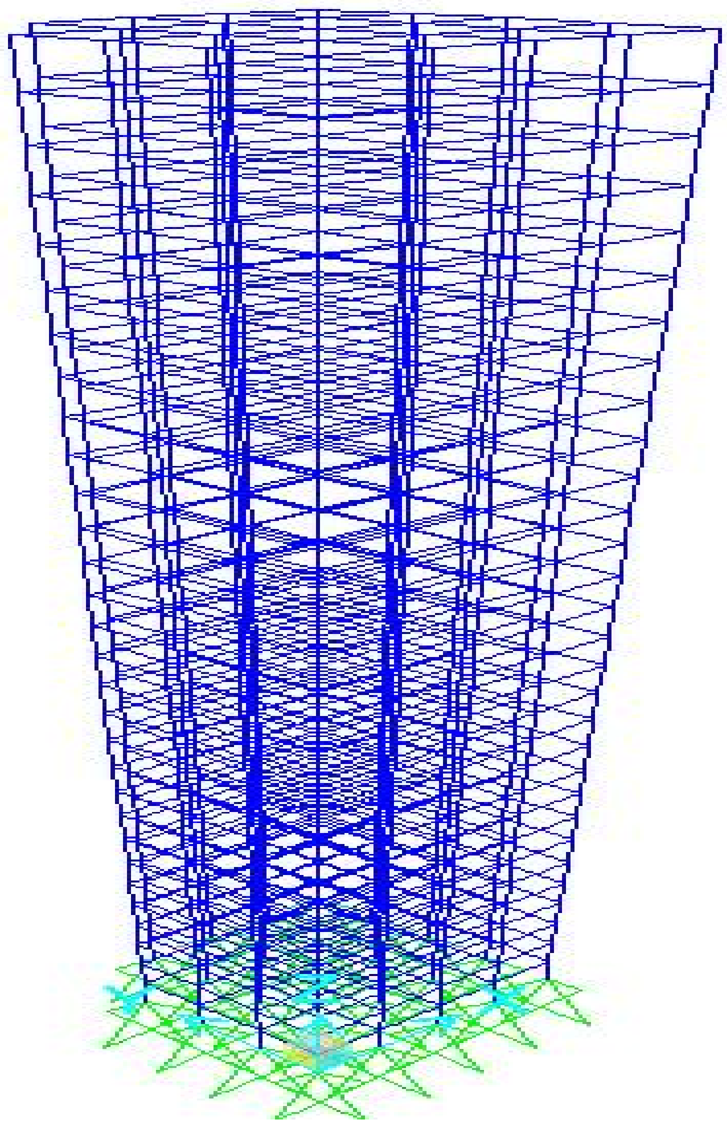

Figure 3.12 3D view of structure used in analyses.

3.4 Soil Properties:

A finite element mesh, composed of (1m.x1m to 6m.x6m) finite elements, models the soil under the structure. The depth of the soil layers is 60m. The mesh width is assumed as 200m. And the mesh length is assumed as 200m taking into account that the boundary conditions should satisfy the free field behavior of the analyzed soil profile.

The Material Properties for strata (or ground) used for analysis are enlisted below in tab.3.5
The analysis is carried out for one soil profiles (SAND) of single layer with the given depth, with average shear-wave velocities Vs, of 92 m/sec and 43 m/sec.
Mechanical properties of the soil types are calculated using the formula,
where , G is the elastic shear modulus, ν is the Poisson ratio and ρ is the mass density.
ν = 0.30
ρ = 1.80 t/m3

ID	1
Name	Sand
Type	MC
Modulus of Elasticity (E) [MPa]	40
Poisson's Ratio (ν)	0.3
Unit Weight (□) [tonf/m3]	1.8
Unit Weight (Saturated) [tonf/m3]	1.85
Cohesion (C) [tonf/m2]	0.1
Friction Angle (□)	30
Damping Ratio	0.05

Table 3.5: Material Properties for the soil under the structure

4 Boundary Conditions

In the finite element method, the finite element mesh must have finite dimensions and only a portion of the virtually infinite soil can be simulated. If the model is not treated appropriately, the waves propagating in the medium to outer regions (which are not simulated in the finite element mesh) will falsely reflect at the mesh boundaries and can introduce numerical instabilities in the simulation.

The finite element software MIDAS-GTS allows the 'surface spring elements' as a transmitting boundary of the model [5].This element consists of a set of spring-dashpot elements in all nodes that belong to the surface

In MIDAS-GTS, the damping coefficients related to P-waves and S-waves are defined as follows

$$C_p = c_p \times A; \text{ where, } c_p = \rho \sqrt{\frac{\lambda + 2G}{\rho}} \quad (3)$$

$$C_s = c_s \times A; \text{ where, } c_s = \rho \sqrt{\frac{G}{\rho}} \quad (4)$$

Where

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

(5)

$$G = \frac{E}{2(1+\nu)} \quad (6)$$

Where c_p and c_s : Multiplication factors that take into account the wave velocity occur at the end of the mesh.

λ : Volumetric elastic modulus

G : Shear elastic modulus

z : Modulus of elasticity

A : Cross-sectional area

ν : Poisson's ratio

□ : is the mass density

Moreover, the vertical and horizontal spring stiffness can be calculated as follows (MIDAS IT. Co. Ltd. 2010) [6]:

$$k_v = k_{vo} \times \left(\frac{B_v}{30}\right)^{-3/4} \quad (7)$$

$$k_h = k_{ho} \times \left(\frac{B_h}{30}\right)^{-3/4} \quad (8)$$

$$k_{vo} = k_{ho} = \frac{1}{30} \alpha E_0 \quad (9)$$

$$B_v = \sqrt{A_v} \quad (10)$$

$$B_h = \sqrt{A_h} \quad (11)$$

Where, k_{vo} or k_{ho} is coefficient of sugared reaction; α is a user defined parameter; and E_0 is the modulus of elasticity; and B_v & B_h are the effective width in vertical and horizontal direction, respectively; A_v & A_h are the area of boundary in the vertical and horizontal plane, respectively.

Furthermore, MIDAS-GTS considers mass and stiffness proportional damping, normally referred to as Rayleigh damping is commonly used in nonlinear dynamic analysis. The generalized equation of Rayleigh damping is as follows:

$$[C] = \alpha\,[M] + \alpha_1\,[K] \tag{12}$$

Where, [C] represents the damping matrix; [M] mass matrix; and [K] stiffness matrix. The parameters α and α_1 are the mass and stiffness proportional damping coefficients respectively. In order to establish α and α_1, it is necessary to relate these parameters to the hysteretic damping coefficient ξ. The parameters α and α_1 can be achieved using the following equations (MIDAS IT. Co. Ltd. 2012):

$$\alpha = \frac{2\cdot\omega_i\,\omega_j\,(\xi_i\cdot\omega_j - \xi_j\cdot\omega_i)}{(\omega_j^2 - \omega_i^2)} \tag{13}$$

$$\alpha_1 = \frac{2\,(\xi_j\cdot\omega_j - \xi_i\cdot\omega_i)}{(\omega_j^2 - \omega_i^2)} \tag{14}$$

where, ξ_i and ξ_j are the damping coefficient of mode-1 and mode-2, respectively; and ω_i and ω_j the natural frequency of mode-1 and mode-2, respectively. In MIDAS-GTS, this natural frequenciesare obtained by Eigenvalue analysisproviding the spring stiffness at the boundary of the layered material mesh.The layouts of the structure are given in Figure 3.13

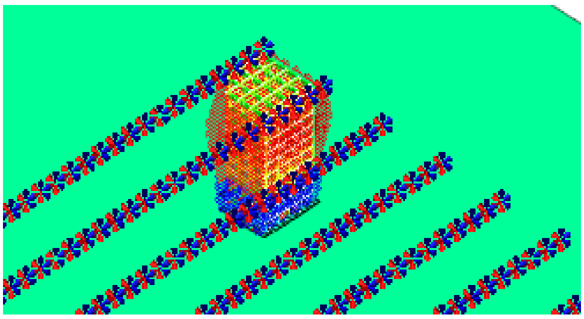

Figure 3.13 3D view of the soil and structure used in analyses.

5 RESULTS AND DISCUSSION

In this research program. Base shear, maximum lateral deflections, axial force, Bending moment and the shear force diagrams, are the main parameters to be checked in order to understand how the soil-structure interaction works. The results of the elastic and inelastic analyses for Dynamic stiffness's of foundations model, hinged model and full models respectively are determined and compared. According to the results showed in figures 5.1 - 5.9, it is observed that the ratios of the base shear of the full model to those of rigid-base in all models are less than one. These results have good conformity to the NEHRP-2003 regulations. Also axial force, Bending moment and the shear force diagrams of the flexible base model are more realistic than of fixed-base models. In addition, it is observed that in the analysis, the maximum lateral deflections of the flexible base model substantially increase when subjected to the mentioned earthquake record in comparison with the fixed base model.

In this study, the spectral displacement may change considerably with changes in natural period due to Dynamic soil structure interaction (DSSI) effects for both elastic and inelastic cases. Therefore, such increases in the natural period may considerably alter the response of the building frames under seismic excitation. This is due to the fact that the natural period lies in the long period region of the response spectrum curve because of the natural period lengthening for such systems. Hence, the displacement response tends to increase.

Therefore, performance level of the structure, especially for the structures analyzed and designed based on the elastic method, may be changed from life safe to near collapse.

2 storey building (This model is supposed to simulate low rise buildings

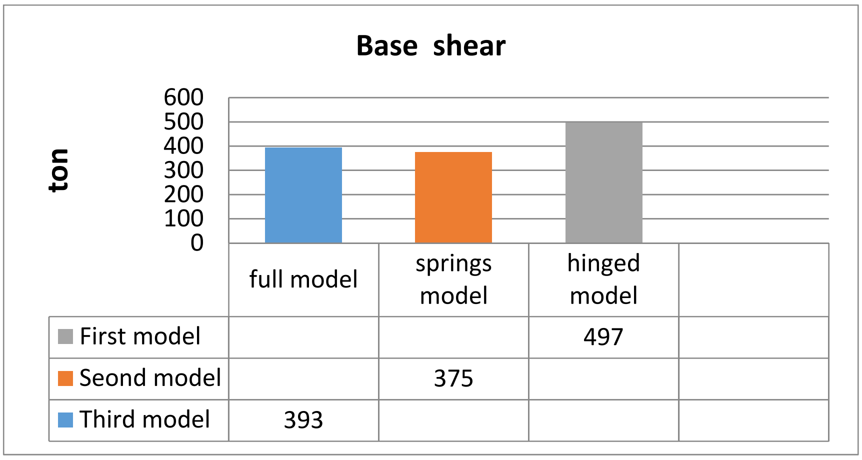

Figure 5.1- results of the base shear for the three analysis models.

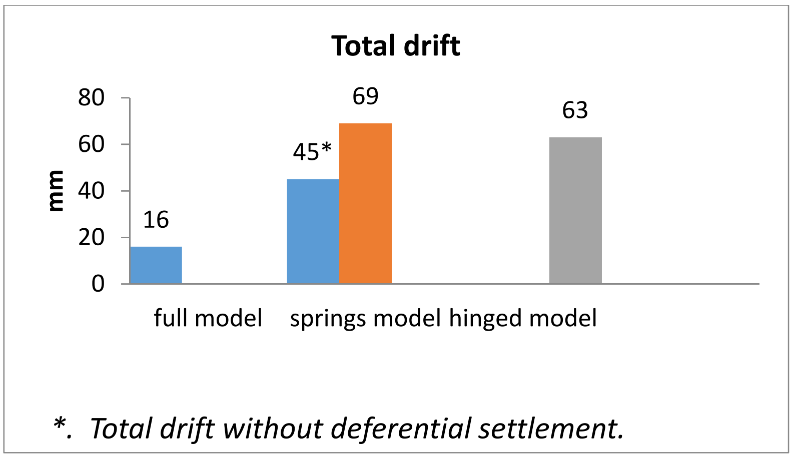

Figure 5.2- results of the total drift for the three analysis models
[Frame at axe (A-A)].

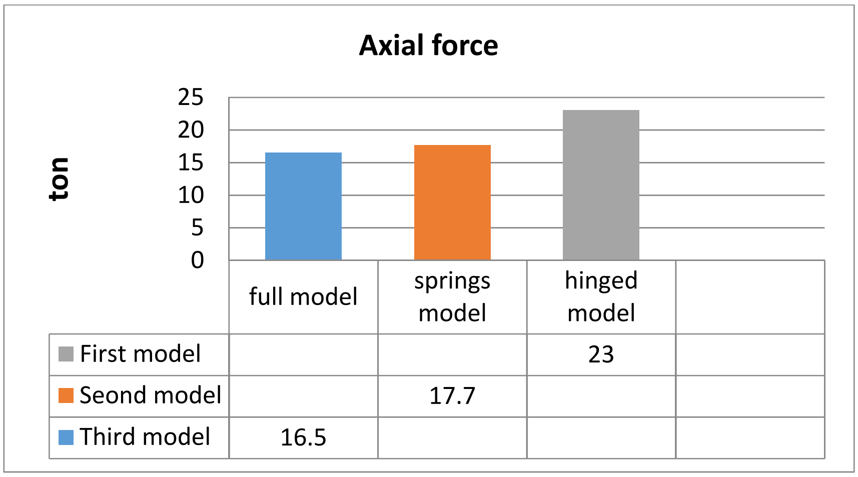

Figure 5.3- results of the axial force for the three analysis models
Exterior frame [Column at axis (E-E and 1-1)].

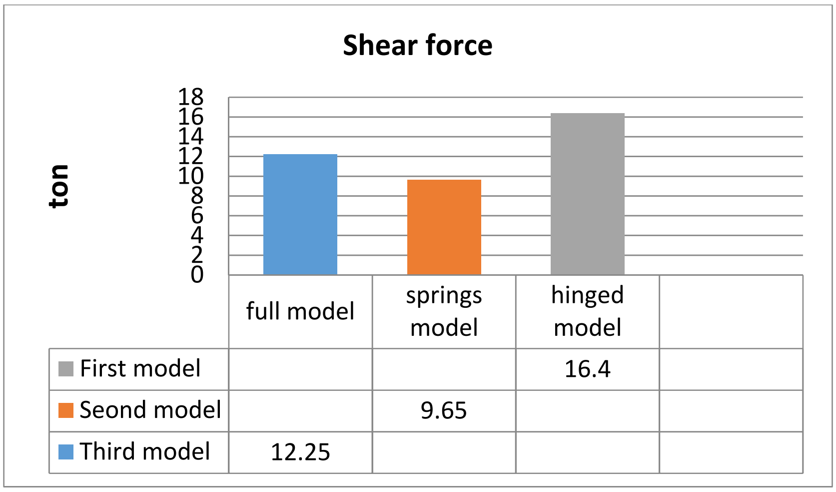

Figure 5.4- results of the shear force for the three analysis models
Exterior frame [Column at axis (E-E and 1-1)].

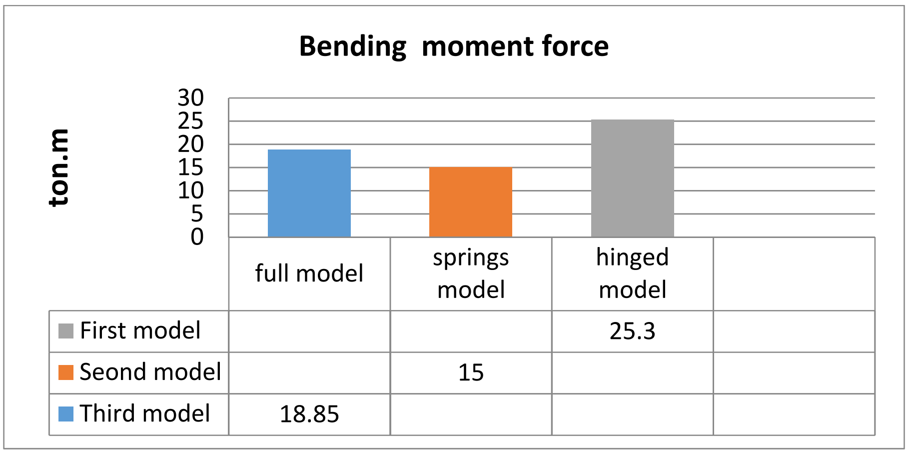

Figure 5.5- results of the bending moment force for the three analysis models Exterior frame [Column at axis (E-E and 1-1)].

30 storey (This group is supposed to simulate high rise buildings)

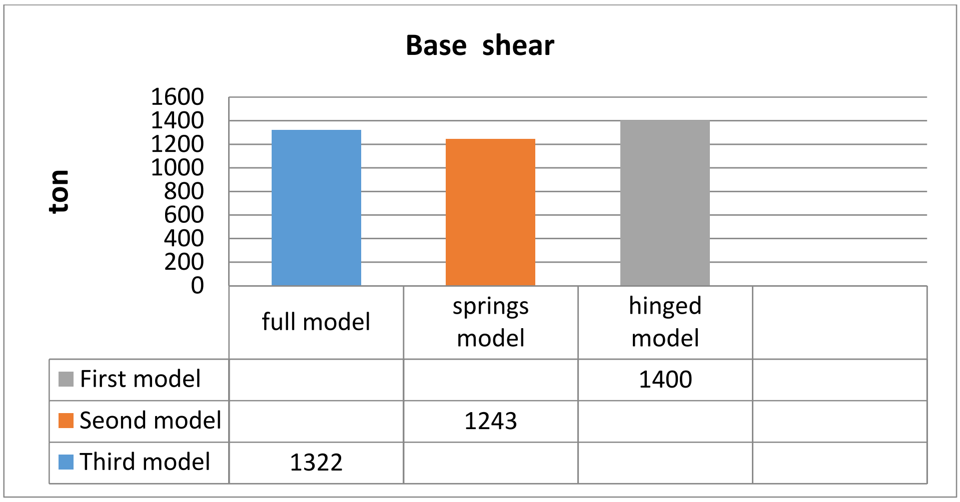

Figure 5.6 - results of the base shear for the three analysis models.

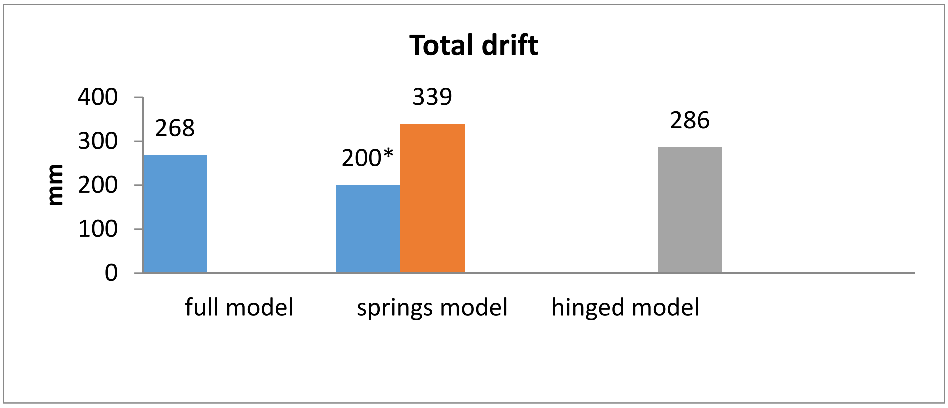

Figure 5.7 - results of the total drift for the three analysis models [Frame at axe (A-A)].

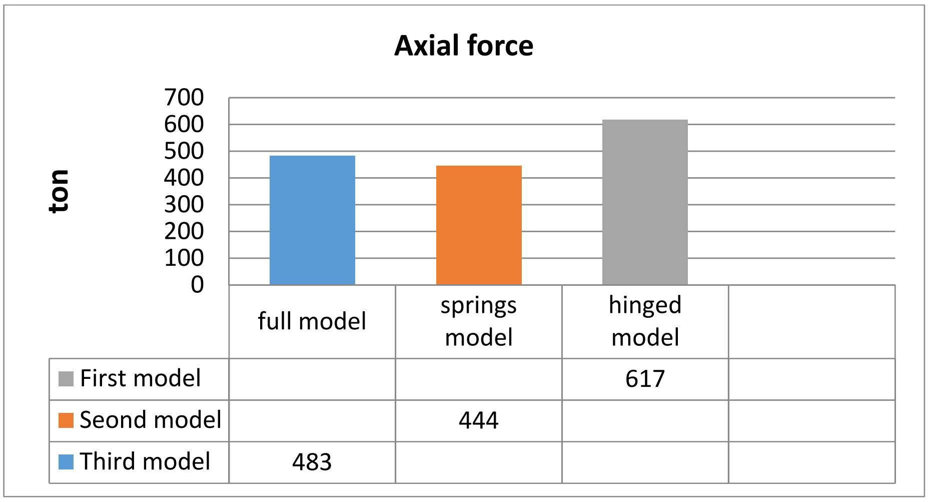

Figure 5.8 - results of the axial force for the three analysis models
Exterior frame [Column at axis (E-E and 1-1)].

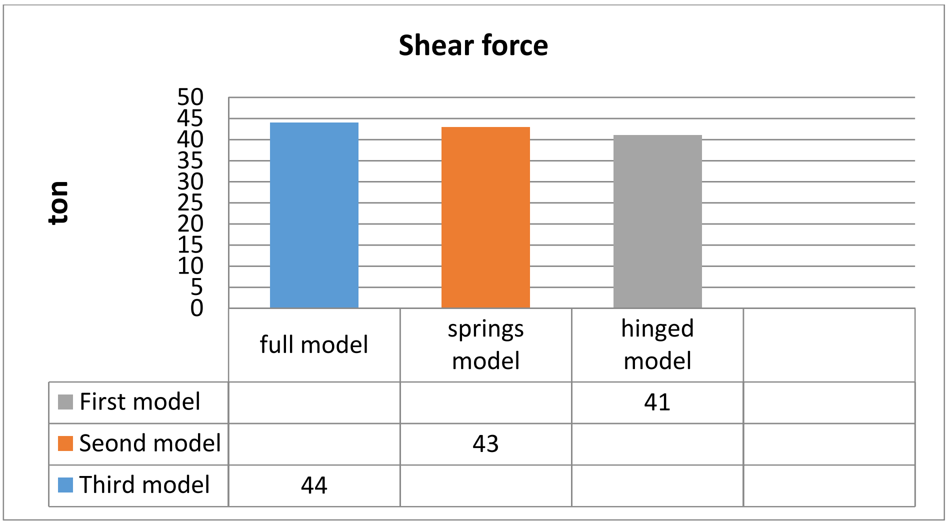

Figure 5.9 - results of the shear force for the three analysis models
Exterior frame [Column at axis (E-E and 1-1)].

6 SUMMARY AND CONCLUSIONS

The numerical investigation conducted in this study is for moment resisting frame MRF buildings with number of stories (Ten stories), flexibility and different boundary conditions at foundation level, a constant beam, slab, column cross-sections with rigid diaphragm assumption are considered in the analysis.

The base shear and the internal forces due to seismic excitation of the structures modeled with the soil deposit are always less than the base shear and internal forces of the structures modeled as fixed-base as expected also axial force, Bending moment and the shear force diagrams of the flexible base model are more realistic than of fixed-base models.

However, the maximum lateral storey drifts of the structures resting on soil deposit substantially increase when the Soil-Structure interaction is considered especially when the soil deposit a clayey soil layers. Considering the results of this study, it can be concluded that the conventional structural analysis methods assuming rigid-base structures is no longer adequate to guarantee the structural safety. The dynamic soil structures interaction (DSSI) effect in seismic design of concrete moment resisting building frames resting on soft soil deposit is essential.

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