



## **ANALYTICAL MODEL OF FLAT SLABS REINFORCED WITH DIFFERENT SHAPES TO RESIST PUNCHING SHEAR**

**Prof. Dr. / Mohamed A. Razek** Civil Engineering Department AL-Azhar University

**Prof. Dr. / Abo Elwafa M. Elthakeb** Civil Engineering Department AL-Azhar University

**Dr. / Amro H. Ryad** Civil Engineering Department AL-Azhar University

**Eng Waled Mahmoud Elsharma M.Sc,BSc.** Civil Engineering Department AL-Azhar University

### **الملخص العربي**

هذا البحث محاولة لفهم سلوك البلاطات المسطحة عند مقاومتها لقوى القص الثاقب وذلك عند استخدام اشكال وانواع مختلفه من التسليح الاضافي الموضوع مسبقا في المراحل الاولي لتنفيذ هذه النوعيه من البلاطات والمخصص لمقاومه القص الثاقب وللوصول لفهم اكبر لهذا الموضوع تم انشاء نموذج تحليلي حسابي باستخدام برامج الكمبيوتر علي عدد ٢١ عينه بمقاييس موحده حيث تبلغ ابعاد البلاطات ٤٠٠\*٥٠٠\*٤٠٠ مم\*٣٠٠ مم باشكال مختلفه من التسليح الاضافي مع ثبات نسب التسليح الرئيسي الطولي والعرضي في كل العينات واطهرت النتائج التأثير المباشر للتسليح الاضافي علي مقدار القص الثاقب المؤثر علي العينات المستخدمه .

### **Abstract**

Finite element results conducted in the current study indicated the effectiveness of the proposed punching shear reinforcement method can be attractive solution for increasing punching strength, ductility enhancement, and increasing carrying load capacity of flat slab. Despite that Egyptian code provisions may be useful for the design purpose of flat slab reinforced against punching stresses. It may not be effective for the assessment purpose of existing reinforcement. Hence. Some important parameters are not included in such provisions. Therefore. This finite element model is employed for studying the effect of several design parameters on the behavior of flat slab subjected to concentric punching load.

### **1-INTRODUCTION**

Finite element method is the most widely used numerical technique in the engineering field. With the advancement in the understanding of material properties of concrete, various constitutive laws and failure criteria have been developed to model the behavior of concrete. Reinforcement against punching for flat slabs plays an important role in producing ductile structures. For earthquake resistance in particular, requires that brittle failure of members should not occur. In the worst case, a structure under extreme loading condition should be able to undergo large deformation and maintain a substantial part of its load-carrying capacity. The large deformation can provide ample warning before failure of structure. If a structure is not designed to perform in a ductile manner, then much higher

elastic inertia forces should be used for design if the failure of the structure is to be avoided. This paper depends on ABAQUS model presents the analytical program conducted to examine the impact of some Parameters on the behavior of flat slabs reinforced against punching shear.

## 2- PUNCHING SHEAR MODELS

2-1 American Building Code for Reinforced Concrete ACI-318 The punching stress by ACI may be calculated by the following expressions[1].

$$v = 0.17 \left( 1 + \frac{2}{\beta} \right) \sqrt{f_c}$$

$$v = 0.17 \left( \frac{\alpha d}{2u} + 1 \right) \sqrt{f_c}$$

$$v = 0.33 \sqrt{f_c}$$

Where  $f_c$  - cylindrical compressive strength in concrete ( $f_c = f_{cu}/1.25$ ),  $f_{cu}$  - cubical compressive strength in concrete,  $u = 2(c_1+c_2)$ ,  $\beta$  is the ratio the long side to the short side of the columns,  $\alpha$  is 40 for interior columns.

When shear reinforcement is used ACI expresses the nominal shear stress as

$$v_n = 0.5 v + v_s \leq 3\sqrt{f_c}/6$$

$$v_s = \frac{A_{sv} f_{yv}}{b_o s} \quad (1)$$

Where  $v_s$  is the nominal shear stress is provided by the shear reinforcement within a radial distance equal to  $d$ ,  $A_{sv}$  is the area of shear reinforcement in one circumferential layer, and  $f_{yv}$  is the specified yield strength of shear reinforcement in  $n/mm^2$  and shall not exceed 400  $n/mm^2$ , and  $s$  is the spacing of shear reinforcement. The upper limit for  $s$  is 0.5  $d$  and the shear reinforcement must be extended for a sufficient distance until the critical section outside the shear – reinforcement zone satisfies Equation 3 with  $v = 0.33 \sqrt{f_c}$

2-2 Egyptian Code of Practice ECCS

In ECCS, the nominal shear stress of the concrete slabs without shear reinforcement is the smallest of

$$v = 0.8 \left( 0.2 + \frac{\alpha d}{b_o} \right) \sqrt{f_{cu}/\gamma_c}$$

$$v = 0.316 \left( 0.5 + \frac{a}{b} \right) \sqrt{f_{cu}/\gamma_c}$$

$$v = 0.316\sqrt{f_{cu}/\gamma_c} \quad (2)$$

2-3 Euro Code, EC-2

For slab with shear reinforcement, the predicted punching capacity according to EC-2 is the lower of the following two expressions

$$vn = 0.75v + 1.5\left(\frac{d}{s_r}\right) A_{sw} f_{ywd,ef} \left(\frac{1}{u1d}\right) \sin\alpha \quad (3)$$

Where

$A_{sw}$  is the area of one perimeter of shear reinforcement around the column mm<sup>2</sup>

$s_r$  is the radial spacing of perimeters of shear reinforcement mm

$f_{ywd,ef}$  is the effective design strength of the punching shear reinforcement, according to

$$f_{ywd,ef} = 250 + 0.25d$$

$d$  is the mean of effect depth in the original direction mm

$\alpha$  is the angle between the shear reinforcement and the plan of the slab

If single line of bent – down bars is provided, then the ratio  $\left(\frac{d}{s_r}\right)$  may be given the value 0.67.

2-4 CEB – FIP Model Code , MC90

In the CEB-FIP Model Code, the design punching shear strength of slabs without shear reinforcement is determined from

$$v = 0.12\varepsilon(100 \rho f_c)^{1/3}$$

$$\text{Where } \varepsilon = 1 + \sqrt{200/d}$$

$$\rho = \sqrt{\rho_x \rho_y} \leq 0.02$$

(4)

Where  $\rho_x$  and  $\rho_y$  are the ratio of flexural reinforcement in x and y directions respectively.

2-5 British Standard Institution BSI

$$v = v = 0.18\varepsilon(100 \rho f_c)^{1/3}$$

Where ,  $\rho$  is the mean flexural reinforcement ratio =  $\frac{\rho_x + \rho_y}{2}$  for a width equal to (3+3d), which be taken not greater than 0.03, nor less than 0.0015. However the critical shear perimeter is taken as a rectangular located at a distance 1.5d from the column faces regardless of whether the column are rectangular or circular in section.

$\gamma_m$  = the partial safety factor for materials, taken as 1.25

$f_c$  = the characteristic concrete cube strength .For concrete strength less than 25 n/mm<sup>2</sup>,

the factor  $\left(\frac{f_c}{25}\right)^{1/3} = 1$  . Shear reinforcement may be provided in slabs over 200 mm depth to increase the shear resistance in accordance with the following equation

$$vn = v + 0.87vs$$

$$vs = 1.5 \frac{A_{sv} f_{yv}}{b_o u1} \quad (5)$$

## 2-6 STR, DIN Codes

In the STR Code, the design punching shear strength of slabs without shear reinforcement is determined from

$$v = 0.18\varepsilon(100 \rho f_c)^{1/3}$$

In the DIN Code, the design punching shear strength of slabs without shear reinforcement is determined from

$$v = 0.14\varepsilon(100 \rho f_c)^{1/3}$$

Where  $\varepsilon = 1 + \sqrt{200/d}$

$$\rho = \sqrt{\rho_x \rho_y} \leq$$

$$0.02$$

(6)

Where  $\rho_x$  and  $\rho_y$  are the ratio of flexural reinforcement in x and y directions respectively.

### 3. Analysis models of flat slabs reinforced with different shapes to resist punching shear

The variables of the analyses presented in this chapter are spacing between bars of studs, diameter of studs, Amount of bent bars steel, spacing between bent bars, Thickness of head plate cap the analyzed slabs are gathered in analysis groups. Within each analysis group the only variable is one of the parameters mentioned.

The analyzed were divided into fourteen groups, **group H** ( FSS1W , FSS2W , FSS3W , ), **group I** ( FSS4W , FSS5W , FSS6W), **group J** ( FS1W , FS2W , FS3W) , **group K** ( FS4W , FS5W,FS6W), **group L** (FB1W,FB2W,FB3W) , **group M** (FB4W,FB5W,FB6W) and **group N** (FH1W,FH2W,FH3W) all analyzed were geometrically identical.

All analyzed reinforced ( for group H to group N) with dimensions (5400mm\*5400mm\*300mm) , (300mm\*300mm\*800mm) column dimension, high grade steel consisted of 10Φ16/ m in each direction and the reinforcement ratio was 0.067 %. The main variables included in this test program were the amount of bent bars, the spacing of bent bars, thickness of head plate cap, number of studs, diameter of bars for studs, spacing between bars for studs and additional reinforcement configuration. It was attempted to keep the other parameters constant Dimension and reinforcement details for all specimens are shown in figure (1)

**Group H** is composed of three analyzed denoted (FSS1W, FSS2W, FSS3W,). For analyzed FSS1W one stud was installed across the column in each direction to resist the punching shear each stud consisted of two flat plates with width 30 mm and three bars with diameter 10 mm, spacing between bars was 64 mm, for analyzed FSS2W spacing between bars was 45 mm, and for analyzed FSS3W spacing between bars was 30 mm. Details of the group H are shown in Figure (2) **Group I** is composed of three analyzed de ( FSS4W , FSS5W , FSS6W) .For analyzed FSS4W. one stud was installed across the column in each direction to resist the punching shear each stud consisted of two flat plates with width 30 mm and three bars with diameter 10 mm, spacing between bars was 64 mm, for analyzed FSS5W diameter was 12 mm, and for analyzed FSS6W diameter was 16 mm. Details of the group I are shown in Figure (3) **Group J** is composed of three analyzed denoted as (FS1W, FS2W, and FS3W) reinforcement two stud was installed across the

column in each direction to resist the punching shear each stud consist of two flat plates with width 30 mm and three bars with diameter 10 mm, spacing between bars was 64 mm, for analyzed FS2W spacing between bars was 45 mm, and for analyzed FS3W spacing between bars was 30 mm. Details of the group J are shown in Figure (4) **Group K** is composed of three analyzed denoted as (FS4W, FS5W, and FS6W) reinforcement two stud was installed across the column in each direction to resist the punching shear each stud consist of two flat plates with width 30 mm and three bars with diameter 10 mm, spacing between bars was 64 mm. for analyzed FS5W diameter was 12 mm, and for analyzed FS6W diameter was 16 mm. Details of the group K are shown in Figure (5) **Group L** is composed of three analyzed denoted as (FB1W, FB2W, and FB3W). For analyzed FB1W diameter of bar was 8 mm , analyzed FB2W diameter of bar was 10mm and for analyzed FB3W diameter was 12mm three rows from bent bars was installed across the column in each direction to resist the punching shear . Details of the group L are shown in Figure (6) **Group M** is composed of three analyzed denoted as (FB4W, FB5W, and FB6W). Reinforcement with bent bars diameter of bars 8mm. For analyzed FB1W spacing between bars was 10 mm , analyzed FB5W spacing between bars was 8 mm for analyzed FB6W spacing between bars was 6 mm five rows from bent bars was installed across the column in each direction to resist the punching shear . Details of the group M are shown in Figure (7) **Group N** is composed of three analyzed denoted as (FH1W, FH2W, and FH3W). For analyzed FH1W one head plate cap steel was installed under the column to resist the punching shear with thickness of 2 mm, , for analyzed FH2W thickness of head plate cap steel was 4 mm, and for analyzed FH3W thickness of head plate cap steel was 6 mm. Details of the group N are shown in Figure (8)

#### 4- Non-linear concrete model

The stress-strain relationships for each concrete grade have been calculated using equations provided in the Model Code 90\* as follows. For the ascending (hardening) branch of the uniaxial curve, the stress strain relationship was given by:

$$\sigma_t = \left( \left( \frac{E_{ci}\xi_c}{E_{cl}\xi_{cl}} - \left( \frac{\xi_c}{\xi_{cl}} \right)^2 \right) / \left( 1 + \left( \frac{E_{cl}}{E_{ci}} - 2 \right) \left( \frac{\xi_c}{\xi_{cl}} \right) \right) \right) * f_{cm} \quad \dots\dots\dots(7)$$

Where,

$E_{ci}$ , is the tangent modulus of elasticity.

$\sigma_c$ , is the compression stress (MPa).

$\xi_c$ , is the compression strain

$\xi_{ci} = -0.0022$

$E_{cl}$ , is the secant modulus from the origin to the peak stress.

The descending branch of the stress-strain curve was approximated by a straight line according to the following expression .

$$\frac{\sigma_{ci}}{\sigma_{if}} = \frac{\xi_{ci}}{\xi_{cif}} \frac{0.8}{n-1} + \frac{n-0.2}{n-1}$$

.....(8)

However, the linear part of the curve is defined in ABAQUS by the elastic modulus  $E_{ci}$ . Beyond the linear part, the strains stress-strain relationship was converted to stress-inelastic strain relationship by subtracting the elastic strain from the total strain as described earlier. In addition, the uniaxial tensile strength for concrete was defined in a simple manner. The following equation was adopted to represent the bilinear tension stiffening relationship for concrete.

$$\sigma_{ct} = f_{cm} (1 - 0.85w/w_i) \quad \text{for } 0.15 f_{cm} \leq \sigma_{ct} \leq f_{cm} \quad \text{.....(9)}$$

$$\sigma_{ct} = (0.15 f_{cm} / w_c - w_i) (w_c - w) \quad \text{for } 0 \leq \sigma_{ct} \leq 0.15 f_{cm} \quad \text{.....(10)}$$

$$w_i = (2 G_f / f_{cm}) - 0.1 \quad \text{.....(11)}$$

$$w_c = \alpha_f \frac{G_f}{f_{ctm}} \quad \text{.....(12)}$$

Where,

- $W$  is the crack opening (mm)
- $w_i$  is the crack opening (mm) for  $\sigma_{ck} = 0.15 f_{cm}$
- $w_c$  is the crack opening for  $\sigma_{ct} = 0$
- $f_{cm}$  is the concrete tensile strength (MPa)
- $\alpha_f$ , coefficient depends on the maximum aggregate size
- $G_f$  is the fracture energy and calculated as follows

$$G_f = G_{fo} \left( \frac{f_{cm}}{f_{cm0}} \right)^{0.7} \quad \text{.....(13)}$$

Where,

$G_{fo}$ , is the base value of fracture energy, depends on the maximum aggregate size

$f_{cm0} = 10$ (MPa).

## 5-Analysis results

### 5-1 Load deflection relationship

Figures (9) shows the load - vertical displacements relationships for all tested specimens. For group H specimens (FSSW1, FSSW2, FSSW3) maximum load varied between ( 872 KN to 990 KN ) with a 13.5 % percent increase and a vertical displacement varied between (93mm to 109mm ) with a 17.2 % percent increase . For group I specimens (FSSW5, FSSW6, FSSW7) maximum load varied between ( 867 KN to 933 KN ) with a 7.6 % percent increase and a vertical displacement varied between (112mm to 124mm ) with a 7.2 % percent increase . For group J specimens (FSW1, FSW2, FSW3) maximum load varied between ( 858 KN to 936 KN ) with a 9 % percent increase and a vertical displacement varied between (112mm to 121mm ) with a 8 % percent increase . For group K specimens (FSW4, FSW5, FSW6) maximum load varied between ( 858 KN to 986 KN ) with a 14.5 % percent increase and a vertical displacement varied between (112mm to

120mm ) with a 7 % percent increase . For group L specimens (FBBW1, FBBW2, FBBW3) maximum load varied between ( 882 KN to 1019 KN ) with a 15.5 % percent increase and a vertical displacement varied between (110mm to 119mm ) with a 8 % percent increase . For group M specimens (FBBW4, FBBW5, FBBW6) maximum load varied between ( 891 KN to 926 KN ) with a 4 % percent increase and a vertical displacement varied between (111mm to 117mm ) with a 5.4 % percent increase .

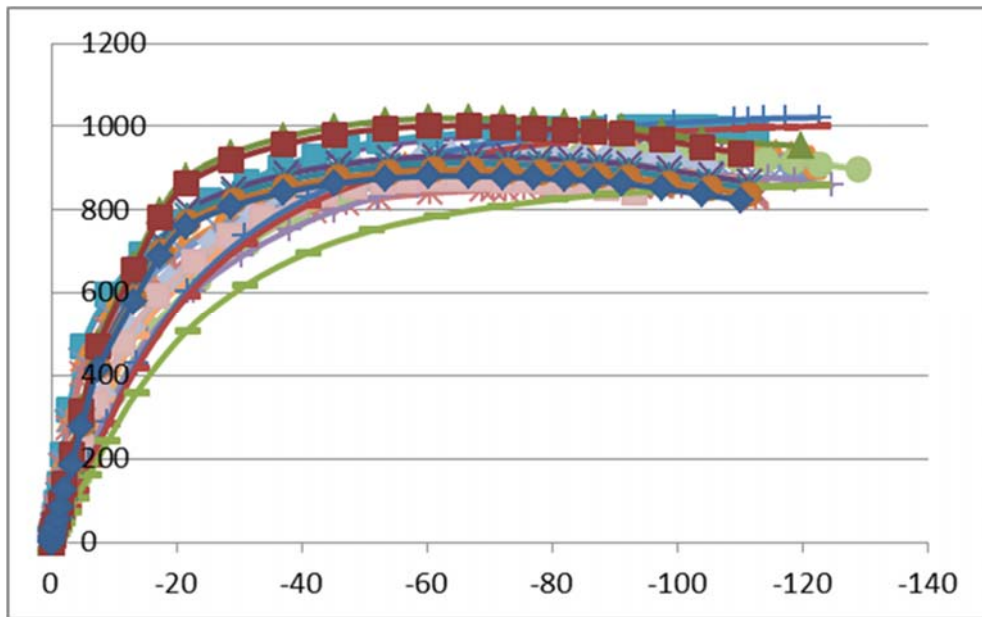
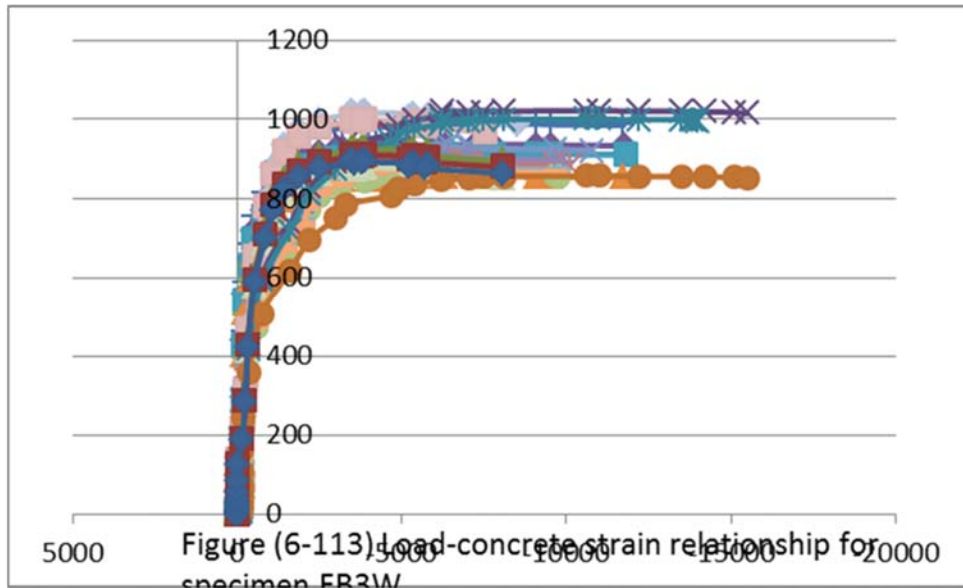


Figure (9) load-deflection relationship

Figure (9) load- concrete strain relationship

## 5-2 Load –compression strain relationship

Figures (10) show For specimens at strain value of 0.002. For group H specimens (FSSW1, FSSW2, FSSW3) load varied between (737 KN to 764 KN) with increase 3.6 % in load capacity. For group I specimens (FSSW4, FSSW5, FSSW6) load varied between (730 KN to 786 KN) with increase 7.6 % in load capacity. For group J specimens (FSW1, FSW2, FSW3) load varied between (828 KN to 882 KN) with increase 6.5 % in load capacity. For group K specimens (FSW4, FSW5, FSW6) load varied between (828 KN to 953 KN) with increase 15 % in load capacity. For group L specimens (FBBW1, FBBW2, FBBW3) load varied between (850 KN to 990 KN) with increase 16.5 % in load capacity. For group M specimens (FBBW4, FBBW5, FBBW6) load varied between (867 KN to 990 KN) with increase 14 % in load capacity. For group N specimens (FHPW1, FHPW2, FHPW3) load varied between (785 KN to 830 KN) with increase 6 % in load capacity.

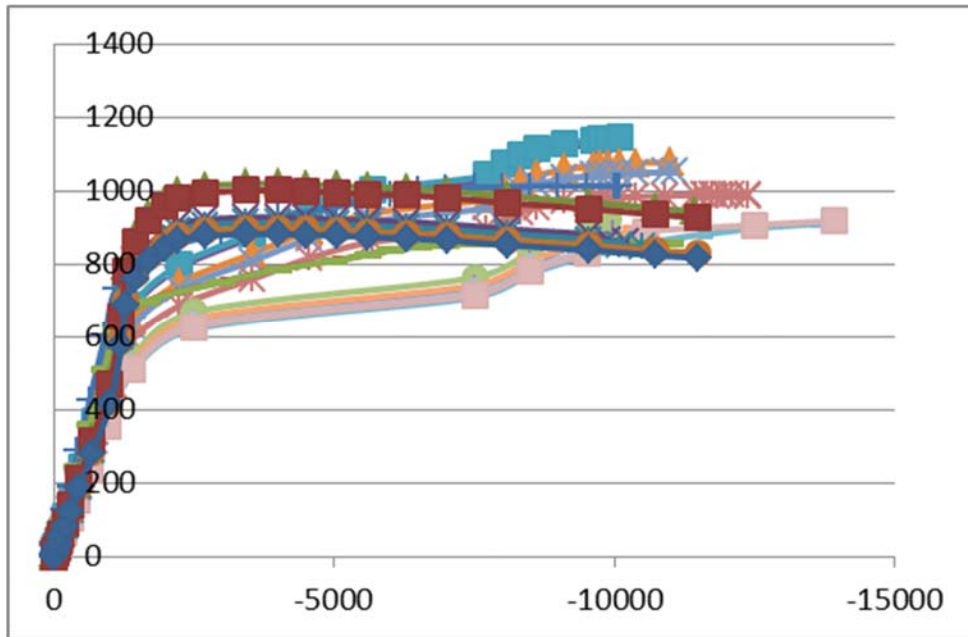


**Figure (10) load-compression strain relationship**

### **5-3 load – tensile strain relationship**

A figure (11) shows a load-tensile strain relationship. For group H specimens (FSSW1, FSSW2, FSSW3) value of tensile strain varied between (13910 micro-strain to 10388 micro-strain indicating 25.3 % decrease ). For group I specimens (FSSW4, FSSW5, FSSW6) value of tensile strain varied between (13505 micro-strain to 9899 micro-strain indicating 26.7 % decrease ). For group J specimens (FSW1, FSW2, FSW3) value of tensile strain varied between (12370 micro-strain to 10990 micro-strain indicating 12 % decrease ). For group K specimens (FSW4, FSW5, FSW6) value of tensile strain varied between (12370 micro-strain to 9877 micro-strain indicating 25 % decrease ). For group L specimens (FBBW1, FBBW2, FBBW3) value of tensile strain varied between (11488 micro-strain to 8205 micro-strain indicating 27 % decrease ). For group M specimens (FBBW4, FBBW5, FBBW6) value of tensile strain varied between (11488 micro-strain to 10210 micro-strain indicating 11 % decrease ). For group N specimens (FHPW1, FHPW2, FHPW3) value of tensile strain varied between (11000 micro-strain to 10001 micro-strain indicating 9 % decrease ).





**Figure (11) load-tensile strain relationship**

## 6- CONCLUSIONS

. - For group H the percentage of gained concrete compression strength due to reinforcement was about 3.6 %. For group IH the percentage of gained concrete compression strength due to reinforcement was about 7.6 %. For group J the percentage of gained concrete compression strength due to reinforcement was about 6.5 %. For group K the percentage of gained concrete compression strength due to reinforcement was about 15 %. For group L the percentage of gained concrete compression strength due to reinforcement was about 16.5 %. For group M the percentage of gained concrete compression strength due to reinforcement was about 14 %. For group N the percentage of gained concrete compression strength due to reinforcement was about 6 %

.- For group H the load carrying capacity due to reinforcement was increase about 13.5 %. For group I the load carrying capacity due to reinforcement was increase about 7.6 %. For group J the load carrying capacity due to reinforcement was increase about 9 %. For group K the load carrying capacity due to reinforcement was increase about 14.5 %. For group L the load carrying capacity due to reinforcement was increase about 15.5%. For group N the load carrying capacity due to reinforcement was increase about 7 % .

## 7-REFERENCES

- 1- ACI Committee 318 (2008) (Building Code Requirements for Structural Concrete ) (ACI 318-08).

- 2- ECCS Egyptian Building Code Requirements for Structural Concrete (ECCS-203-2014)
- 3- EURO –Code2 ( Design of Concrete Structures part 1-1 General Rules and Rules for Building 1992)
- 4- MC90- CEB – FIP Model Code Design Code 1990 (Comite Euro International –Du Beton
- 5- BSI British Standards Institution, the structural use of concrete part 1,( Code of practice for design and construction (BSI-8110).
- 6- DIN 1045 (1998) Beton und Stahlbeton / Bemessung und Ausführung. Deutsches Institut for Normung,
- 7- Menetrey Ph. Analytical Computation of the Punching strength of reinforced concrete. ACI structural journal 1996, vol 93.no5
- 8- Alexander S.D.B. Simmonds S.H. Test column-flat plate connections. ACI structural journal 1992, vol 893.no5
- 9- Lovrovich J.S. Mclean D.J Punching shear behavior of slab whit varying span – depth ratio ACI structural journal 1990, vol 87.no5

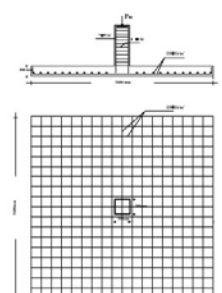


Figure (1) dimension and reinforcement details for all specimens

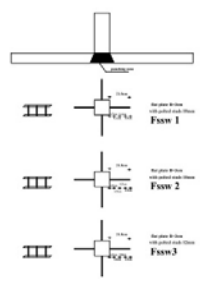


Figure (2) details of group II

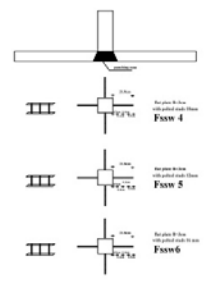


Figure (3) details of group I

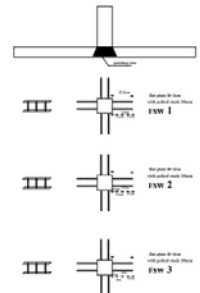


Figure (4) details of group 2

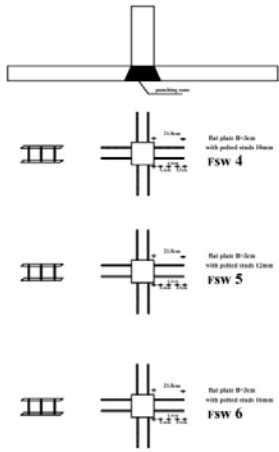


Figure (5) details of group K

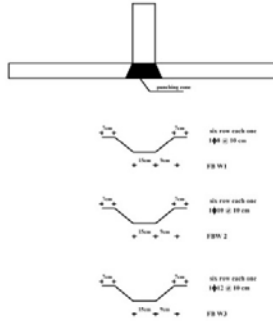


Figure (6) details of group L

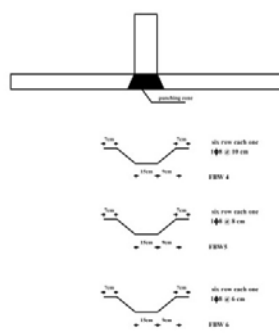


Figure (7) details of group M

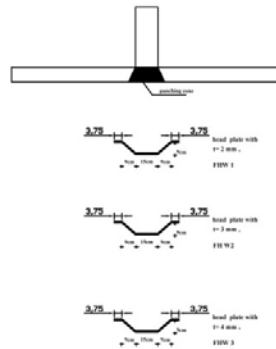


Figure (8) details of group N